

**Mathematics**  
**Calculus: Honors**

**Unit 4: Integration & Applications**

<b>Essential Understandings</b>	<ul style="list-style-type: none"> <li>▪ The concept of an “integral” as the operational inverse of a derivative and as a summation model is introduced using anti-derivatives.</li> <li>▪ Students find integration shortcuts for the basic functions.</li> <li>▪ Students find integrals of composite functions.</li> <li>▪ Students apply integrals to area &amp; motion problems.</li> <li>▪ Students learn the Fundamental Theorem of Calculus.</li> <li>▪ Students perform numerical integration.</li> </ul>
<b>Essential Questions</b>	<ul style="list-style-type: none"> <li>▪ How do you find an integral using anti-derivatives?</li> <li>▪ What is an integral and how is it used in calculus?</li> <li>▪ How can an integral be used in the real world?</li> <li>▪ How do integrals apply to motion problems?</li> <li>▪ How do integrals apply to area problems?</li> </ul>
<b>Essential Knowledge</b>	<ul style="list-style-type: none"> <li>▪ An integral can be thought of as “undoing” a derivative problem.</li> <li>▪ Applying an integral to motion problems allows for quick computational methods to finding total distance traveled and the position shift of an object in motion.</li> <li>▪ Applying an integral can find areas under curves.</li> </ul>
<b>Vocabulary</b>	<ul style="list-style-type: none"> <li>▪ <u>Terms:</u> <ul style="list-style-type: none"> <li>○ area under a curve; average value; definite &amp; indefinite integrals; Fundamental Theorem of Calculus; Rectangle approximation method for areas; mean value theorem for integrals; net vs total area; Riemann Sums; the Trapezoid &amp; Simpson’s rules</li> </ul> </li> </ul>
<b>Essential Skills</b>	<ul style="list-style-type: none"> <li>▪ Find and evaluate an integral.</li> <li>▪ Apply integrals to various motion problems.</li> <li>▪ Apply integrals to various geometry problems.</li> </ul>

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<p><b>Related Maine Learning Results</b></p>	<p><u>Mathematics</u></p> <p>A. Number</p> <p>Real Number</p> <p>A1.Students will know how to represent and use real numbers.</p> <ol style="list-style-type: none"><li>Use the concept of nth root.</li><li>Estimate the value(s) of roots and use technology to approximate them.</li><li>Compute using laws of exponents.</li><li>Multiply and divide numbers expressed in scientific notation.</li><li>Understand that some quadratic equations do not have real solutions and that there exist other number systems to allow for solutions to these equations.</li></ol> <p>B. Data</p> <p>Measurement and Approximation</p> <p>B1.Students understand the relationship between precision and accuracy.</p> <ol style="list-style-type: none"><li>Express answers to a reasonable degree of precision in the context of a given problem.</li><li>Represent an approximate measurement using appropriate numbers of significant figures.</li><li>Know that most measurements are approximations and explain why it is useful to take the mean of repeated measurements.</li></ol> <p>Data Analysis</p> <p>B2.Students understand correlation and cause and effect.</p> <ol style="list-style-type: none"><li>Recognize when correlation has been confused with cause and effect.</li><li>Create and interpret scatter plots and estimate correlation and lines of best fit.</li><li>Recognize positive and negative correlations based on data from a table or scatter plot.</li><li>Estimate the strength of correlation based upon a scatter plot.</li></ol>
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**Related  
Maine Learning  
Results**

- B3.Students understand and know how to describe distributions and find and use descriptive statistics for a set of data.
- a. Find and apply range, quartiles, mean absolute deviation, and standard deviation (using technology) of a set of data.
  - b. Interpret, give examples of, and describe key differences among different types of distributions: uniform, normal, and skewed.
  - c. For the sample mean of normal distributions, use the standard deviation for a group of observations to establish 90%, 95%, or 99% confidence intervals.

- B4.Students understand that the purpose of random sampling is to reduce bias when creating a representative sample for a set of data.
- a. Describe and account for the difference between sample statistics and statistics describing the distribution of the entire population.
  - b. Recognize that sample statistics produce estimates for the distribution of an entire population and recognize that larger sample sizes will produce more reliable estimates.
  - c. Apply methods of creating random samples and recognize possible sources of bias in samples.

Probability

- B5.Students understand the relationship of probability to relative frequency and know how to find the probability of compound events.
- a. Find the expected frequency of an event.
  - b. Find the expected value of events.
  - c. Find the probability of compound events including independent and dependent events.

C. Geometry

Geometric Figures

- C1.Students justify statements about polygons and solve problems.
- a. Use the properties of triangles to prove theorems about figures and relationships among figures.
  - b. Solve for missing dimensions based on congruence and similarity.
  - c. Use the Pythagorean Theorem in situations where right triangles are created by adding segments to figures.
  - d. Use the distance formula.
- C2.Students justify statements about circles and solve problems.
- a. Use the concepts of central and inscribed angles to solve problems and justify statements.
  - b. Use relationships among arc length and circumference, and areas of circles and sectors to solve problems and justify

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<p><b>Related Maine Learning Results</b></p>	<p>C3.Students understand and use basic ideas of trigonometry.</p> <ol style="list-style-type: none"> <li>a. Identify and find the value of trigonometric ratios for angles in right triangles.</li> <li>b. Use trigonometry to solve for missing lengths in right triangles.</li> <li>c. Use inverse trigonometric functions to find missing angles in right triangles.</li> </ol> <p>D. Algebra</p> <p>Symbols and Expressions</p> <p>D1.Students understand and use polynomials and expressions with rational exponents.</p> <ol style="list-style-type: none"> <li>a. Simplify expressions including those with rational numbers.</li> <li>b. Add, subtract, and multiply polynomials.</li> <li>c. Factor the common term out of polynomial expressions.</li> <li>d. Divide polynomials by <math>(ax+b)</math>.</li> </ol> <p>Equations and Inequalities</p> <p>D2.Students solve families of equations and inequalities.</p> <ol style="list-style-type: none"> <li>a. Solve systems of linear equations and inequalities in two unknowns and interpret their graphs.</li> <li>b. Solve quadratic equations graphically, by factoring in cases where factoring is efficient, and by applying the quadratic formula.</li> <li>c. Solve simple rational equations.</li> <li>d. Solve absolute value equations and inequalities and interpret the results.</li> <li>e. Apply the understanding that the solution(s) to equations of the form <math>f(x) = g(x)</math> are x-value(s) of the point(s) of intersection of the graphs of <math>f(x)</math> and <math>g(x)</math> and common outputs in table of values.</li> <li>f. Explain why the coordinates of the point of intersection of the lines represented by a system of equations is its solution and apply this understanding to solving problems.</li> </ol> <p>D3.Students understand and apply ideas of logarithms.</p> <ol style="list-style-type: none"> <li>a. Use and interpret logarithmic scales.</li> <li>b. Solve equations in the form of <math>x + b^y</math> using the equivalent form <math>y = \log_b x</math>.</li> </ol>
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<b>Related Maine Learning Results</b>	<p>Functions and Relations</p> <p>D4. Students understand and interpret the characteristics of functions using graphs, tables, and algebraic techniques.</p> <ol style="list-style-type: none"> <li>a. Recognize the graphs and sketch graphs of the basic functions.</li> <li>b. Apply functions from these families to problem situations.</li> <li>c. Use concepts such as domain, range, zeros, intercepts, and maximum and minimum values.</li> <li>d. Use the concepts of average rate of change (table of values) and increasing and decreasing over intervals, and use these characteristics to compare functions.</li> </ol> <p>D5. Students express relationships recursively and use iterative methods to solve problems.</p> <ol style="list-style-type: none"> <li>a. Express the <math>(n+1)</math>st term in terms of the <math>n</math>th term and describe relationships in terms of starting point and rule followed to transform one terms to the next.</li> <li>b. Use technology to perform repeated calculations to develop solutions to real life problems involving linear, exponential, and other patterns of change.</li> </ol>
<b>Sample Lessons And Activities</b>	<ul style="list-style-type: none"> <li>▪ Students must find the area under a curve.</li> </ul>
<b>Sample Classroom Assessment Methods</b>	<ul style="list-style-type: none"> <li>▪ Homework</li> <li>▪ Chapter quizzes and chapter exams: with and without graphing calculator – some to be done in-class, some to be done as take-home</li> <li>▪ AP exam problems: multiple choice and free-response</li> <li>▪ Student presentations to the class and to the teacher</li> </ul>
<b>Sample Resources</b>	<ul style="list-style-type: none"> <li>▪ <u>Publications:</u> <ul style="list-style-type: none"> <li>○ <u>Calculus: Graphical, Numerical, Algebraic</u>, third edition</li> </ul> </li> <li>▪ <u>Other Resources:</u> <ul style="list-style-type: none"> <li>○ Various AP exam prep books</li> <li>○ Past AP exam questions: multiple choice and free-response</li> <li>○ Graphing calculator</li> </ul> </li> </ul>