

**Mathematics**  
**Calculus: Honors**  
**Unit 2: Limits & Continuity**

<b>Essential Understandings</b>	<ul style="list-style-type: none"> <li>▪ The concept of a “limit” as approaching infinitely small or infinitely large quantities is explored.</li> <li>▪ Average rate of change versus instantaneous rate of change is explored.</li> <li>▪ The concept of “continuity” is explored to assist in the analysis of modeling real-life situations in which certain values can not exist.</li> <li>▪ Various types of discontinuities are explored.</li> </ul>
<b>Essential Questions</b>	<ul style="list-style-type: none"> <li>▪ What is a limit and how it used calculus?</li> <li>▪ What is an end behavior model?</li> <li>▪ What is an instantaneous rate of change?</li> <li>▪ What does it mean for a function to be discontinuous?</li> <li>▪ What do limits &amp; continuity have to do with rates of change?</li> </ul>
<b>Essential Knowledge</b>	<ul style="list-style-type: none"> <li>▪ Rates of change are the real-life meaning of slopes (from algebra).</li> <li>▪ Using limits, slopes can be used to describe rates of change at an instant in time, rather than over an interval of time.</li> <li>▪ Using limits and continuity, slopes can be used to describe rates of change (slopes) at a point on a curve.</li> <li>▪ Limits and continuity can be used to formally describe how a function behaves anywhere over its graph.</li> <li>▪ The various types of discontinuities can be used to describe real-life situations as they apply to functions.</li> </ul>
<b>Vocabulary</b>	<ul style="list-style-type: none"> <li>▪ <u>Terms:</u> <ul style="list-style-type: none"> <li>○ increment / displacement as a change in position; average vs instantaneous speed; free-fall; limits – one-sided &amp; two-sided; slope as a rate of change with units; the sandwich theorem; vertical, horizontal &amp; oblique asymptotes; infinity; end behavior models and asymptotes; continuity; types of discontinuities: removable, jump, infinite &amp; oscillating; Intermediate value theorem; difference quotient; “tangents” and “normals” to a line</li> </ul> </li> </ul>
<b>Essential Skills</b>	<ul style="list-style-type: none"> <li>▪ Evaluate limit problems – with &amp; without a calculator.</li> <li>▪ Find &amp; identify discontinuities – supply reasons for discontinuity.</li> <li>▪ Find average &amp; instantaneous rates of change.</li> <li>▪ Find end behavior models &amp; asymptotes.</li> </ul>
<b>Related Maine Learning Results</b>	<p><u>Mathematics</u>  A. Number  Real Number  A1.Students will know how to represent and use real numbers.</p> <ol style="list-style-type: none"> <li>a. Use the concept of nth root.</li> <li>b. Estimate the value(s) of roots and use technology to approximate them.</li> <li>c. Compute using laws of exponents.</li> <li>d. Multiply and divide numbers expressed in scientific notation.</li> <li>e. Understand that some quadratic equations do not have real</li> </ol>

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	<p>solutions and that there exist other number systems to allow for solutions to these equations.</p>
<p><b>Related Maine Learning Results</b></p>	<p>B. Data  Measurement and Approximation  B1.Students understand the relationship between precision and accuracy.</p> <ol style="list-style-type: none"> <li>a. Express answers to a reasonable degree of precision in the context of a given problem.</li> <li>b. Represent an approximate measurement using appropriate numbers of significant figures.</li> <li>c. Know that most measurements are approximations and explain why it is useful to take the mean of repeated measurements.</li> </ol> <p>Data Analysis  B2.Students understand correlation and cause and effect.</p> <ol style="list-style-type: none"> <li>a. Recognize when correlation has been confused with cause and effect.</li> <li>b. Create and interpret scatter plots and estimate correlation and lines of best fit.</li> <li>c. Recognize positive and negative correlations based on data from a table or scatter plot.</li> <li>d. Estimate the strength of correlation based upon a scatter plot.</li> </ol> <p>B3.Students understand and know how to describe distributions and find and use descriptive statistics for a set of data.</p> <ol style="list-style-type: none"> <li>a. Find and apply range, quartiles, mean absolute deviation, and standard deviation (using technology) of a set of data.</li> <li>b. Interpret, give examples of, and describe key differences among different types of distributions: uniform, normal, and skewed.</li> <li>c. For the sample mean of normal distributions, use the standard deviation for a group of observations to establish 90%, 95%, or 99% confidence intervals.</li> </ol> <p>B4.Students understand that the purpose of random sampling is to reduce bias when creating a representative sample for a set of data.</p> <ol style="list-style-type: none"> <li>a. Describe and account for the difference between sample statistics and statistics describing the distribution of the entire population.</li> <li>b. Recognize that sample statistics produce estimates for the distribution of an entire population and recognize that larger sample sizes will produce more reliable estimates.</li> <li>c. Apply methods of creating random samples and recognize possible sources of bias in samples.</li> </ol>

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**Related  
Maine Learning  
Results**

- Probability
- B5.Students understand the relationship of probability to relative frequency and know how to find the probability of compound events.
- Find the expected frequency of an event.
  - Find the expected value of events.
  - Find the probability of compound events including independent and dependent events.
- C. Geometry
- Geometric Figures
- C1.Students justify statements about polygons and solve problems.
- Use the properties of triangles to prove theorems about figures and relationships among figures.
  - Solve for missing dimensions based on congruence and similarity.
  - Use the Pythagorean Theorem in situations where right triangles are created by adding segments to figures.
  - Use the distance formula.
- C2.Students justify statements about circles and solve problems.
- Use the concepts of central and inscribed angles to solve problems and justify statements.
  - Use relationships among arc length and circumference, and areas of circles and sectors to solve problems and justify statements.
- C3.Students understand and use basic ideas of trigonometry.
- Identify and find the value of trigonometric ratios for angles in right triangles.
  - Use trigonometry to solve for missing lengths in right triangles.
  - Use inverse trigonometric functions to find missing angles in right triangles.
- D. Algebra
- Symbols and Expressions
- D1.Students understand and use polynomials and expressions with rational exponents.
- Simplify expressions including those with rational numbers.
  - Add, subtract, and multiply polynomials.
  - Factor the common term out of polynomial expressions.
  - Divide polynomials by  $(ax+b)$ .

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<p><b>Related Maine Learning Results</b></p>	<p>Equations and Inequalities  D2.Students solve families of equations and inequalities.</p> <ol style="list-style-type: none"> <li>a. Solve systems of linear equations and inequalities in two unknowns and interpret their graphs.</li> <li>b. Solve quadratic equations graphically, by factoring in cases where factoring is efficient, and by applying the quadratic formula.</li> <li>c. Solve simple rational equations.</li> <li>d. Solve absolute value equations and inequalities and interpret the results.</li> <li>e. Apply the understanding that the solution(s) to equations of the form <math>f(x) = g(x)</math> are x-value(s) of the point(s) of intersection of the graphs of <math>f(x)</math> and <math>g(x)</math> and common outputs in table of values.</li> <li>f. Explain why the coordinates of the point of intersection of the lines represented by a system of equations is its solution and apply this understanding to solving problems.</li> </ol> <p>D3.Students understand and apply ideas of logarithms.</p> <ol style="list-style-type: none"> <li>a. Use and interpret logarithmic scales.</li> <li>b. Solve equations in the form of <math>x + b^y</math> using the equivalent form <math>y = \log_b x</math>.</li> </ol> <p>Functions and Relations  D4.Students understand and interpret the characteristics of functions using graphs, tables, and algebraic techniques.</p> <ol style="list-style-type: none"> <li>a. Recognize the graphs and sketch graphs of the basic functions.</li> <li>b. Apply functions from these families to problem situations.</li> <li>c. Use concepts such as domain, range, zeros, intercepts, and maximum and minimum values.</li> <li>d. Use the concepts of average rate of change (table of values) and increasing and decreasing over intervals, and use these characteristics to compare functions.</li> </ol> <p>D5.Students express relationships recursively and use iterative methods to solve problems.</p> <ol style="list-style-type: none"> <li>a. Express the <math>(n+1)</math>st term in terms of the <math>n</math>th term and describe relationships in terms of starting point and rule followed to transform one terms to the next.</li> <li>b. Use technology to perform repeated calculations to develop solutions to real life problems involving linear, exponential, and other patterns of change.</li> </ol>
<p><b>Sample Lessons And Activities</b></p>	<ul style="list-style-type: none"> <li>▪ Students are given a piece-wise defined function and are asked to find &amp; identify any discontinuities within the function. They must use the vocabulary and notation from limits and continuity to supply reasons.</li> </ul>

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<b>Sample Classroom Assessment Methods</b>	<ul style="list-style-type: none"> <li>▪ Homework</li> <li>▪ Chapter quizzes and chapter exams: with and without graphing calculator – some to be done in-class, some to be done as take-home</li> <li>▪ AP exam problems: multiple choice and free-response</li> <li>▪ Student presentations to the class and to the teacher</li> </ul>
<b>Sample Resources</b>	<ul style="list-style-type: none"> <li>▪ <u>Publications:</u> <ul style="list-style-type: none"> <li>○ <u>Calculus: Graphical, Numerical, Algebraic</u>, third edition</li> </ul> </li> <li>▪ <u>Other Resources:</u> <ul style="list-style-type: none"> <li>○ Various AP exam prep books</li> <li>○ Past AP exam questions: multiple choice and free-response</li> <li>○ Graphing calculator</li> </ul> </li> </ul>