

Mathematics
Calculus: Honors
Unit 2: Limits & Continuity

<p style="text-align: center;">Essential Understandings</p>	<ul style="list-style-type: none"> ▪ The concept of a “limit” as approaching infinitely small or infinitely large quantities is explored. ▪ Average rate of change versus instantaneous rate of change is explored. ▪ The concept of “continuity” is explored to assist in the analysis of modeling real-life situations in which certain values can not exist. ▪ Various types of discontinuities are explored.
<p style="text-align: center;">Essential Questions</p>	<ul style="list-style-type: none"> ▪ What is a limit and how it used calculus? ▪ What is an end behavior model? ▪ What is an instantaneous rate of change? ▪ What does it mean for a function to be discontinuous? ▪ What do limits & continuity have to do with rates of change?
<p style="text-align: center;">Essential Knowledge</p>	<ul style="list-style-type: none"> ▪ Rates of change are the real-life meaning of slopes (from algebra). ▪ Using limits, slopes can be used to describe rates of change at an instant in time, rather than over an interval of time. ▪ Using limits and continuity, slopes can be used to describe rates of change (slopes) at a point on a curve. ▪ Limits and continuity can be used to formally describe how a function behaves anywhere over its graph. ▪ The various types of discontinuities can be used to describe real-life situations as they apply to functions.
<p style="text-align: center;">Vocabulary</p>	<ul style="list-style-type: none"> ▪ <u>Terms:</u> <ul style="list-style-type: none"> ○ increment / displacement as a change in position; average vs instantaneous speed; free-fall; limits – one-sided & two-sided; slope as a rate of change with units; the sandwich theorem; vertical, horizontal & oblique asymptotes; infinity; end behavior models and asymptotes; continuity; types of discontinuities: removable, jump, infinite & oscillating; Intermediate value theorem; difference quotient; “tangents” and “normals” to a line
<p style="text-align: center;">Essential Skills</p>	<ul style="list-style-type: none"> ▪ Evaluate limit problems – with & without a calculator. ▪ Find & identify discontinuities – supply reasons for discontinuity. ▪ Find average & instantaneous rates of change. ▪ Find end behavior models & asymptotes.
<p style="text-align: center;">Related Maine Learning Results</p>	<p><u>Mathematics</u> A. Number Real Number A1.Students will know how to represent and use real numbers.</p> <ol style="list-style-type: none"> a. Use the concept of nth root. b. Estimate the value(s) of roots and use technology to approximate them. c. Compute using laws of exponents. d. Multiply and divide numbers expressed in scientific notation. e. Understand that some quadratic equations do not have real

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	<p>solutions and that there exist other number systems to allow for solutions to these equations.</p>
<p>Related Maine Learning Results</p>	<p>B. Data Measurement and Approximation B1.Students understand the relationship between precision and accuracy.</p> <ol style="list-style-type: none"> a. Express answers to a reasonable degree of precision in the context of a given problem. b. Represent an approximate measurement using appropriate numbers of significant figures. c. Know that most measurements are approximations and explain why it is useful to take the mean of repeated measurements. <p>Data Analysis B2.Students understand correlation and cause and effect.</p> <ol style="list-style-type: none"> a. Recognize when correlation has been confused with cause and effect. b. Create and interpret scatter plots and estimate correlation and lines of best fit. c. Recognize positive and negative correlations based on data from a table or scatter plot. d. Estimate the strength of correlation based upon a scatter plot. <p>B3.Students understand and know how to describe distributions and find and use descriptive statistics for a set of data.</p> <ol style="list-style-type: none"> a. Find and apply range, quartiles, mean absolute deviation, and standard deviation (using technology) of a set of data. b. Interpret, give examples of, and describe key differences among different types of distributions: uniform, normal, and skewed. c. For the sample mean of normal distributions, use the standard deviation for a group of observations to establish 90%, 95%, or 99% confidence intervals. <p>B4.Students understand that the purpose of random sampling is to reduce bias when creating a representative sample for a set of data.</p> <ol style="list-style-type: none"> a. Describe and account for the difference between sample statistics and statistics describing the distribution of the entire population. b. Recognize that sample statistics produce estimates for the distribution of an entire population and recognize that larger sample sizes will produce more reliable estimates. c. Apply methods of creating random samples and recognize possible sources of bias in samples.

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**Related
Maine Learning
Results**

- Probability
- B5.Students understand the relationship of probability to relative frequency and know how to find the probability of compound events.
- Find the expected frequency of an event.
 - Find the expected value of events.
 - Find the probability of compound events including independent and dependent events.
- C. Geometry
- Geometric Figures
- C1.Students justify statements about polygons and solve problems.
- Use the properties of triangles to prove theorems about figures and relationships among figures.
 - Solve for missing dimensions based on congruence and similarity.
 - Use the Pythagorean Theorem in situations where right triangles are created by adding segments to figures.
 - Use the distance formula.
- C2.Students justify statements about circles and solve problems.
- Use the concepts of central and inscribed angles to solve problems and justify statements.
 - Use relationships among arc length and circumference, and areas of circles and sectors to solve problems and justify statements.
- C3.Students understand and use basic ideas of trigonometry.
- Identify and find the value of trigonometric ratios for angles in right triangles.
 - Use trigonometry to solve for missing lengths in right triangles.
 - Use inverse trigonometric functions to find missing angles in right triangles.
- D. Algebra
- Symbols and Expressions
- D1.Students understand and use polynomials and expressions with rational exponents.
- Simplify expressions including those with rational numbers.
 - Add, subtract, and multiply polynomials.
 - Factor the common term out of polynomial expressions.
 - Divide polynomials by $(ax+b)$.

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Related Maine Learning Results	<p>Equations and Inequalities</p> <p>D2.Students solve families of equations and inequalities.</p> <ol style="list-style-type: none"> a. Solve systems of linear equations and inequalities in two unknowns and interpret their graphs. b. Solve quadratic equations graphically, by factoring in cases where factoring is efficient, and by applying the quadratic formula. c. Solve simple rational equations. d. Solve absolute value equations and inequalities and interpret the results. e. Apply the understanding that the solution(s) to equations of the form $f(x) = g(x)$ are x-value(s) of the point(s) of intersection of the graphs of $f(x)$ and $g(x)$ and common outputs in table of values. f. Explain why the coordinates of the point of intersection of the lines represented by a system of equations is its solution and apply this understanding to solving problems. <p>D3.Students understand and apply ideas of logarithms.</p> <ol style="list-style-type: none"> a. Use and interpret logarithmic scales. b. Solve equations in the form of $x + b^y$ using the equivalent form $y = \log_b x$. <p>Functions and Relations</p> <p>D4.Students understand and interpret the characteristics of functions using graphs, tables, and algebraic techniques.</p> <ol style="list-style-type: none"> a. Recognize the graphs and sketch graphs of the basic functions. b. Apply functions from these families to problem situations. c. Use concepts such as domain, range, zeros, intercepts, and maximum and minimum values. d. Use the concepts of average rate of change (table of values) and increasing and decreasing over intervals, and use these characteristics to compare functions. <p>D5.Students express relationships recursively and use iterative methods to solve problems.</p> <ol style="list-style-type: none"> a. Express the $(n+1)$st term in terms of the nth term and describe relationships in terms of starting point and rule followed to transform one terms to the next. b. Use technology to perform repeated calculations to develop solutions to real life problems involving linear, exponential, and other patterns of change.
Sample Lessons And Activities	<ul style="list-style-type: none"> ▪ Students are given a piece-wise defined function and are asked to find & identify any discontinuities within the function. They must use the vocabulary and notation from limits and continuity to supply reasons.

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Sample Classroom Assessment Methods	<ul style="list-style-type: none"> ▪ Homework ▪ Chapter quizzes and chapter exams: with and without graphing calculator – some to be done in-class, some to be done as take-home ▪ AP exam problems: multiple choice and free-response ▪ Student presentations to the class and to the teacher
Sample Resources	<ul style="list-style-type: none"> ▪ <u>Publications:</u> <ul style="list-style-type: none"> ○ <u>Calculus: Graphical, Numerical, Algebraic</u>, third edition ▪ <u>Other Resources:</u> <ul style="list-style-type: none"> ○ Various AP exam prep books ○ Past AP exam questions: multiple choice and free-response ○ Graphing calculator