Advanced Placement
Calculus BC

Course Overview:

I cover all of the topics in the AP Calculus BC topic outline as they appear in the AP Calculus Course Description. In addition, I always go beyond the BC curriculum and cover topics in a traditional third semester college calculus course. We cover these calculus topics in a way that allows each of my students the opportunity to work with all types of functions and concepts in the following ways: graphically, numerically, analytically and verbally.

On a regular basis, I require my students to explain to the class (and occasionally to me on an individual basis) their solutions, their methods of solution and the calculus justifications for those solutions. We also discuss, on a regular basis, other options to the explanations presented in class. By the end of the school year, each student can use their graphing calculator for its intended purposes on the AP exam: 1) finding a root, 2) sketching a function in a specified window, 3) approximating the derivative at a point using numerical methods, and 4) approximating the value of a definite integral using numerical methods. However, this list is incomplete, as we utilize the graphing calculator well beyond the four previously mentioned functions.

Each of my students owns or has year-round access to a TI-89 graphing calculator. Graphing calculators are used extensively at my high school, not just in AP calculus. By the time students get to my AP Calculus BC class, they are quite proficient in using the basic functional programs of a calculator. Each student has use of a calculus text book that includes material from the first semester college calculus through a third semester college calculus course. In addition, the school's library has a large selection of books on mathematics.

Students are always assigned problems from the textbook following each section within a unit of study. The course expectations are that students show their work along with the answer, even if a calculator is allowed. Many of the problems require a written explanation to the solution. For example, to find the dimensions of a rectangular field that would have a maximum area with a fixed perimeter of fencing on three of the sides. Also, determine if a given infinite series converges or diverges. While the final answer is important it may not verify a complete knowledge of the appropriate concept and skills. The justification of the solution best illustrates one's knowledge along with a written explanation using a few sentences. For the first example above, a student would be required to show knowledge of the first derivative test and/or the second derivative test. As for the second example above, a student would be required to illustrate knowledge of the various tests for convergence, like the Ratio Test or the n^{th} Term Test.

The AP Calculus BC course is an option for all students who feel they are mathematically qualified, as we have no entrance requirements. But the vast majority of our AP Calculus BC students have come through our school system's accelerated mathematics program, which includes studying Calculus AB

during their junior year, and at times during their sophomore year. All of my students in Calculus BC are strongly encouraged to take the AP Calculus Exam, but our school's policy does not require that students actually take the AP Exam. Nonetheless, well over 80% of my previous students have chosen to take the exam.

Teaching strategy:

I encourage students to learn calculus independently, like at the collegiate level. Students are expected to come to class prepared with assignments complete. Although lecture style teaching is used for part of each class, interactive group work is a common approach to enhance learning. Students are often asked to present solutions at the board or on the overhead projector. A lot of the learning involves an interaction between students and teacher over the concepts of calculus. However, I provide direction and guidance for students while learning the concepts of BC calculus.

Assessment:

Students earn a grade based on their performance on written assignments, quizzes, and tests. However, the largest percentage of their grade is based on tests' scores. A project is expected on one of the concepts of calculus, but it is a small percent of their total assessment.

Sample of Student Activities:

A. My students perform the following calculus graphing calculator experiments.
   1) We find the derivatives of functions, as well as the value at a given point.
   2) We find the limit of functions at x = a and at x = infinity.
   3) We find the area of regions bounded by two or more functions.
   4) We find the value of a definite integral.
   5) We find the value of an improper integral.
   6) We find the integral of a function.
   7) We find the Taylor Series of a function.
   8) We find the Maclaurin Series of a function.
   9) We find the finite sum of a series.
   10) We find the infinite sum of a series.
   11) We graph parametric equations.
   12) We graph polar equations.
   13) We graph equations in 3-D.
   14) We find the derivatives of functions, as well as the value at a given point.
   15) We find the limit of functions at x = a and at x = infinity.
   16) We find the area of regions bounded by two or more functions.
   17) We find the value of a definite integral.
   18) We find the value of an improper integral.
   19) We find the integral of a function.

20) We find the Taylor Series of a function.
21) We find the Maclaurin Series of a function.
22) We find the finite sum of a series.
23) We find the infinite sum of a series.
24) We graph parametric equations.
25) We graph polar equations.
26) We graph equations in 3-D.

B. My students interpret results and support conclusions of graphing calculator experiments.
1) When finding the derivatives of functions, as well as the value at a given point.
2) When finding the limit of functions at x = a and at x = infinity.
3) When finding the area of regions bounded by two or more functions.
4) When finding the value of a definite integral.
5) When finding the value of an improper integral.
6) When finding the integral of a function.
7) When finding the Taylor Series of a function.
8) When finding the Maclaurin Series of a function.
9) When finding the finite sum of a series.
10) When finding the infinite sum of a series.
11) When graphing parametric equations.
12) When graphing polar equations.
13) When graphing equations in 3-D.
14) When finding the derivatives of functions, as well as the value at a given point.
15) When finding the limit of functions at x = a and at x = infinity.
16) When finding the area of regions bounded by two or more functions.
17) When finding the value of a definite integral.
18) When finding the value of an improper integral.
19) When finding the integral of a function.
20) When finding the Taylor Series of a function.
21) When finding the Maclaurin Series of a function.
22) When finding the finite sum of a series.
23) When finding the infinite sum of a series.
24) When graphing parametric equations.
25) When graphing polar equations.
26) When graphing equations in 3-D.

C. My students go to the chalkboard or overhead and illustrate their solution.

D. My students share strategies in solving problems.

E. My students find alternative methods in solving problems.

F. My students assist each other in using the graphing calculator to solve problems.

G. My students see me for extra help on a one on one basis.

Post AP Calculus BC Exam:

(3 weeks)

Following the AP Examination, each AP calculus student is selected to present to the class their solution to one of the Free-Response problems from this year's exam. Students may select a partner for this assignment since my average class size is around twelve. Their presentation to the class needs to be clear and accurate, including methods of solving the problem and any formulas used to find a particular solution.

After the presentations I teach topics involving multiple integrals that may not have been covered prior to the exam. The number of teaching days, prior to the exam, at my school varies substantially from year to year. The major factor in this variation is snow days, ranging from two days to six days over the past six years.

Course Planner

Note: Students in my BC calculus course are expected to review the topics in units 1A through 7A during their participation in BC calculus. In addition, I frequently revisit many of these topics in class as we extent our studies of calculus. This course begins with unit 1B and ends with unit 9B.

Unit 1A: Prerequisites of Calculus

A. Lines
   a. Slope of a line as a rate of change
   b. Slopes of parallel and perpendicular lines
   c. Three forms of linear equations and their proper applications
      1. Standard Form
      2. Point-Slope Form
      3. Slope-Intercept Form

B. Functions
   a. Relations
      1. Domain and Range
      2. Interval notation
      3. Implicit and explicit
   b. Properties of functions
      1. Vertical line test
      2. Symmetry
      3. Even, odd, or neither
      4. One-to-one test
   c. Transformations
      1. Translations
      2. Dilation
      3. Reflection
   d. Graphing by using transformations
   e. Piece-wise functions
   f. Operations of functions
   g. Composition of functions

C. Exponential and logarithmic functions

a. Properties of each function
b. Applications of each function
c. Inverse relationship of the functions
d. Natural function compared to the basic function

D. Trigonometric Functions
   a. Definition of the six functions using the unit circle
   b. Degrees versus radian measure
   c. Graphs of the six trigonometric functions
      1. Domain and range
      2. Periodicity
      3. Transformations
      4. Inverse trigonometric functions
   d. Trigonometric identities
      1. Pythagorean identities
      2. Double angle identities
      3. Half angle identities
      4. Symmetric identities
   e. Applications of trigonometry
      1. Right triangle problems
      2. Triangle problems
      3. Other polygons
      4. Graphing calculator problems using real-world data

E. Graphing calculator skills
   a. Graphing functions
   b. Evaluating functions at a given value
   c. Solving equations
   d. Exploring solutions
   e. Approximating solutions

Unit 2A: Limits and Continuity of Functions

A. Rates of change
   a. Average rate of change
   b. Instantaneous rate of change

B. Limits functions
   a. Properties of limits
   b. Two-sided limits
   c. One-sided limits
   d. Formal definition

C. Limits of functions with infinity
   a. Asymptotic behavior
      1. Horizontal
      2. Vertical
      3. Oblique
   b. Properties of limits
   c. Limits by using a graphing calculator
   d. Discussions, with an emphasis on using correct vocabulary, about limits

D. Continuity of functions
   a. Continuity at a point
      1. Definition
      2. Removable discontinuities
      3. Jump discontinuities
      4. Infinite discontinuities
   b. Continuity on an interval
      1. Open interval
      2. Closed interval
   c. Intermediate Value Theorem

Unit 3A: Derivatives of Functions

A. Definition of a derivative
   a. Tangent to a curve
   b. Slope of a curve at a point
   c. Different notations
   d. Using a graphing calculator
B. Differentiability
   a. Exists at a point or interval
   b. Numeric derivatives
   c. Differentiability versus continuity
   d. Illustrations using a graphing calculator
C. Rules for algebraic functions
   a. Constant, power, product, and quotient rules
   b. First, second, and higher order derivatives
   c. Chain Rule
   d. Using a graphing calculator
D. Applications
   a. Velocity and acceleration of a particle on a line
   b. Motion due to gravity
   c. Rectilinear motion and simple harmonic motion
   d. Using the graphing calculator to illustrate motion by using parametric equations
E. Applications to parallel and perpendicular (normal) lines
F. Derivatives of the six trigonometric functions
G. Derivatives of implicit relations
   a. Differential notation
   b. Related rates word problems
H. Derivatives of inverse trigonometric functions
I. Derivatives of exponential and logarithmic functions
   a. Natural functions or base e
   b. Functions with other bases
   c. Using a graphing calculator
J. Logarithmic differentiation

Unit 4A: Applications of the First and Second Derivatives

A. Extreme values and critical points
   a. Local (relative) extrema
   b. Global (absolute) extrema
   c. The Extreme Value Theorem
B. Using the derivatives for graphing
   a. Increasing and decreasing intervals for functions
   b. The Mean Value Theorem
   c. Rolle's Theorem
   d. Graphing functions using first and second derivatives
   e. Finding points of inflection
   f. Intervals of concavity
   g. Using a graphing calculator
C. Optimization problems
D. Using linear approximations and differentials for estimations
E. Indeterminate forms and L'Hopital's Rule

Unit 5A: The Integral

A. Approximating areas numerically
    a. Riemann Sums
    b. Rectangle Approximation Methods (LRAM, RRAM, and MRAM)
        i. Graphing calculator program
        ii. Use with data from a table
    c. The Trapezoid Rule and Simpson's Rule
        i. Graphing calculator program
        ii. Use with data from a table
    d. The definite integral definition and Riemann Sums
B. The Fundamental Theorem of Calculus – First and Second
    a. Finding the integral of a function
    b. Rules of integrals
C. Indefinite integrals and antiderivatives
    a. The Mean Value Theorem for definite integrals
    b. The significance of a constant of integration
D. Finding numerical integration with a graphing calculator

Unit 6A: Integration Techniques

A. Deriving formulas for integgation
B. Integration using the method of **u**-substitution
C. Solving separable differential equations
    a. Growth problems
    b. Decay problems
D. Differential equations and antiderivatives
    a. General and particular solutions
    b. First and second order differential equations
    c. Solving for a particular solution
E. Slope fields
    a. Drawing slope fields onto lattice grids from differential equations
    b. Finding a particular curve using given values
    c. Matching slope fields and their differential equations
    d. Matching slope fields and their general solution equations

Unit 7A: Applications of Definite Integrals

A. Accumulating rates of change
B. Particle motion
    a. Total distance traveled
    b. Velocity and speed of a particle
C. Area of an enclosed region in a plane
    a. Net and total areas
    b. Area between two curves
    c. Using integrals with geometry formulas
    d. Integrals with respect to **x**
    e. Integrals with respect to **y**
D. Volumes
    a. Volumes of solids with known cross-sections using slicing method
    b. Volumes of solids of revolution
        i. Disc method
        ii. Washer method
        iii. Shell Method
        iv. Using axes other than the **x**- and **y**-axes
    c. Volume integrals with respect to **x** and with respect to **y**

Unit 1B: Techniques of Integration: (4 weeks)

F. Basic Integration Formulas
    a. Power Rule
    b. Trigonometric Functions
    c. Inverse Trigonometric Functions
    d. Exponential Functions

G. Integration by "U Substitution"
    a. Single Substitution
    b. Double Substitution

H. Integration by Parts

I. Integration by Trigonometric Identity Substitutions
    a. Sine and Cosine Integrands
    b. Tangent and Secant Integrands

J. Integration by Partial Fractions

K. Integration by Trigonometric Substitutions
    a. $a^2 + u^2$ Expressions
    b. $a^2 - u^2$ Expressions
    c. $u^2 - a^2$ Expressions

L. Integration of Rational Functions
    a. Completing the Square
    b. Using Long Division

M. Improper Integrals
    a. Discontinuous on the open interval
    b. Undefined at the endpoints of an interval

N. Graphing Calculator Integration
    a. Indefinite Integrals
    b. Definite Integrals
    c. Improper Integrals

Unit 2B: Infinite Series (6 weeks)

A. Sequences and Their Limits

B. Convergence and Divergence of Sequences

C. Infinite Series
    a. Geometric
    b. Harmonic
    c. Telescoping
    d. Finite Sum
    e. Infinite Sum
    f. Sums with a Calculator

D. Tests for Convergence
    a. nth Term Test
    b. Ratio Test
    c. Comparison Test
    d. Limit Comparison Test
    e. nth Root Test
E. Alternating Series
    a. Absolute Convergence
    b. Conditional Convergence
F. Power Series
    a. Center of Convergence
    b. Radius of Convergence
G. Taylor Series
    a. Taylor Polynomials
        i. Without a Calculator
        ii. With a Calculator
    b. Error in Approximation
H. Maclaurin Series
    a. For sin x
    b. For cos x
    c. For e^x
    d. With a Calculator

Unit 3B: Parametric Equations (3 weeks)

E. Using a graphing calculator
F. Transforming Systems of Equations
    a. Parametric to Rectangular
    b. Rectangular to Parametric
G. Derivatives of Parametric Equations
    a. First Derivative
    b. Second Derivative
    c. Equation of a Tangent Line
H. Length of a Parametric Curve
    a. Using a Calculator
    b. Using Integration
I. Area of a Surface of Revolution
    a. Formed by Rotation about the x-axis
    b. Formed by Rotation about the y-axis
    c. Using a Graphing Calculator

Unit 4B: Polar Coordinates (3 weeks)

K. Polar Coordinate System
    a. Graphing Points
    b. Graphing Regions
    c. Calculator Uses in Polar Mode
L. Types of Polar Graphs
    a. Circles
    b. Lines
    c. Limacons
        i. Cardioid
        ii. Inner Loop
        iii. Convex
        iv. Dimpled
    d. Rose Curves
        i. n is even
        ii. n is odd
    e. Lemniscates
    f. Spirals
    g. Conic Sections
        i. Parabola
        ii. Ellipse
        iii. Hyperbola
        iv. Circle
M. Calculus of Polar Coordinates
    a. First Derivative
    b. Length of a Polar Curve
    c. Area of a Region Bounded by Two Curves
    d. Surface Area of Revolution
        i. Formed by Rotation About the Polar Axis
        ii. Formed by Rotation About the vertical Axis

Unit 5B: Cartesian Coordinates in Space: (3 weeks)

F. Coordinate System in 3-D
    a. Graphing Points
    b. Graphs of Equations with Use of a Calculator
G. Graphs of Linear Equations
    a. Planes Parallel to an axis

b. Planes Intersecting each axis

H. Graphing of Quadratic Equations

a. Parallel to an axis

i. Parabolic Sheet

ii. Elliptical Sheet

iii. Hyperbolic Sheet

iv. Circular Cylindrical Sheet

b. Quadric Surfaces

i. Sphere

ii. Ellipsoid

1. Circular

2. Elliptical

iii. Paraboloid

1. Circular

2. Elliptical

iv. Hyperboloid

1. Of One Sheet

2. Of Two Sheets

v. Hyperbolic Paraboloid

Unit 6B: Vectors and Analytic Geometry: (4 weeks)

E. Vectors in a Plane

a. Graphing Vectors

b. Direction

c. Magnitude

d. Sum

e. Difference

F. Vectors in Space

a. Graphing Vectors

b. Direction

c. Magnitude

d. Sum

e. Difference

f. Area of a Parallelogram

G. Products of Vectors

a. Scalar Product

b. Cross Product

H. Lines in Space

a. Parametric Equation of a Line
b. Distance From a Point to a Line
c. Finding Orthogonal Lines to Planes
d. Finding Equations of Parallel Lines

I. Planes in Space
    a. Equations of Planes
    b. Equations of Parallel Planes
    c. Equations of Perpendicular Planes
    d. Equations of a Plane Perpendicular to a Line
    e. Angle Between Intersecting Planes

Unit 7B: Vector-Valued Functions and Motion in Space: (2 weeks)

F. Vector-Valued Functions
    a. Limits
    b. Continuity
    c. Derivatives
    d. Integrals
G. Modeling Projectile Motion
H. Directed Distance
    a. Distance Along a Curve
    b. Distance Along a Line
    c. Unit Tangent Vector
I. Curvature of a Plane Curve
J. Tangent and Normal Components of Acceleration
K. Vector Equations for Motion in Polar Coordinates
    a. Planetary Motion
    b. Kepler's First Three Laws

Unit 8B: Functions of Two or More Variables: (3 weeks)

E. Function of Three Variables
F. Level Curves of Functions
G. Level Surfaces of Functions
H. Limits and Continuity of Functions
I. Partial Derivatives
    a. First Order Derivatives
    b. Second Order Derivatives
J. The Chain Rule for Functions of Two Variables
K. The Chain Rule for Functions of Three Variables

L. Directional Derivatives
M. Gradient of a Function
N. Rules for Gradients
O. Tangent Planes to a Level Surface
P. Normal Lines to a Level Surface
Q. Tangent Lines to a Level Surface
R. Linearization of a Function
S. Differentials
T. Maxima and Minima of Functions
U. Saddle Points of a Function
V. Lagrange Multipliers

Unit 9B: Multiple Integrals: (2 weeks)

A. Double Integrals
    a. Finding Areas of Regions
    b. Finding Volumes of Solids
B. Triple Integrals
    a. Volume of a Region in Space
    b. Average Value of a Function in Space

AP Exam Review

The above course planner allows 2 to 3 weeks in April and early May with which to review the major concepts of the AB Calculus course, as well as the BC Calculus course. We use previously released AP type problems and problems from various AP Prep books. We carefully analyze the scoring rubric for the Free-Response problems and the overall scoring scale. In addition, we discuss multiple-choice strategies and practice solving previous exam problems. Furthermore, throughout the school year, the students and I go through the samples of AP released problems. An alternative method of review involves students presenting their solutions in front of the class, including all necessary calculations. Collectively, these different forms of preparation for the AP Exam provide more than adequate knowledge of the concepts of calculus.

Major Text:

Finney, Ross L., Thomas, George B., *Calculus*, Addison-Wesley Publishing Co., 1992

Other resources: (AP Examination Preparation books)

Finney, Ross L. Franklin D. Demana; Bert K. Waits; Daniel Kennedy, AP Test Prep Series: *Calculus,* Pearson; Addison-Wesley, 2007

Lifshitz, Maxine, Martha Green, *Calculus AB / BC – Preparing For The Advanced Placement Examinations*, AMSCO Publications, 2004

Kahn, David, *Princeton Review: Cracking The AP Calculus AB & BC*, Random House, 1999

Released Exams, AP Calculus AB and Calculus BC, The College Board, 1997

Released Exams, AP Calculus AB and Calculus BC, The College Board, 1998

Previously Released Exams of AB Calculus and BC Calculus from years ranging from 1974 to present, with solutions to many of these.

Technology:

Graphing Calculator: TI-89

Overhead Projection for Graphing Calculator: TI-89

Window XP – Graphing Application for Polar Coordinates and 3-D Surfaces