

**Example #1** “Don’t break the ice”

In the children’s game Don’t Break the Ice, small plastic ice cubes are squeezed into a square frame. Each child takes turns tapping out a cube of “ice” with a plastic hammer hoping that the remaining cubes don’t collapse. For the game to work correctly, the cubes must be big enough so that they hold each other in place in the plastic frame but not so big that they are too difficult to tap out. The machine that produces the plastic ice cubes is designed to make cubes that are 29.5 millimeters (mm) wide, but the actual width varies a little. To make sure the machine is working well, a supervisor inspects a random sample of 50 cubes every hour and measures their width. The Fathom output below summarizes the data from a sample taken during one hour.

- Do these data give convincing evidence that the mean width of cubes produced this hour is not 29.5 mm?

Collection 1

	29.4874 mm
	50
	0.0934676 mm
	0.0132183 mm
Width	29.2717 mm
	29.4225 mm
	29.4821 mm
	29.5544 mm
	29.7148 mm

S1 = mean ( )  $\rightarrow \bar{x}$   
 S2 = count ( )  $\rightarrow n$   
 S3 = stdDev ( )  $\rightarrow s_x$   
 S4 = stdError ( )  
 S5 = min ( )  
 S6 = Q1 ( )  
 S7 = median ( )  
 S8 = Q3 ( )  
 S9 = max ( )

- Parameter of Interest  $\mu = \text{TRUE mean width of plastic ice cubes}$
- Null Hypothesis  $H_0: \mu = 29.5$
- Alternative Hypothesis  $H_A: \mu \neq 29.5$
- Level of Significance  $\alpha = 0.05$
- Choice of Test 1 Sample t test for means
- Conditions of Test. Assume random and independent conditions have been met.
  - Check the Normal Condition  $n = 50 > 30 \rightarrow \text{CLT applies}$

- Sampling Distribution (Sample statistics & sketch normal graph under  $H_0$ )

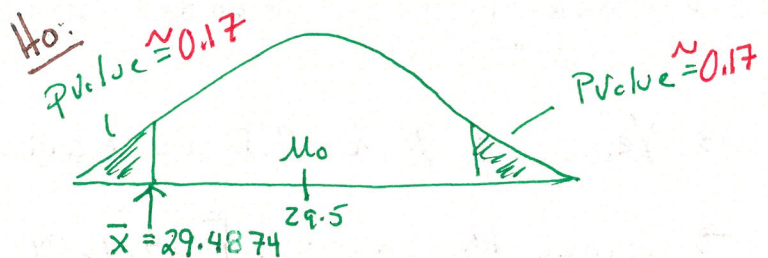
$$n = 50$$

$$\bar{x} = 29.4874$$

$$s_x = 0.093$$

- Test Statistic (OPTIONAL calculation)

$$t = \frac{29.4874 - 29.5}{0.093 / \sqrt{50}} = -0.958$$



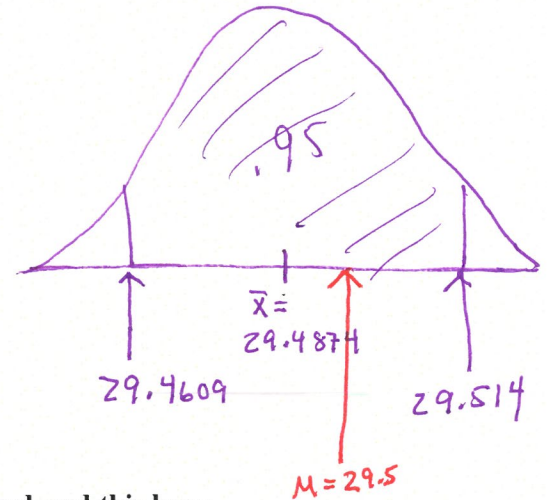
- P-value (Use correct probability notation.)  $p\text{-value} = 2 \cdot P(t \leq -0.958) = 0.3428$

- Conclusions (in context) Since the p-value (0.3428) is Greater than  $\alpha = 0.05$ , We fail to reject  $H_0$ . We do NOT have convincing evidence the true mean width of plastic ice cubes is different than 29.5mm.

## Example #2 "Don't break the ice" Confidence intervals for Means -

Here is computer output for a 95% confidence interval for the true mean width of plastic ice cubes produced this hour.

Estimate of Collection 1		Estimate Mean
Attribute (numeric): Width		
Interval estimate for population mean of Width		
Count:	50	
Mean:	29.4874 mm	
Std dev:	0.0934676 mm	
Std error:	0.0132183 mm	
Confidence level:	95.0 %	
Estimate:	29.4874 mm +/- 0.0265632 mm	
Range:	29.4609 mm to 29.514 mm	



a) Interpret the confidence interval.

$\mu$  = true mean width of plastic ice cubes produced this hour.

CI [ 29.4609, 29.514 ]

We are 95% confident the true mean width of plastic ice cubes is in the interval 29.4609 mm to 29.514 mm.

b) Would you make the same conclusion with the confidence interval as you did with the significance test in the previous example? Explain in context.

→ Yes. A 95% CI is equivalent to a 2 tail Test  $\alpha=0.05$

rewrite → Since the interval [29.4609, 29.514] contains the True mean ( $\mu$ ) 29.5 mm as a plausible value for the ice cube width, we would again FAIL to Reject  $H_0$ .

c) [REVIEW QUESTION] Interpret the confidence level.

d) [REVIEW QUESTION] Interpret the standard deviation and the standard error provided by the computer output.



**Step I:** Set up your Test of Hypothesis (TOH).

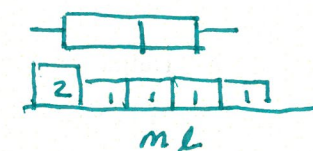
- Define parameter:  $\mu = \text{TRUE MEAN AMOUNT OF COLA BEING FILLED}$
- Define hypothesis:  $H_0: \mu = 300 \text{ ml}$   
 $H_A: \mu \neq 300 \text{ ml}$  Words: Bottles not filled to 300 ml
- Define your Level of Significance:  $\alpha = 0.05$

**Step II:** Check the conditions for carrying out this significance test.

R: random sample of 6 bottles

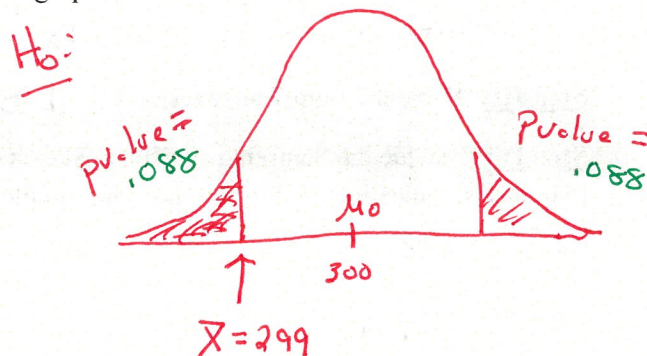
I:  $n = 6 \leq \frac{1}{10}$  (all bottles)

N: stated distribution of cola amounts is Normal

Practice sketching  
Graphs**Step III:** Name the significance test: 1 Sample t test for means**Step IV:** Provide the Sampling statistics. Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the PE, p-value, and sample statistic.

- $n = 6$
- $\bar{x} = 299.0$
- $s_x = 1.50$
- $\mu_0 = 300$

Sketch the graph:

**Step V:** Calculate the test statistic:

$$t = -1.58$$

**Step VI:** Calculate the p-value (write as a probability statement):

$$p\text{value} = 2 \cdot P(t \leq -1.58) = \underline{\underline{0.176}}$$

**Step VII:** Interpret your significance test decision in context.

Since the pvalue (0.176) is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ . We do not have convincing evidence the mean amount of cola filled in bottles is different than 300 ml.

9.3A ACTIVITY

Example #?? "Less music?"

A classic rock radio station claims to play an average of 50 minutes of music every hour. However, every time you turn to this station it seems like there is a commercial playing. To investigate their claim, you randomly select 12 different hours during the next week and record what the radio station plays in each of the 12 hours.

→ Here are the results: 44 49 45 51 49 53 49 44 47 50 46 48

Step I: Set up your Test of Hypothesis (TOH).

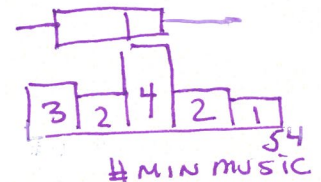
- Define parameter:  $\mu = \text{TRUE AVERAGE PLAY TIME OF MUSIC PER HOUR}$
- Define hypothesis:  $H_0: \mu = 50$   
 $H_A: \mu < 50$  Words: STATION PLAYS LESS THAN 50 MIN
- Define your Level of Significance:  $\alpha = 0.05$

Step II: Check the conditions for carrying out this significance test.

R: randomly selected sample of 12 hrs

I:  $n = 12 \leq \frac{1}{10}$  (all hours)

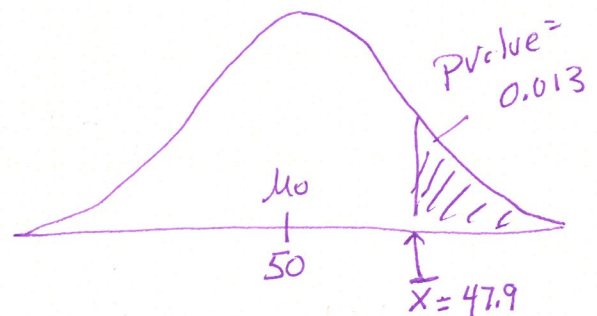
N: small sample. The graph shows no outliers + is roughly symmetric



Step III: Name the significance test: 1 sample t-test for means

Step IV: Provide the Sampling statistics. Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the PE, p-value, and sample statistic.

- $n = 12$
- $\bar{x} = 47.9$
- $s_x = 2.8$
- $\mu_0 = 50$
- Sketch the graph:



Step V: <sup>Provide</sup> Calculate the test statistic:

$$t = -2.57$$

Step VI: <sup>Provide</sup> Calculate the p-value (write as a probability statement):

$$P_{\text{value}} = P(t \leq -2.57) = 0.013$$

Step VII: Interpret your significance test decision in context.

since the  $p_{\text{value}}$  (0.013) is less than  $\alpha = 0.05$ , we reject  $H_0$ .  
We have convincing evidence, the radio station plays less than 50 min music per hour



**Step I:** Set up your Test of Hypothesis (TOH).

- Define parameter:  $p = \text{TRUE PROPORTION OF TEENS THAT PASS DRIVING TEST ON FIRST ATTEMPT.}$
- Define hypothesis:  $H_0: p = 0.60$   
 $H_A: p \neq 0.60$  Words: 60% pass is incorrect
- Define your Level of Significance:  $\alpha = 0.05$

**Step II:** Check the conditions for carrying out this significance test.

R: random sample of 125 teens

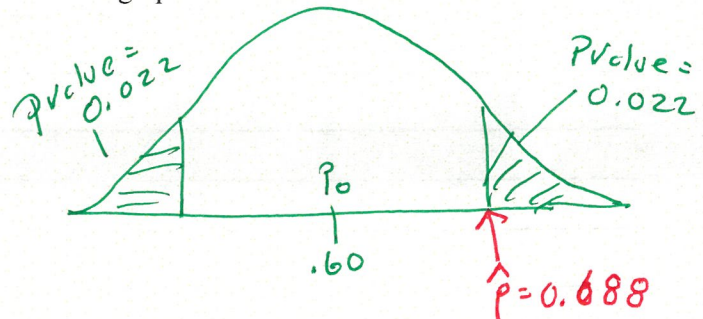
I:  $n = 125 \leq \frac{1}{10}$  (all 1ST ATTEMPT teen drivers)

N:  $125(.6) = 75 \geq 10 \checkmark$   
 $125(.4) = 50 \geq 10 \checkmark$

 $p_0 = .6$ **Step III:** Name the significance test: 1 sample Z test for proportions**Step IV:** Provide the Sampling statistics. Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the PE, p-value, and sample statistic.

- $n = 125$
- $\hat{p} = \frac{86}{125} = 0.688$
- $\underline{\hspace{2cm}}$
- $p_0 = 0.60$

- Sketch the graph:

**Step V:** ~~Calculate~~ <sup>Provide</sup> the test statistic:

$$t = 2.01$$

**Step VI:** ~~Calculate~~ <sup>Provide</sup> the p-value (write as a probability statement):

$$p\text{-value} = 2 \cdot P(t \geq 2.01) = 0.044$$

**Step VII:** Interpret your significance test decision in context.

Since the p-value (0.044) is less than  $\alpha = 0.05$ , we reject  $H_0$ . We have convincing evidence the proportion of teens who pass their drivers test on the first attempt is Not 60%

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94. Significance and sample size A study with 5000 subjects reported a result that was statistically significant at the 5% level. Explain why this result might not be particularly large or important.

#'s 94-97 answer here

The study may have rejected  $H_0$ , But with such a large sample size, such a rejection might occur even if the actual differs only slightly from the hypothesized value. For example, the difference between  $\mu=10$  and  $\mu=10.5$  might have no practical importance.

95. Sampling shoppers A marketing consultant observes 50 consecutive shoppers at a supermarket, recording how much each shopper spends in the store. Explain why it would not be wise to use these data to carry out a significance test about the mean amount spent by all shoppers at this supermarket.

Any number of things could go wrong with a convenience sample. Depending on the time of day or the day of the week, certain types of shoppers would or would not be present.

Remember! THE ONLY WAY TO SHOW CAUSE AND EFFECT IS WITH A WELL-DESIGNED, WELL-CONTROLLED EXPERIMENT! 3 COMPONENTS ① Randomization ② CONTROL ③ REPLICATION

96. Ages of presidents Joe is writing a report on the backgrounds of American presidents. He looks up the ages of all the presidents when they entered office. Because Joe took a statistics course, he uses these numbers to perform a significance test about the mean age of all U.S. presidents. Explain why this makes no sense.

We have information about the whole population of interest.

97. Do you have ESP? A researcher looking for evidence of extrasensory perception (ESP) tests 500 subjects. Four of these subjects do significantly better ( $P < 0.01$ ) than random guessing.

(a) Is it proper to conclude that these four people have ESP? Explain your answer.

(b) What should the researcher now do to test whether any of these four subjects have ESP?

<sup>would</sup>  
① No we expect about 5 of the 500 subjects who don't have ESP to do better than randomly guessing just by chance.  
 $500(.01)=5$

② The researcher should repeat the procedure on these 4 to see if they again perform well

