

### Chapter 3 Guided LSRL MODEL "Carbonation of Concrete Structures"

Consider the following data from the article, "The Carbonation of Concrete Structures in the Tropical Environment of Singapore" (Magazine of Concrete Research (1996)): which discusses how the corrosion of steel (caused by carbonation) is the biggest problem affecting concrete strength, where  $x$ =

$x$ = carbonation depth in concrete (mm) and  $y$ = strength of concrete (Mpa)

$X$	8	20	20	30	35	40	50	55	65
$Y$	22.8	17.1	21.5	16.1	13.4	12.4	11.4	9.7	6.8

- Investigate:** What are the mean and standard deviation for each variable?

(Using correct notations)...

$$\bar{x} = 35.89$$

$$S_x = 18.53$$

$$\bar{y} = 14.58$$

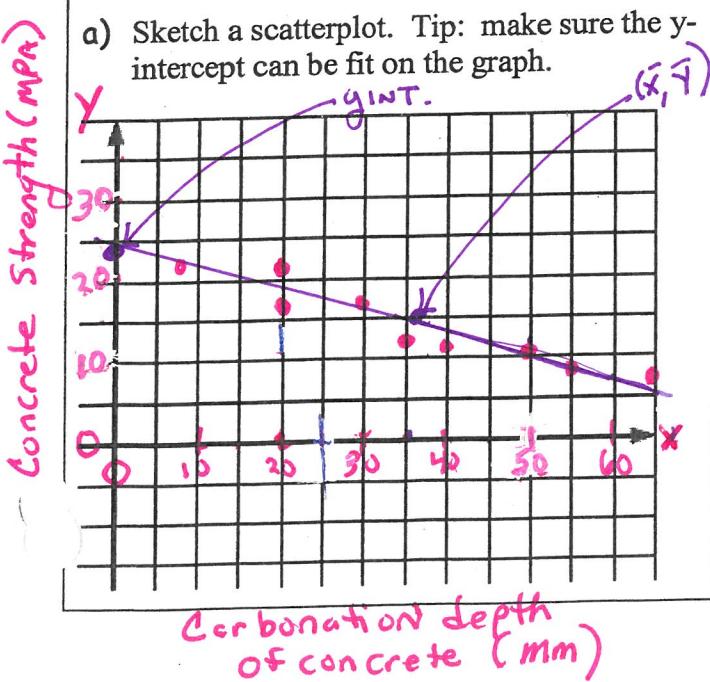
$$S_y = 5.29$$

(In context)...

$x$ : The mean <sup>carbonation</sup> depth of concrete is about 36mm with a std dev of about 19mm.

$y$ : The mean strength of concrete is about 15MPA with a std dev of about 5MPA

- a) Sketch a scatterplot. Tip: make sure the y-intercept can be fit on the graph.



- b) Describe the correlation coefficient in context.

$$r = -0.97$$

There is a strong, negative, linear association between Carbonation depth of concrete (mm) and strength of concrete (MPa).

• Develop Model:

- a. Provide the linear regression equation:

$$\text{STRENGTH} = 24.52 - 0.28(\text{DEPTH})$$

- b. Sketch the regression line on your scatter plot and label the y-intercept and  $(\bar{x}, \bar{y})$   
 $\curvearrowleft (0, 24.52)$   $\curvearrowright (35.89, 14.58)$
- c. Describe the slope of the line in context.  $B = -0.28$

The predicted strength of concrete decreases about 0.28 MPa FOR EVERY INCREASE OF 1 mm<sup>carbonation</sup> depth of concrete corrosion.

- d. Describe the y-intercept in context  $A = 24.52$

carbonation

when the<sup>^</sup> depth of concrete is 0mm, the baseline strength of concrete is about 25 MPa. (the starting point is about 25 MPa)

use:  
baseline,  
starting pt,  
initial

- e. Is the model appropriate? TIP: Create a residual plot

1) The scatter plot shows a linear association.

2) Residual Calc:

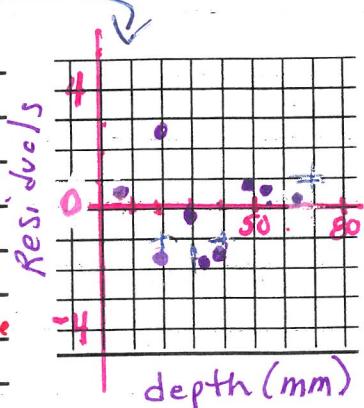
scatterplot:

X LIST: [L1]

Y LIST: [resid]

$[2ND] \cdot [LIST] \text{ Resid}$

IN Reviewing the residual plot, there is NO apparent pattern in the residuals, therefore a linear model is appropriate



- f. Describe the strength of the model

$$r^2 = 0.9375$$

About 94% of the variability in predicted strength can be explained by the depth of concrete based on using this linear model.

Note: about 6% of the variability can be explained by other variables, <sup>Page 2</sup> Therefore this is a strong linear model.

• Using the Model:

a. What is the predicted strength of concrete with a corrosion depth of 40mm?

i.) Use your regression equation and show your work. Then write your prediction in context.

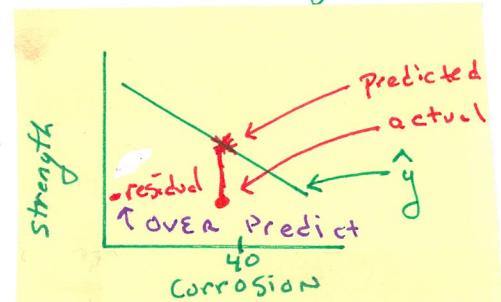
$$\text{Strength} = 24.52 - 0.28(40) = \underline{\underline{13.32 \text{ MPa}}}$$

We calculated the predicted strength of the concrete when the corrosion depth is 40mm to be about 13.32 MPa.

ii.) What is the error in your prediction (show your work and answer in context)?

See Table  $\rightarrow x(\text{concrete depth}) = 40\text{mm} \rightarrow \text{ACTUAL } Y(\text{strength}) = 12.4 \text{ MPa}$

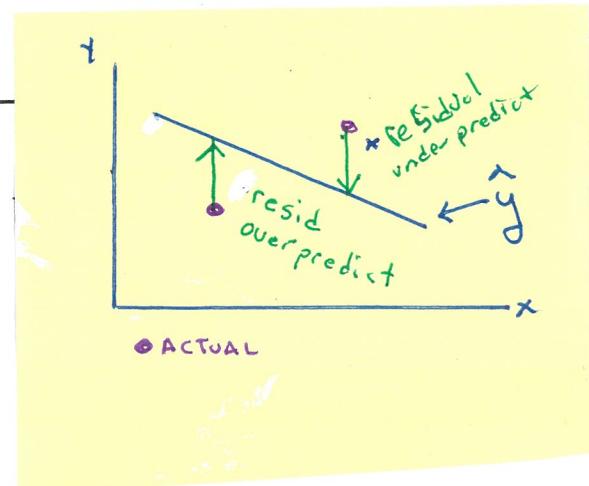
$$\begin{aligned}\text{Residual} &= \text{Actual } y - \text{predicted } Y \\ &= 12.4 \text{ (table)} - 13.32 \text{ (above)} \\ &= -0.92\end{aligned}$$



When the actual corrosion depth is 40mm, the error is about -0.92 MPa

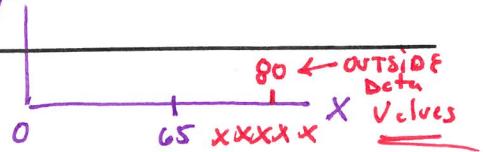
$\Rightarrow$  The error between actual and predicted concrete strength is about -0.92 MPa; which means the regression equation is OVER PREDICTING the concrete strength by about 1 MPa.

What is this error called? Residual



- b. What is the predicted strength of concrete with a corrosion depth of 80mm? Explain in context.

We cannot make this prediction. This is EXTRAPOLATION!! Since 80mm, is outside the interval of X values used in our analysis, it is not appropriate to use this model to make predictions.



- c. For this model  $s=.03\text{mm}$ . What is "s" called?

$s = \text{STANDARD DEVIATION of the residuals}$

- d. Interpret "s" in context.

"s" tells us how far off the predicted values of concrete strength from the actual concrete strength is about 0.03m, on average.

OR

"s" is the typical or average prediction error from the actual concrete strength

OR

"s" is the standard deviation of the residuals  
(actual - predicted)

Memorize these!!

$r^2 \rightarrow$  About  $r^2$  % OF THE VARIABILITY PREDICTING  $y_{context}$  IS ACCOUNTED FOR BY  $x_{context}$  USING THIS LSRL MODEL.

$s \rightarrow$  The actual  $y_{context}$  is typical about  $s$  (use  $y$  units) AWAY FROM THE NUMBER PREDICTED BY THE LSRL.