

9.8

Factor Polynomials Completely

Academic

Goal • Factor polynomials completely.

Your Notes

VOCABULARY

Factor by grouping → NOT COVERED IN ACADEMIC

Factor

SPECIAL CASES :

[2A] DIFFERENCE OF 2 SQUARES

$$4x^2 - 25 \xrightarrow{\text{PSQ}} (2x-5)(2x+5)$$

[2B] PERFECT BINOMIAL SQUARES

$$4x^2 - 20x + 25 \xrightarrow{\text{PSQ}} (2x-5)(2x-5) \text{ OR } (2x-5)^2$$

VOCABULARY

Factor completely

FACTOR A POLYNOMIAL AS MUCH AS YOU CAN UNTIL YOU ARE UNABLE TO FACTOR ANY MORE.

GUIDELINES FOR FACTORING POLYNOMIALS COMPLETELY

To factor a polynomial completely, you should try each of these steps. the GCF

- Factor out the GREATEST common monomial factor.
- Look for a difference of two squares or a Perfect Binomial Square (see above)
- Factor a trinomial of the form $ax^2 + bx + c$ into a product of BINOMIAL factors. $(x \quad)(x \quad)$
- ~~Factor a polynomial with four terms by 2~~
NOT COVERED IN ACADEMIC

Example 4 Factor completely

Factor the polynomial completely.

a. $x^2 + 3x - 1$

b. $3r^3 - 21r^2 + 30r$

c. $9d^4 - 4d^2$

Solution

- a. The terms of the polynomial have no common monomial factor. Also, there are no factors of -1 that have a sum of 3. This polynomial CAN NOT be factored. (called Prime)

b. $3r^3 - 21r^2 + 30r = 3r(r^2 - 7r + 10)$
 $= 3r(r-5)(r-2)$

c. $9d^4 - 4d^2 = d^2(9d^2 - 4)$
 $= d^2(3d-2)(3d+2)$

[A] $x^2 + 3x - 1 =$
 ~~$(x+ \quad)(x- \quad)$~~
PRIME

① FACTOR GCF!

② KEEP FACTORING, IF POSSIBLE AND Circle final answer.

FACTOR

✓ Checkpoint Factor the polynomial.

5. $\frac{-2x^3}{-2x} + \frac{6x^2}{-2x} + \frac{108x}{-2x}$

IF A is Negative then
FACTOR out (-1)

GCF

$-2x(x^2 - 3x - 54)$

154
227
318
69

KEEP FACTORING
A=1 B=-3 C=-54

Completely
Factored

$-2x(x+6)(x-9)$

6. $12y^4 - 75y^2$

SPECIAL CASE

GCF

$3y^2(4y^2 - 25)$

$3y^2(2y-5)(2y+5)$

You want

$_ \cdot _ = C$

$_ + _ = B$



$-9 \cdot 6 = -54 (C)$

$-9 + 6 = -3 (B)$

Example 5

Solve a polynomial equation

by factoring

$$\text{Solve } 5x^3 - 25x^2 = -30x.$$

Solution

mental or
CALC
CHECK ✓

Remember that
you can check
your answers by
substituting each
solution for x in the
original equation.

$$5x^3 - 25x^2 = -30x$$

Write original
equation.

$$5x^3 - 25x^2 + 30x = 0$$

ADD 30x
to each side.

The EQ
MUST
= zero

$$5x(x^2 - 5x + 6) = 0$$

Factor
out GCF

$$5x(x-3)(x-2) = 0$$

Factor
trinomial.

$$\frac{5x}{5} = 0 \text{ or } \frac{x-3}{+3 +3} = 0 \text{ or } \frac{x-2}{+2 +2} = 0$$

Zero-product
property

set all
factors to
= 0

$$x = 0$$

$$x = 3$$

$$x = 2$$

Solve for x .

$$C: 0 = 0 \checkmark \quad C: -90 = -90 \checkmark \quad C: -60 = -60 \checkmark$$

Solve by Factoring

Checkpoint Complete the following exercises.

$$7. \text{ Solve } 2x^3 + 2x^2 = 40x.$$

$$-40x \quad -40x$$

$$2x^3 + 2x^2 - 40x = 0$$

GCF →

$$2x(x^2 + x - 20) = 0$$

Keep Factoring →

$$2x(x+5)(x-4) = 0$$

Now Solve +
check:

$$\frac{2x}{2} = 0 \quad \frac{2}{2} \quad \frac{0}{2}$$

$$C: 0 = 0 \checkmark$$

$$x+5 = 0 \quad -5 \quad -5$$

$$C: -200 = -200 \checkmark$$

$$x-4 = 0 \quad +4 \quad +4$$

$$C: 160 = 160 \checkmark$$

