

7.5

Solve Special Types of Linear Systems

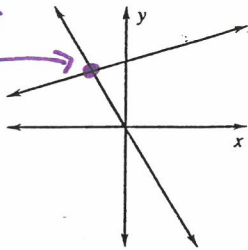
Goal • Identify the number of solutions of a linear system.

GRAPHING SYSTEMS:

NUMBER OF SOLUTIONS OF A LINEAR SYSTEM

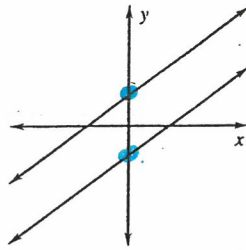
One solution

The solution is an ordered pair



- * The lines INTERSECT.
- * The lines have DIFFERENT slopes.

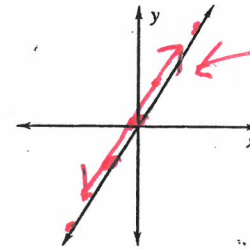
No solution



- * The lines are parallel (//).
- * The lines have the same slope and DIFFERENT y-intercepts.

* When both variables are eliminated and the STMT is FALSE → NO SOLUTION

Infinitely many solutions



- * The lines are the Same.
- * The lines have the same slope and the Same y-intercept.

* When both variables are eliminated and the STMT IS TRUE →

∞ SOLUTIONS

Solutions are all the points on the line.

∞ solutions
∞ number of points on the line.
~~all real numbers~~

Example 1 A linear system with no solutions

Show that the linear system has no solution.

$$-2x + y = 1 \quad \text{Equation 1}$$

$$-2x + y = -3 \quad \text{Equation 2}$$

Solution

Method 2 Elimination (subtraction)

Subtract the equations.

$$\begin{array}{r} -2x + y = 1 \\ -1(-2x + y = -3) \\ \hline + = 4 \end{array} \quad \begin{array}{l} \longrightarrow \\ \longrightarrow \end{array} \quad \begin{array}{r} -2x + y = 1 \\ 2x - y = 3 \\ \hline 0 = 4 \end{array} \quad \begin{array}{l} + \\ \downarrow \\ - \\ \downarrow \\ \textcircled{F} \end{array}$$

The variables are ELIMINATED and you are left with a FALSE STATEMENT. This tells you that the system has

NO SOLUTION

Example 2 A linear system with infinitely many solutions

Show that the linear system has infinitely many solutions.

$$x + 3y = -3 \quad \text{Equation 1}$$

$$3x + 9y = -9 \quad \text{Equation 2}$$

Solve

$$\begin{array}{rcl} (x + 3y = -3) \times 3 & \rightarrow & -3x - 9y = 9 \\ 3x + 9y = -9 & \rightarrow & \underline{3x + 9y = -9} \\ & & 0 = 0 \quad \text{T} \end{array}$$

The variables are ELIMINATED and you are left with a statement that is TRUE

This tells you that the system has

∞ SOLUTIONS

INFINITE SOLUTIONS

✓ **Checkpoint** Tell whether the linear system has no solution or infinitely many solutions. SOLVE WITH SUBSTITUTION

1. $y = 2x - 7$

$$4x - 2y = 14$$

$$4x - 2(2x - 7) = 14$$

$$4x - 4x + 14 = 14$$

$$14 = 14 \quad \text{T}$$

INFINITE SOLUTIONS

Both variables dropped out +
STMT IS TRUE.

2. $2y = 8x + 4$

$$-4x + y = 4$$

ISOLATE 1 variable

$$\frac{2y}{2} = \frac{8x + 4}{2}$$

$$y = 4x + 2$$

Solve for x →

$$-4x + (4x + 2) = 4$$

$$-4x + 4x + 2 = 4$$

$$2 \neq 4$$

NO SOLUTION

Both variables dropped out +
STMT IS FALSE