2.1B ACTIVITY

Introduction to Z-Scores
And the Normal Distribution

A) INTRODUCTION TO Z-SCORES

- One way to describe relative position in a data set is to tell how many standard deviations above or below the mean the observation is.

Standardized Value: "z-score"
If the mean and standard deviation of a distribution are known, the "z-score" of a particular observation, x, is:

$$z = \frac{x - \text{mean}}{\text{standard deviation}}$$

DATAVALUE

Sample:

$$Z = \frac{X - \bar{X}}{S_x}$$

n = Sample size
$\bar{X}$ = Sample mean
S_x = Sample S.D

Population:

$$Z = \frac{X - \mu}{\sigma}$$

Example 1: Consider the following test scores for a small class -

79	81	80	77	73	83	74	93	78	80	75	67	73
77	83	86	90	79	85	83	89	84	82	77	72	

Minitab

Session

Descriptive Statistics: Test 1 scores

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Test 1 scores	25	80.00	80.00	80.00	6.07	1.21

Variable	Minimum	Maximum	Q1	Q3
Test 1 scores	67.00	93.00	76.00	83.50

Reading the Minitab output, the mean is __80__ **and standard deviations is** __6.07__.

$\bar{X} = 80$ $S_x = 6.07$

EXAMPLE 1 - USE Z-SCORES TO JUSTIFY YOUR ANSWERS TO THE FOLLOWING QUESTIONS :

1. Julia's score was 86. How did she perform on this test relative to her peers? Her score is "above average"...but how far above average is it?
2. Kevin's score was 72. How did he perform on this test relative to her peers?
3. Katie's score was 80. How did she perform on this test relative to her peers?

EXAMPLE 1: ANSWER

79	81	80	77	73	83	74	93	78	80	75	67	73
77	83	86	90	79	85	83	89	84	82	77	72	

Julia: z=(86-80)/6.07
z= 0.99

Julia is about 1 S.D. above avg.
{above average = +z}

Kevin: z=(72-80)/6.07
z= -1.32

Kevin is about 1.3 SD below avg.
{below average = -z}

Katie: z=(80-80)/6.07
z= 0

{average z = 0}

EXAMPLE 2: COMPARING SCORES USING ZSCORES

- **Standardized values can be used to compare scores from two different distributions.**
  - Statistics Test: mean = 80, std dev = 6.07
  - Chemistry Test: mean = 76, std dev = 4
  - Jenny got an 86 in Statistics and 82 in Chemistry.
  - On which test did she perform better?

EXAMPLE 2: ANSWER

So which test did she perform better?

Statistics

$$z = \frac{86 - 80}{6.07} = \boxed{0.99}$$

Chemistry

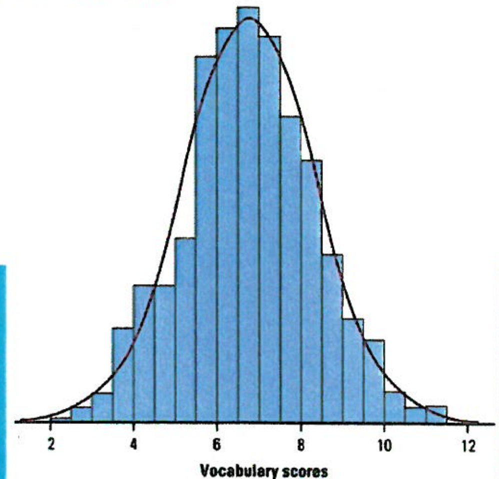
$$z = \frac{82 - 76}{4} = \boxed{1.5}$$

Although she had a lower score, she performed relatively better in Chemistry.

6

B) INTRODUCTION TO DENSITY CURVE

- In Chapter 1, you learned how to plot a dataset to describe its shape, center, spread, etc.
  - Sometimes, the overall pattern of a large number of observations is so regular that we can describe it using a smooth curve.



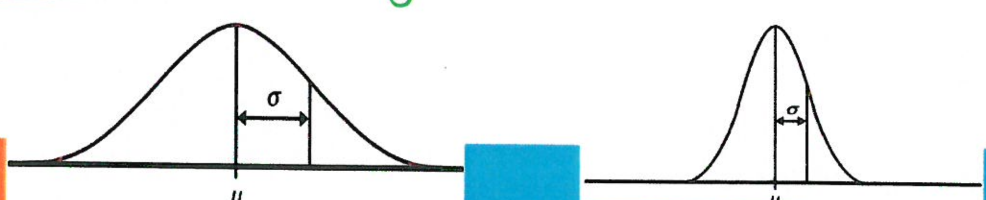
Density Curve:
An idealized description of the overall pattern of a distribution.
- Area underneath = 1
- representing 100% of observations.

7

C) NORMAL DENSITY DISTRIBUTIONS

- Normal Curves are symmetric, single-peaked, bell-shaped.
- The μ and median are the same.
- Size of the σ will affect the spread of the normal curve.

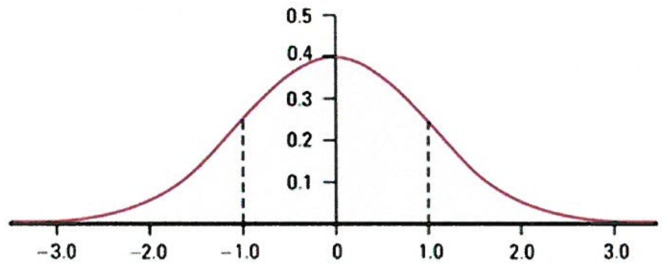
QUESTION: which graph has $\sigma = 2$ vs $\sigma = 3$?



Standard Normal Distribution

The **standard Normal distribution** is the Normal distribution N(0, 1) with mean 0 and standard deviation 1 (Figure 2.15).

Standard Normal distribution.



If a variable x has any Normal distribution $N(\mu, \sigma)$ with mean μ and standard deviation σ, then the standardized variable

$$z = \frac{x - \mu}{\sigma}$$

has the standard Normal distribution.

STANDARD NORMAL DISTRIBUTION:

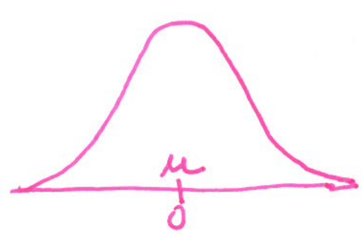
① Short hand $N(0, 1)$

Normal distribution

mean μ

S.D σ

② Graph



	Sample	Population
CENTER	$\bar{x}$	μ (mu)
SPREAD	$s = s_x$	σ (sigma)
SIZE	n	N