Lesson 9.3a Day 1– Significance Test for μ

The level of dissolved oxygen (DO) in a stream or river is an important indicator of the water's ability to support aquatic life. A researcher measures the DO level at 30 randomly chosen locations along a stream. Here are the results in milligrams per liter (mg/l): $\bar{x} = 4.77$ and $s_x = 0.939$. An average dissolved oxygen level below 5 mg/l puts aquatic life at risk. Do the data provide convincing evidence at the α = 0.05 significance level that aquatic life in this stream is at risk?

1. What are the sample mean and sample standard deviation (using correct notations)?

$\bar{x} = 4.77$ mg/L $\qquad s_x = 0.939$ mg/L

2. State the appropriate hypotheses for a significance test. Be sure to define the parameter of interest.

μ = TRUE MEAN DO LEVEL

$H_0: \mu = 5$

$H_A: \mu < 5$ (FISH LIFE AT RISK)

$\alpha = 0.05$

3. What conditions must be met? Check them.

- Randomly chosen locations
- 10%: $n = 30 \le \frac{1}{10}$(all locations)
- NORMAL: $n = 30 \ge 30$ (CLT)

under H_0

4. Give the formulas for the mean and standard deviation of the sampling distribution of $\bar{x}$ and calculate the values.

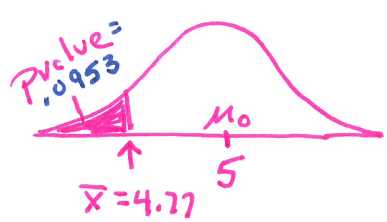
$\mu_{\bar{x}} = \mu = 5$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \approx \frac{s_x}{\sqrt{n}} = \frac{0.939}{\sqrt{30}} = 0.1714$

5. Draw a picture and then calculate the test statistic. Remember, since we are working with means, the test statistic is a *t* value

$\boxed{\text{1 sample } t\text{-test for } \mu}$

H_0:



$t = \frac{\bar{x} - \mu}{s_x/\sqrt{n}} = \frac{4.77 - 5}{0.939/\sqrt{30}} = -1.34 \quad \boxed{t = -1.34}$

6. . Find the P-value for your test statistic

Pvalue = $P(t \le -1.34) = .0953$ ← tcdf(−∞, −1.34, 29)

$df = 30 - 1 = 29$

$\boxed{\text{Pvalue} = 0.0953}$

7. What is the interpretation of the p-value?

ASSUMING THE MEAN DO LEVEL IS 5mg/l, there is a .095 probability of getting a sample mean of 4.77 mg/l OR LESS purely by chance.

8. What conclusion can we make?

Since the pvalue (.095) is greater than $\alpha = 0.05$, We fail to reject Ho. We do not have convincing evidence the mean Do Level is less than 5mg/l.

AP FRQ Requirements:

Section 1:	Define Parameter:	Name test
	Hypotheses:	α Level:
Section 2:	Check conditions (random, normal, 10%-condition/Independent)	
	Plug-ins	Work
	Test Statistic:	P-value:
Section 3:	Conclude	
	p-value, alpha, decision, conclusion in context	

Important ideas:

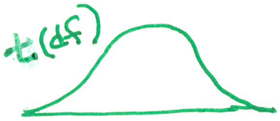
#1 CONDITIONS

* Random
* INDEPENDENT
  $10\%: n < \frac{1}{10}N$
* NORMAL
  - STATED
  - CLT
  - GRAPH - NO STRONG SKEWNESS OR OUTLIERS

#2 TEST STATISTIC

$$t = \frac{\bar{x} - \mu}{sx/\sqrt{n}}$$

t(df)



#3 PVALUE

tail probability
W/ DF = n-1

tcdf(LB, UB, df)

Example #2 *"Construction zones"*

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27 33 32 21 30 30 29 25 27 34

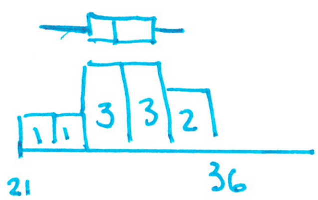
(a) Can we conclude that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

- *Parameter of Interest:* μ = TRUE MEAN SPEED OF DRIVERS IN A 25 MPH CONSTRUCTION ZONE
- *Null Hypothesis:* $H_0: \mu = 25$ (POSTED SPEED LIMIT)
- *Alternative Hypothesis:* $H_A: \mu > 25$ (ARE DRIVERS SPEEDING)
- *Level of Significance:* $\alpha = .05$
- *Choice of Test:* 1 SAMPLE t-test for means
- *Conditions of Test:*

① Random SAMPLE OF 10 DRIVERS

② INDEPENDENT: 10% $n = 10 < \frac{1}{10}$ (all drivers)

③ NORMAL: SMALL SAMPLE. THE GRAPH SHOW NO APPARANT OUTLIER OR LARGE SKEWNESS ⟶ t-test OKAY ✓



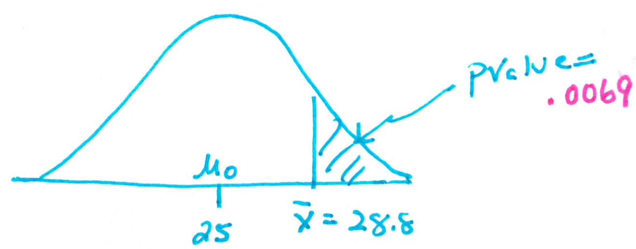
- *Sampling Distribution (Sample statistics and sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean)*

$n = 10$

$\bar{x} = 28.8$

$S_x = 3.94$

H_0:



- *Test Statistic (clearly show calculation)*

$$t = \frac{28.8 - 25}{3.94/\sqrt{10}} = 3.05$$

$\underline{\underline{df = 9}}$

- *P-value (Use correct probability notation.)* pvalue = $P(t \geq 3.05) = .0069$

Example #2 (CONTINUED) *"Construction zones"*

- *Conclusions (decision in context)*

Since the pvalue (.0069) is less than $\alpha = .05$, we Reject H_0.

WE HAVE CONVINCING EVIDENCE THE MEAN DRIVING SPEED IS GREATER THAN 25mph IN CONSTRUCTION ZONES.

(b) Given your conclusion in part (a), which kind of mistake—a Type I or a Type II error—could you have made? Explain what this mistake means in this context.

SINCE WE REJECTED H_0, WE MAY HAVE MADE A TYPE I ERROR.

MISTAKE: IT IS POSSIBLE THAT WE CONCLUDED THE AVERAGE SPEED WAS MORE THAN 25 mph, WHEN IN FACT THE AVERAGE SPEED IS 25mph.

A POSSIBLE CONSEQUENCE IS MORE POLICE ARE HIRED WASTING TAX PAYERS MONEY

Example #3 *"Don't break the ice"* **2-Sided Test of Significance for Means**

In the children's game Don't Break the Ice, small plastic ice cubes are squeezed into a square frame. Each child takes turns tapping out a cube of "ice" with a plastic hammer hoping that the remaining cubes don't collapse. For the game to work correctly, the cubes must be big enough so that they hold each other in place in the plastic frame but not so big that they are too difficult to tap out. The machine that produces the plastic ice cubes is designed to make cubes that are 29.5 millimeters (mm) wide, but the actual width varies a little. To make sure the machine is working well, a supervisor inspects a random sample of 50 cubes every hour and measures their width. The Fathom output below summarizes the data from a sample taken during one hour.

Collection 1

Width	29.4874 mm	mean
	50	n
	0.0934676 mm	SD
	0.0132183 mm	SE
	29.2717 mm	MIN
	29.4225 mm	Q1
	29.4821 mm	MEDIAN
	29.5544 mm	Q3
	29.7148 mm	MAX

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )
S5 = min ( )
S6 = Q1 ( )
S7 = median ( )
S8 = Q3 ( )
S9 = max ( )

Key

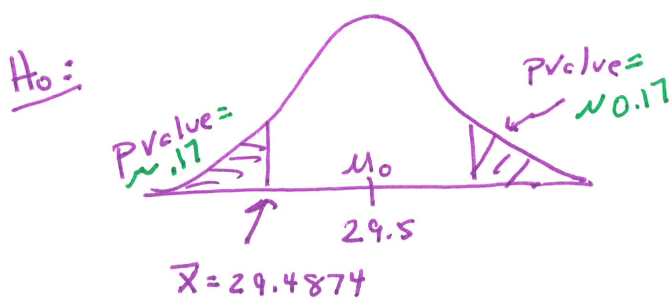
a) **Do these data give convincing evidence that the mean width of cubes produced this hour is not 29.5 mm?**

- **Parameter of Interest** μ = TRUE MEAN WIDTH OF PLASTIC ICE CUBES
- **Null Hypothesis** $H_0: \mu = 29.5$
- **Alternative Hypothesis** $H_A: \mu \neq 29.5$
- **Level of Significance** $\alpha = 0.05$
- **Choice of Test** 1 SAMPLE t-test for μ (means)
- **Conditions of Test (assume conditions have been met)** – SRS ✓, CLT ✓, 10% ✓
- **Sampling Distribution** *(Sample statistics and sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean)*

$\bar{x} = 29.4874$

$S_x = .093 \longrightarrow SE = 0.132183$

$n = 50 \longrightarrow df = 49$



- **Test Statistic** (clearly show calculation)

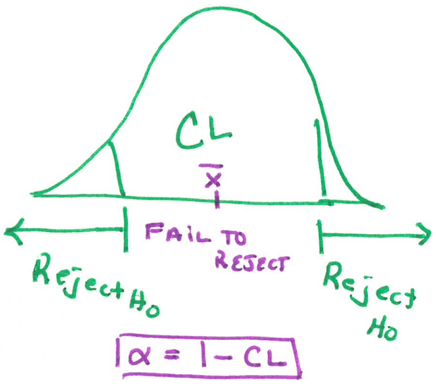
$$t = \frac{29.4874 - 29.5}{.093/\sqrt{50}} = -0.948 \qquad \boxed{t = -0.948}$$

- **P-value** (Use correct probability notation.) PVALUE $= 2 \cdot P(t \leq -0.948) = \underbrace{.3428}_{2(.1714)}$
- **Conclusions (in context)**

SINCE THE PVALUE (.3428) IS GREATER THAN $\alpha = .05$, WE FAIL TO REJECT H_0. WE DO NOT HAVE CONVINCING EVIDENCE THE TRUE MEAN WIDTH OF PLASTIC ICE CUBE IS DIFFERENT THAN 29.5MM.

Example #4 *"Don't break the ice"* **Confidence intervals for Means**

Here is computer output for a 95% confidence interval for the true mean width of plastic ice cubes produced this hour.



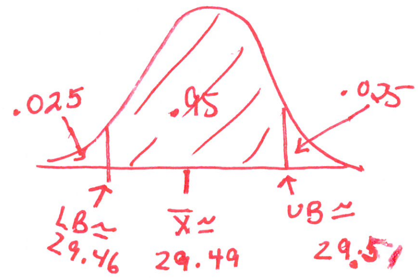
Estimate of Collection 1   Estimate Mean

Attribute (numeric): Width

Interval estimate for population mean of **Width**

Count:	50
Mean:	29.4874 mm
Std dev:	0.0934676 mm
Std error:	0.0132183 mm
Confidence level:	95.0 %
Estimate:	29.4874 mm +/- 0.0265632 mm
Range:	29.4609 mm to 29.514 mm

LB   UB



Problem:

a) Interpret the confidence interval. [Would you make the same conclusion with the confidence interval as you did with the significance test in the previous example?]

→ WE ARE 95% CONFIDENT THE TRUE MEAN WIDTH OF PLASTIC ICE CUBES IS IN THE INTERVAL 29.46 TO 29.51 mm.

A 95% CI IS EQUIVALENT TO A TOH w/ $\alpha = .05$

SINCE THE INTERVAL [29.46, 29.51] CONTAINS 29.5 mm AS A PLAUSIBLE VALUE FOR THE TRUE MEAN WIDTH ICE CUBE, WE WOULD MAKE THE SAME DECISION TO FAIL TO REJECT H_0

b) **[REVIEW QUESTION]** Interpret the confidence level.

→ 95% OF ALL POSSIBLE SAMPLES OF SIZE 50 FROM THE [PRODUCTION MACHINE] (Population) WILL RESULT IN AN INTERVAL THAT CAPTURES THE TRUE MEAN WIDTH OF PLASTIC ICE CUBES

c) **[REVIEW QUESTION]** Interpret the standard deviation and the standard error provided by the computer output.

SD = 0.093   SE = 0.013 $\bar{x}$ = 29.49   n = 50

S.D. → THE WIDTHS OF THE ICE CUBES ARE ABOUT 0.093 mm FROM THE MEAN WIDTH OF 29.49 mm, ON AVERAGE.

S.E. → IN RANDOM SAMPLES OF SIZE 50, THE SAMPLE MEAN WILL BE ABOUT 0.013 mm FROM THE TRUE MEAN, ON AVERAGE.