Section A - REVIEW Type I and Type II Errors

Test Yourself:
Fill in the table: Reject Ho, FTR Ho, Ho true, Ho False, Type I error, Type II error, Power, α, β

Truth about the population

		Ho TRUE	Ho FALSE
Conclusion based on sample	Reject Ho	TYPE I ERROR (α)	CORRECT (power = 1 − β)
	FAIL TO REJECT Ho	CORRECT	TYPE II ERROR (β)

Example "*Faster fast food?*" The manager of a fast-food restaurant wants to reduce the proportion of drive-through customers who have to wait to receive their food once their order was placed. Based on store records, 63% of customers had to wait when they got to the cashier's window. To reduce this wait time proportion, the manager assigns an additional employee to assist with drive-through orders. During the next month the manager will collect a random sample of drive-through times.

$$H_0: p = 0.63 \quad H_a: p < 0.63$$

— IMPROVED CUSTOMER SATISFACTION

where *p* = the true proportion of drive-through customers who have to wait more than 2 minutes after their order is placed to receive their food.

1) Describe **Type I Error** in this setting. Explain the financial consequence(include the impact with new employee)

   THE MANAGER FINDS CONVINCING EVIDENCE CUSTOMER SATISFACTION HAS IMPROVED WITH THE EXTRA EMPLOYEE, WHEN IN FACT 63% OF CUSTOMERS WAIT LONGER THAN 2 MINUTES. CONSEQUENCE: MGR IS SPENDING MORE $'s FOR AN ADDITIONAL EMPLOYEE AND C.S. NOT IMPROVED.

2) Describe **Type II Error** in this setting. Explain the financial consequence(include the impact with new employee)

   THE MANAGER DOES NOT FIND CONVINCING EVIDENCE CUSTOMER SERVICE HAS IMPROVED WITH EXTRA EMPLOYEE, WHEN IN FACT 63% OF CUSTOMERS WAIT LESS THAN 2 MINUTES. CONSEQUENCE: MGR FIRES ADDITIONAL EMPLOYEE AND UPSETS CUSTOMERS WITH POOR SERVICE

   WHICH IS WORSE?

   TYPE I ERROR ⟶ LOWER α
   TYPE II ERROR ⟶ INCREASE α

3) Suppose that the manager decided to carry out this test using a significance level **of α= 0.10.** He takes a random sample of **250** orders, the manager found **135** customers (about 53%) who had to wait when they got to the cashier's window to receive their food.

- Make a graph labeling the population parameter, α, the rejection region and fail to reject region.

← REJECT H₀   α=0.10   FAIL TO REJECT H₀ →

NOTES
- Research set α Before Study
- α is willingness to make TYPE I ERROR

P | .63
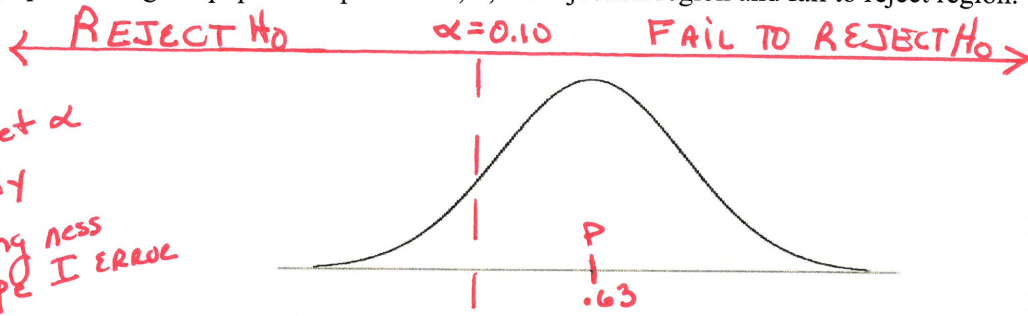
- What is the probability of a making a Type I error? $P(\text{TYPE I ERROR}) = \alpha = 0.10$
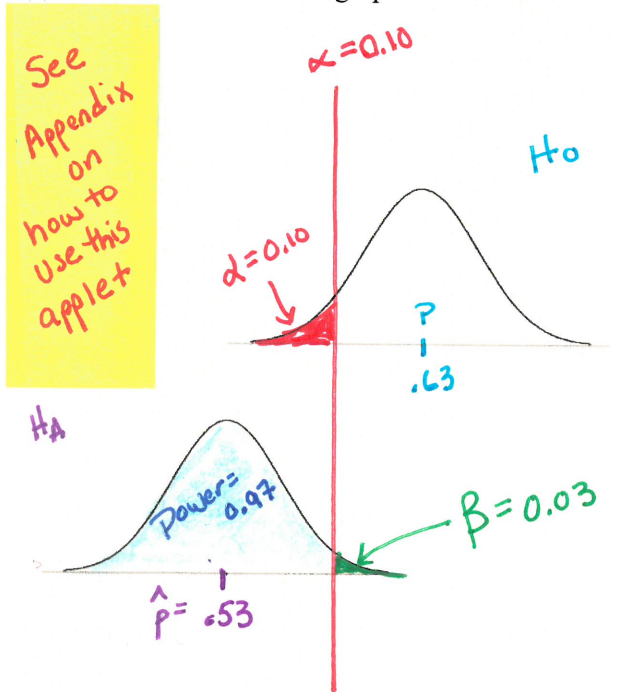
- What is the p-value for this test?

Hypothesis	Sample Results USE [1-PROP Z TEST]
$H_0: p = 0.63$	n=250 x=133 $\hat{p} = \frac{133}{250} \approx .53$
$H_a: p < 0.63$	**Find the P-value =** $P(z < -3.21) = 0.001$

- What decision would the manager make)? Since the p-value (0.001) is LESS THAN α=0.10, WE REJECT H₀. WE HAVE CONVINCING EVIDENCE THE CUSTOMER WAIT TIME

Section B - NEW CONCEPTS - Understanding Relationship between Type I and Type 2 Errors AND POWER

DEMONSTRATION: Launch applet "Improved Batting Averages (Power)" www.rossmanchance.com/applets

(a) Sketch the Ho and Ha graphs for **α= 0.10** :

See Appendix on how to use this applet

α=0.10

Ho
P | .63

α=0.10

HA
Power= 0.97
β=0.03
$\hat{p} = .53$

→ n=250 α=0.10   Power= .97   β= 0.03

(b) Significance level: To reduce the possibility of a Type I error and avoid the possibility of unnecessarily paying an extra employee, the manager **reduces the significance level from 0.10 to 0.01**
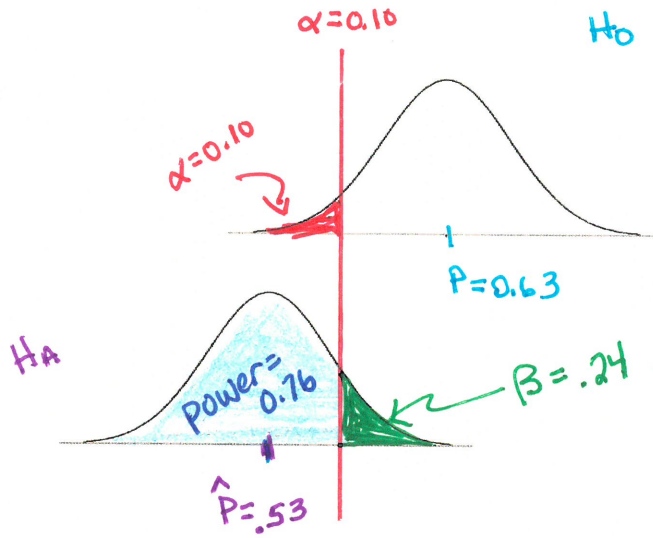
α=0.01

Ho
P | 0.63

α=0.01

HA
Power= 0.81
β=0.19
$\hat{p} = .53$
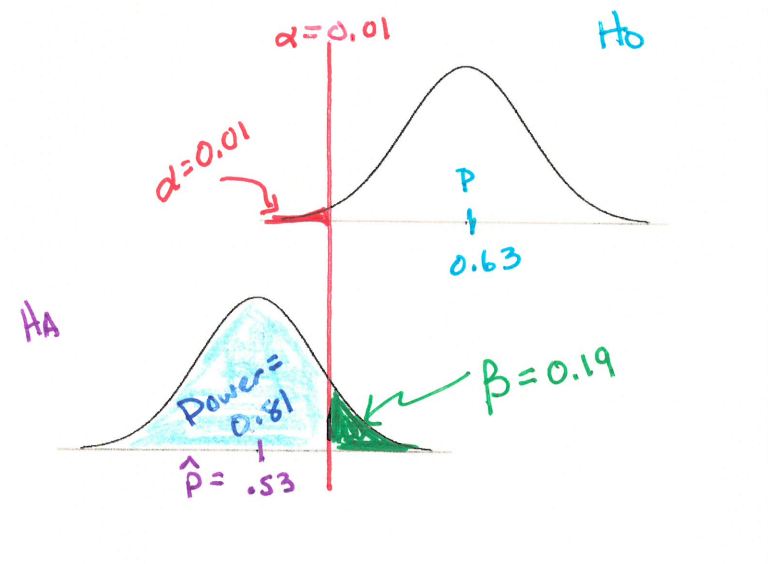
→ n=250 α= 0.01   Power= 0.81 β= 0.19

Conclusion: reduce α → α↓ β↑ Power↓

Notes
- α + β have inverse relationships
- α + Power go in same direction

(c) To get faster results, the manager reduces the sample size from 250 **to 100**. Use **α=.10**

α=0.10   Ho

α=0.10 →

P=0.63

HA

power= 0.76

$\hat{P}=.53$

β=.24

NOTICE:
n=250 ——→ POWER= 0.97
n=100↓ ——→ Power= 0.76 ↓

→ n=100 α=0.10   Power= 0.76   β= 0.24

(d) The manager takes 2 random sample of **100** customers. His first sample, he found **50** customers (**50%**) who had to wait. Use **α=.10**

α=.10   Ho

P=.63

HA   power= .91

$\hat{P}=.50$

β=.09
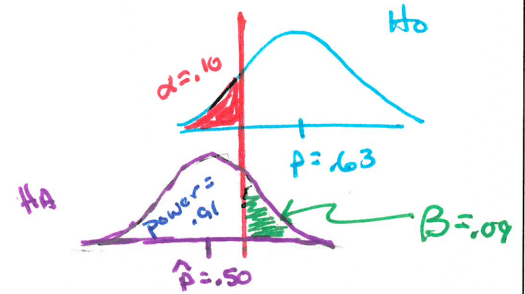
→ n=100   α=0.10   Power= 0.91   β= 0.09

(e) His next sample, He found **45** out **100** customers (**45%**) who had to wait. Use **α=.10**

α=.10   Ho

P=.63

HA   power= .99

$\hat{P}=.45$

.01
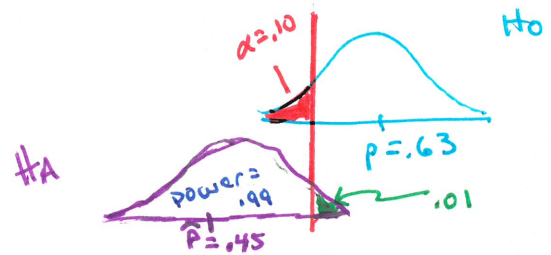
→ n=100   α=0.10   Power= 0.99   β= 0.01

CONCLUSION: NOTICE, when $\hat{p}$ is further away from p=.63, the Power gets bigger

- **Summarize what you have learned about the factors that affect the power of a test.**

What happens when you **increase α**?
α↑   THEN   β↓   AND   Power↑

What happens when you **increase sample size (n)**?
n↑   THEN   power ↑

What can researcher control to reduce errors?
③ Good EXPERIMENTAL DESIGN PRACTICES
① Sample Size   * REDUCE EXTRANEOUS FACTORS
② Predetermine α   * USE OF CONTROL/BLOCKING/STRATIFYING
* REDUCE BIAS ADMINISTER SURVEYS

- **What do you do to maximize the POWER of a test?**

1) Sample Size: ↑ POWER THEN INCREASE "n" (decreases variability)

2) Alpha Level: To ↑ POWER THEN INCREASE α (makes rejecting Ho easier)

3) Alternative P-value: Power is large as the Ha value is more extreme + further away from Ha (pop parameter)

- Definitions for **POWER** of a test

  o **Power is a correct decision**

  o **Power interpretation**: assuming the Pop. Parameter is _____, there is a ____ probability of finding convincing evidence of the alternative _____

  o **Power Probability Statement** → P(Reject Ho | Ha is true)

- **Probability Statements**

  o **P(Type I Error) = α** → P(Reject Ho | Ho is true)

  o **P(Type II Error) = β** → P(Fail to Reject Ho | Ho is false)

  o **β= 1- Power**

Part E – Power of a Test – Past Multiple Choice Questions

2002 Question #35

In a test of the hypothesis H_0: μ = 100 *versus* H_a: μ > 100, the power of the test when μ =101 would be greatest for which of the following choices of sample size *n* and significance level α?

A) *n* = 10, α = 0.05
B) *n* = 10, α = 0.01
C) *n* = 20, α = 0.05
D) *n* = 20, α = 0.01
E) It cannot be determined from the information given.

ANSWER Ⓒ — select the largest sample size (n)
— Select the largest α

2012 Admin version

20. Suppose that on a hypothesis test for a single population mean. H_a: μ < 10. Assume that H_a is true. For a fixed sample size and significance level α. the power of the test will be greatest if the actual mean is which of the following?

(A) 8
(B) 9
(C) 10
(D) 11
(E) 13

Ho
HA
$\mu = 10$
8
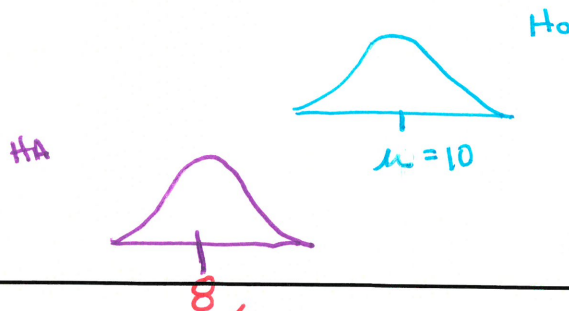
Select the mean the furthest away for Ho ($\mu=10$)

ANSWER Ⓐ

Example "*Faster fast food?*" $H_0 : p = 0.63$ versus $H_a : p < 0.63$

He found from the random sample of 250 orders, that 53% of customers waited more than 2 minutes to receive their food once their order is placed.

(a) **Reduces the significance level:** $\alpha=0.10 \rightarrow \alpha=0.01$.

Launch applet
- *Improved Batting Averages (Power)*
- www.rossmanchance.com/applets

For our test of:
Ho: $p = 0.63$
Ha: $p < 0.63$
We assume:
phat = 0.53, n = 250, α = 0.10

Step 1 - Enter
- 0.63 for the hypothesized value of p or π
- 0.53 for the alternative hypothesis
- 250 for the sample size, and
- 10,000 for the number of samples.
- **Press Draw Samples**.

Step 2 - Enter — Select "PROPORTION OF SUCCESSES"
- In the drop down menu that says "Choose option," choose Level of Significance and enter 0.10 for α.
- Press "count" — □ ← ✓ "show alternative"
- **Result: Power of the test is ~97% and β=.03**

α=0.10 [part a]

Power Simulation n = 250

Hypothesized value of π: 0.63
Alternative value of π: 0.53
Sample size: 250

Number of samples: 10000 Total = 10000

Draw Samples

Level of Significance: ▼

α = 0.10

Count

Reset

Empirical Level of Significance: 917/10000 = 0.0917

Approximate Power: 9738/10000 = 0.9738

Step 3 -
- Change the value of α = 0.01 and
- Press Count.

How does the power change?
- **Result: Power of the test is ~82% and β=.18**

* RESET AFTER EVERY SIMULATION
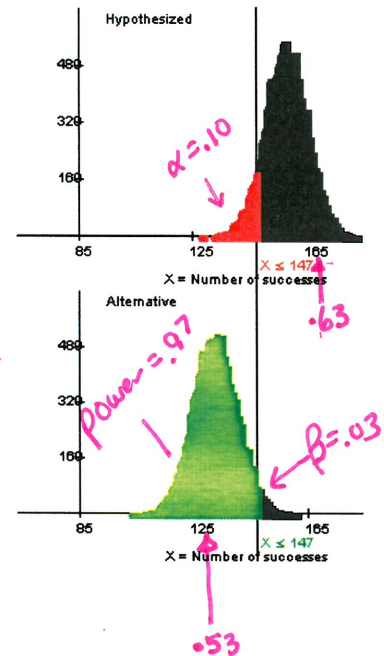

α=0.01 [part a]2

Power Simulation n = 250

Hypothesized value of π: 0.63
Alternative value of π: 0.53
Sample size: 250

Number of samples: 10000 Total = 10000

Draw Samples

Level of Significance: ▼

α = .01

Count

Reset

Empirical Level of Significance: 96/10000 = 0.0096

Approximate Power: 8155/10000 = 0.8155
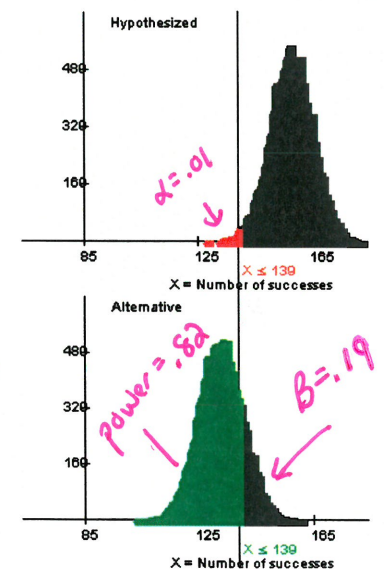
(b) Reduces the sample size: n=250 ➔ n=100. Use α=0.10

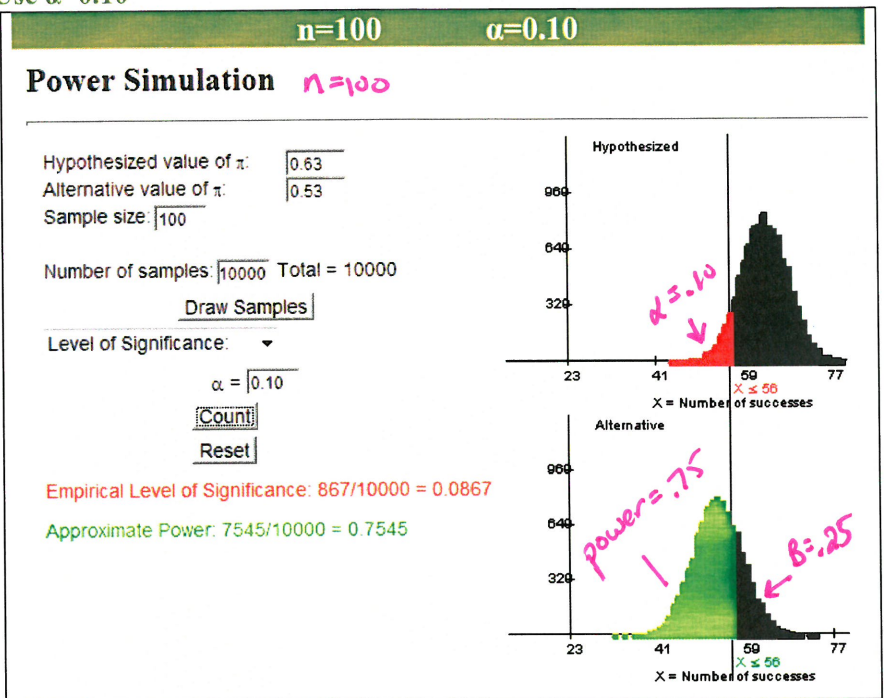| Step 4 - • Reset • Change the sample size to 100 • Draw Samples • Choose α=.10 • Press "count" **How does the power change?** • **Result: Power of the test is ~75% and β=.25**	n=100 α=0.10 **Power Simulation** n=100 Hypothesized value of π: 0.63 Alternative value of π: 0.53 Sample size: 100 Number of samples: 10000 Total = 10000 Draw Samples Level of Significance: ▼ α = 0.10 Count Reset Empirical Level of Significance: 867/10000 = 0.0867 Approximate Power: 7545/10000 = 0.7545

(c) Reduces the sample size: n=250 ➔ n=100. Use α=0.01

Step 5 - • Change the value of α = 0.01 • Press "count" **How does the power change?** • **Result: Power of the test is ~37% and β=.63**	n=100 α=0.01 **Power Simulation** n=100 Hypothesized value of π: 0.63 Alternative value of π: 0.53 Sample size: 100 Number of samples: 10000 Total = 10000 Draw Samples Level of Significance: ▼ α = 0.01 Count Reset Empirical Level of Significance: 94/10000 = 0.0094 Approximate Power: 3727/10000 = 0.3727
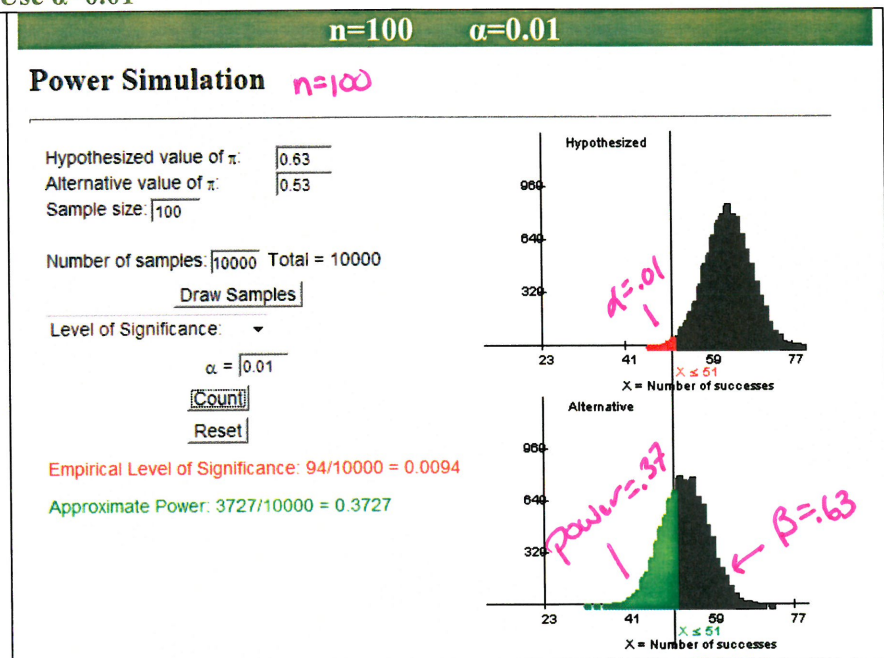
Power Simulation ▢D   n=100

Hypothesized probability of success: `0.63`
Alternative probability of success: `0.50`
Sample size: `100`
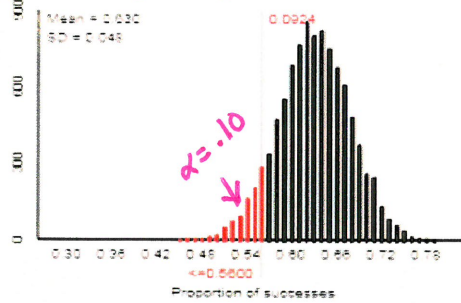Number of samples: `10000`

[Draw Samples]
Total =10000

○ Number of successes
◉ Proportion of successes
Choose one: [Level of significance ▼]
α = `0.10`
[Count]

Hypothesized: Proportion of samples:
924 / 10000 = 0.0924

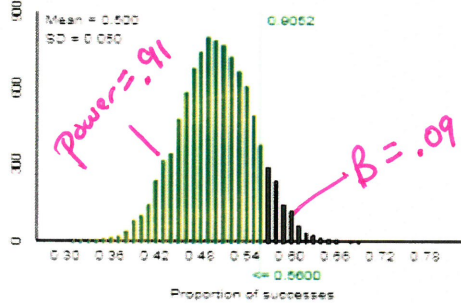**Alternative:** Proportion of samples:
9052 / 10000 = 0.9052

☐ Two-sided
☐ Exact Binomial
☐ Normal Approximation
[Reset]

Hypothesized (π = 0.5)

Mean = 0.630
SD = 0.048

0.0924

α=.10

<=0.5600

Proportion of successes

☑ Show alternative
Alternative (π = 0.5)

Mean = 0.500
SD = 0.050

0.9052

Power=.91

β = .09

<= 0.5600

Proportion of successes

☐ Show previous
☑ Summary Stats

Power Simulation ▢E   n=100

Hypothesized probability of success: `0.63`
Alternative probability of success: `0.45`
Sample size: `100`
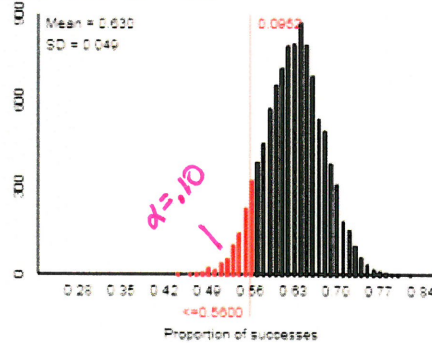Number of samples: `10000`

[Draw Samples]
Total =10000

○ Number of successes
◉ Proportion of successes
Choose one: [Level of significance ▼]
α = `0.10`
[Count]

Hypothesized: Proportion of samples:
952 / 10000 = 0.0952

Alternative: Proportion of samples:
9893 / 10000 = 0.9893

☐ Two-sided
☐ Exact Binomial
☐ Normal Approximation
[Reset]

Hypothesized (π = 0.5)

Mean = 0.630
SD = 0.049

0.0952

α=.10

<=0.5600

Proportion of successes

☑ Show alternative
Alternative (π = 0.45)

Mean = 0.450
SD = 0.050

0.9893

Power=.99

β=.01

<= 0.5600

Proportion of successes