AP Statistics – 8.2	Name: 2020 KEY
Goal: Understanding Confidence Interval (CI) for Proportions and **8.2 Guided Notes Answer Key**	Date:

I. **Definitions for: Confidence Intervals for Proportions**

1) What are the **3 conditions** that must be checked before finding a CI for proportions? %'s

RANDOM – Random Sample (SRS) OR EXPERIMENT (Random ASSIGNMENT)

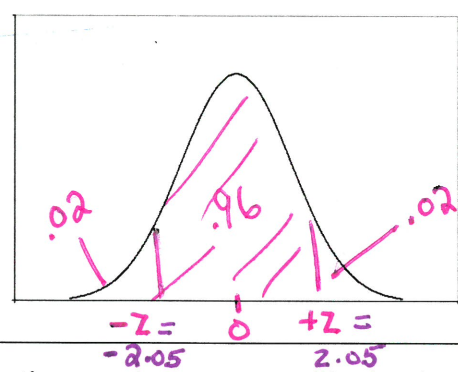
INDEPENDENT – 10% CONDITION (SAMPLING w/o REPLACEMENT)

NORMAL – $n\hat{p} \geq 10$ AND $n(1-\hat{p}) \geq 10$

2) **Critical Values**:

a) What **statistic** is used for the **critical value** to find a CI for proportions? z^* ALWAYS

b) Find the critical value for a 96% confidence Interval.

Sketch the graph.	Use calculator to find the critical value. Explain.
$N(0,1)$ 	INVNORM(tail%, 0, 1) TAIL % $= \frac{1-CL}{2} = \frac{1-.96}{2} = .02$ INVNORM(.02, 0, 1) $z^* = \pm 2.05$

c) Memorize the most common critical values. Find critical values for:

- 90% → $z^* = \pm 1.645$ INVNORM(.05, 0, 1)
- 95% → $z^* = \pm 1.96$ INVNORM(.025, 0, 1)
- 99% → $z^* = \pm 2.58$ INVNORM(.005, 0, 1)

3) What is the point estimator for proportions? the statistic $\hat{p}$

What is the point estimate for proportions? the value of $\hat{p}$

4) What is the **standard deviation** for proportions vs. **standard error** for proportions?

$SD(p) = \sqrt{\frac{p(1-p)}{n}}$ → $SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Standard error *of the* sample *proportion (*$\hat{p}$*) describes how far* $\hat{p}$ *will be from* p*, on* average*, in repeated* SRS's *of size* n.

5) What is the margin of error (ME) for proportions?

$ME = z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

ME *tells how close the* estimate *tends to be to the unknown* parameter (Pop.) *in* repeated *random samples*

6) What is formula to find CI for proportions?

1 sample z-interval for proportions: $\boxed{\hat{p} \pm z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$

II. Conditions for Confidence Intervals for Proportions (CYU on page 487):

1) Random: NOT MET. THIS IS A CONVENIENCE SAMPLE.

Independent: WE MUST ASSUME THE SCHOOL HAS MORE THAN 10(100) = 1,000 STUDENTS

Normal: $\hat{p} = 17/100 \rightarrow .17(100) = 17 \geq 10 \checkmark$

$.83(100) = 83 \geq 10 \checkmark$

2) Random: Random Sample

Independent: $n = 25 \leq \frac{1}{10}$ (thousands of bags of chips)

Normal: $\hat{p} = 3/25 \rightarrow .12(25) = 3 \geq 10$ X (NOT MET)

III. Create Confidence Intervals for Proportions (CYU on page 490):

1) p = true proportion of U.S college students who binge drink

2) Conditions:

Random sample of 10,904 U.S college students

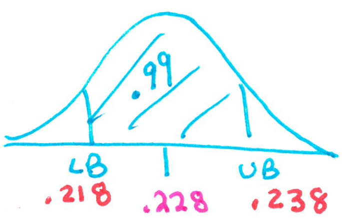
Independent: $n = 10,904 \leq \frac{1}{10}$ (population of U.S college students)

Normal: $.228(10,904) = 2486 \geq 10 \checkmark$

$.772(10,904) = 8418 \geq 10 \checkmark$

3) $CL \rightarrow 99\% \rightarrow z^* = \pm 2.58$ [INV NORM (.005, 0, 1)]

$\hat{p} = \frac{2486}{10,904} = .228$



TEST: 1 Sample Z interval for proportions

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.228 \pm 2.58 \sqrt{\frac{(.228)(.772)}{10904}} \leftarrow \boxed{SE(\hat{p}) = .004}$$

$$0.228 \pm 0.0104 \leftarrow \boxed{ME}$$

$$[0.218, 0.238]$$

Check with Calc: [STAT] [TESTS] [A: 1 PROP Z INTERVAL] X = 2486 n = 10904

Calc provides: (.21764, .23834) C-LEVEL = .99

4) WE ARE 99% CONFIDENT THAT THE INTERVAL 0.218 to 0.238 CAPTURES THE TRUE PROPORTION OF U.S. College students who binge drink.

IV. "Teen's online profiles (TPS4e #36 and 38)" Required Steps for Proportion CI:

TPS4e #36a Over half of all American Teens have an online profile, mainly Facebook. A Youth Survey asked a random sample of 487 teens with profiles found 385 included photos of themselves. Construct and interpret a 95% confidence interval.

Section 1

a) Clearly define parameters in context and confidence level (CL = 95%)

p = true proportion of American teens with online profiles WITH photos of themselves

b) Identify test by name or formula

1 sample Z-interval for proportions OR $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

c) Check conditions:

- Random: RANDOM Sample of U.S. Teens ✓
- Independent (**10% condition**): $n = 487 \leq \frac{1}{10}$ (US TEEN POP.) ✓
- Normal: $.791(487) = 385 \geq 10$ ✓ AND $.209(487) = 102 \geq 10$ ✓

Section 2 Mechanics

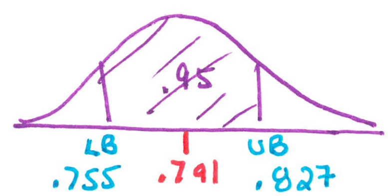
a) Plug-ins:

n = 487

$\hat{p} = 385/487 = .791$

z* = 1.96

b) Sketch the normal graph:



c) General and specific formula:

$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \Rightarrow .791 \pm 1.96 \sqrt{\frac{(.791)(.209)}{487}}$

$.791 \pm .036$

$SE(\hat{p}) = .0184$

d) Confidence interval: [0.755 , 0.827]

Section 3 **Interpret in context** using the stem of the problem

(and may be asked to just claim based on CI for population proportion)

WE ARE 95% CONFIDENT THAT THE INTERVAL 0.755 AND 0.827 CAPTURES THE TRUE PROPORTION OF AMERICAN TEENS WITH ONLINE PROFILES WITH PHOTOS OF THEMSELVES (continued) **"Teen's online profiles"**

TPS4e #36b Based on your CI calculated, is it plausible that the true proportion of American teens with online profile who have posted photos of themselves is .75?

Since 0.75 is NOT included in our 95% CI [.755, .827]
IT WOULD BE SURPRISING IF THE TRUE PROPORTION
OF TEEN OLINE USERS WITH PHOTOS is 0.75.

TPS4e #38 Describe a possible source of error that is not included in the margin of error for the 95% CI you calculated.

THE margin of Error was 0.018.
ME DOES NOT INCLUDE ANY BIAS THAT
OCCURS FROM DATA COLLECTION.
IN THIS EXAMPLE, SOURCES OF BIAS COULD
RESULT FROM UNDER COVERAGE OR NON-RESPONSE.

V. Sample Sizes for Proportions Examples –

a) What is the formula to calculate sample size for **proportions**? $ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

b) What do you use if $\hat{p}$ is not given? $\hat{p} = .5$

This is called the Conservative sample size because it is the largest "n"

c) How do you round sample size calculations in AP Stats? Round up ↑

Why? always round up to insure ME is met.

d) CYU on page 494:

1)	2)
95% CL	99% CL
$ME = .03 \quad \hat{p} = 0.8 \quad z^* = 1.96$	$ME = 0.03 \quad \hat{p} = 0.8 \quad z^* = 2.58$
$.03 \geq 1.96\sqrt{\frac{(.8)(.2)}{n}}$	$.03 \geq 2.58\sqrt{\frac{(.8)(.2)}{n}}$
$\frac{.03\sqrt{n}}{.03} \geq \frac{1.96\sqrt{(.8)(.2)}}{.03}$	$\frac{.03\sqrt{n}}{.03} \geq \frac{2.58\sqrt{(.8)(.2)}}{.03}$
$(\sqrt{n})^2 \geq (26.13)^2$	$(\sqrt{n})^2 \geq (34.4)^2$
$n \geq 682.95$ (round up)	$n \geq 1{,}183.36$ (round up)
Sample size $n = 683$	Sample size $n = 1{,}184$

IMPORTANT: Increasing the CL ↑ THEN Sample size increases ↑

APPENDIX: 8.2 Guided Notes Answer Key

Example on PG491

"Teens Say Sex Can Wait" The Gallup Youth Survey asked a random sample of 439 U.S. teens aged 13 to 17 whether they thought young people should wait to have sex until marriage. Of the sample, 246 said "Yes." Construct and interpret a 95% confidence interval for the proportion of all teens who would say "Yes" if asked this question.

Section 1

a) Clearly define parameters in context and confidence level (CL= 95%)

p= true proportion US teens who thought young people should wait to have sex UNTIL MARRIAGE.

b) Identify test by name or formula

$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ OR 1 sample Z interval for proportions

c) Check conditions:

- Random: Random sample of 439 U.S. teens ✓
- Independent (**10% condition**): $n = 439 \leq \frac{1}{10}$ (U.S teen population) ✓
- Normal: $.56(439) = 246 \geq 10$ ✓ AND $.44(439) = 193 \geq 10$ ✓

Section 2 Mechanics

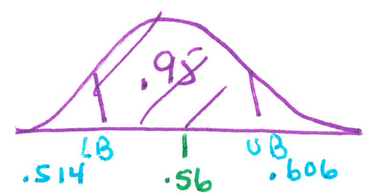
a)Plug-ins:

n= 439

$\hat{p}$ = $246/439 = .56$

z*= ± 1.96

b)Sketch the normal graph:



c) General and specific formula:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.56 \pm 1.96 \sqrt{\frac{(0.56)(0.44)}{439}}$$

$$0.56 \pm 0.046$$

d)Confidence interval: [0.514 , 0.606]

Section 3 **Interpret in context** using the stem of the problem

(and may be asked to just claim based on CI for population proportion)

WE ARE 95% CONFIDENT THAT THE INTERVAL 0.514 TO 0.606 CAPTURES THE TRUE PROPORTION OF TEENS (13-17) IN THE U.S. would say that YOUNG PEOPLE SHOULD WAIT TO HAVE SEX UNTIL MARRIAGE

Example on PG493

A company has received complaints about its customer service. The managers intend to hire a consultant to carry out a survey of customers. Before contacting the consultant, the company president wants some idea of the sample size that they will be required to pay for. One critical question is the degree of satisfaction with the company's customer service, measured on a 5-point scale. The president wants to estimate the proportion p of customers who are satisfied (that is, who choose either "satisfied" or "very satisfied," the 2 highest levels on the 5-point scale).

The president wants the estimate to be within 3% (.03) at a 95% confidence level. How large a sample is needed?

$ME = .03$

95% CL $\longrightarrow$ $z^* = 1.96$

$\hat{p}$ NOT Given $\longrightarrow$ $\hat{p} = 0.5$

$$\boxed{ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}}$$

$$.03 \geq 1.96 \sqrt{\frac{(.5)(.5)}{n}}$$

$$\frac{\cancel{.03}\sqrt{n}}{\cancel{.03}} \geq \frac{1.96\sqrt{(.5)(.5)}}{.03}$$

$$(\sqrt{n})^2 \geq (32.6\overline{6})^2$$

$$n \geq 1067.11 \text{ (ROUND UP)}$$

$\boxed{\text{We need a sample size of 1,068.}}$