Review ToH + CI (Chapters 8-10)

	ToH	CI
Means		
1 Sample	✓	✓
2 Sample	✓	✓
Paired data	✓	
Proportions		
1 sample	✓	✓
2 Sample	✓	✓

Practice Free Response:

⑤ 2005 Q4 — 1 Sample ToH for p

③ 2009B Q5 — 1 Sample ToH for μ

① 2009B Q3 — 2 Sample ToH for $p_1 - p_2$

② 2006 Q4 — 2 Sample — Interval for $\mu_1 - \mu_2$

④ 2007B Q5 — 2 Sample — ToH for $\mu_1 - \mu_2$

⑥ 2007 Q4 — Paired t-test

"FRAPPY"
{Free Response AP Problem...Yay!}

A French study was conducted in the 1990s to compare the effectiveness of using an instrument called a cardiopump with the effectiveness of using traditional cardiopulmonary resuscitation (CPR) in saving lives of heart attack victims. Heart attack patients in participating cities were treated with either a cardiopump or CPR, depending on whether the individual's heart attack occurred on an even-numbered or an odd-numbered day of the month. Before the start of the study, a coin was tossed to determine which treatment, a cardiopump or CPR, was given on the even-numbered days. The other treatment was given on the odd-numbered days. In total, 754 patients were treated with a cardiopump, and 37 survived at least one year; while 746 patients were treated with CPR, and 15 survived at least one year.

Scoring:

(a) The conditions for inference are satisfied in the study. State the conditions and indicate how they are satisfied.

E   P   I

(b) Perform a statistical test to determine whether the survival rate for patients treated with a cardiopump is significantly higher than the survival rate for patients treated with CPR.

E   I

E   I

E   I

Total:___/4

Cardio Pump vs CPR

#1 2009 B Q3 – 2 Sample TOH for $P_1 - P_2$

(A) P_1 = Proportion of Patients surviving w/ Cardio Pump
P_2 = Proportion of Patients surviving w / CPR

$H_0: P_1 = P_2$ or $H_0: P_1 - P_2 = 0$ $\boxed{\alpha = .05}$
$H_A: P_1 > P_2$ $H_A: P_1 - P_2 > 0$

(B) 2 Sample Z-Test for $P_1 - P_2$

Conditions
  Random – Treatments Randomly Assigned
        By Tossing A Coin
  Independent – Treatments are independent
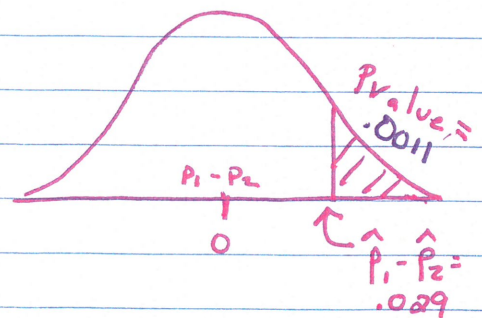   Because This is a well designed experiment
  Normal –   Cardio Pump   vs   CPR
              $37 \geqslant 10$ ✓ $15 \geqslant 10$ ✓
              $717 \geqslant 10$ ✓ $731 \geqslant 10$ ✓

(C) $\hat{P}_1 = .049$ $\hat{P}_2 = .020$
    $n_1 = 754$ $n_2 = 746$

$\hat{P}_1 - \hat{P}_2 = .049 - .020 = .029$

$\hat{P}_c = \dfrac{37 + 15}{754 + 746} = .035$ | must use pooled $\hat{P}$

P value = .0011
$P_1 - P_2$
0
$\hat{P}_1 - \hat{P}_2 = .029$

$Z = \dfrac{(.049 - .020) - 0}{\sqrt{(.035)(.965)(1/754 + 1/746)}} = 3.06$ ← | GET FROM CALC

$P(Z \geqslant 3.06) = .0011$

(D) SINCE THE PVALUE (.0011) IS LESS THAN $\alpha = .05$, WE REJECT H_0.

WE HAVE CONVINCING EVIDENCE TO SUPPORT THE CONCLUSION THAT THE PROPORTION OF PATIENTS WHO SURVIVE WHEN TREATED WITH THE CARDIO PUMP IS HIGHER THAN THE PROPORTION OF PATIENTS WHO SURVIVE WHEN TREATED WITH CPR.

① Additional Questions For 2009B Q3

QUESTIONS

Ⓐ FOR YOUR TOH, CAN YOU COMPLETE A COMPARABLE CI?

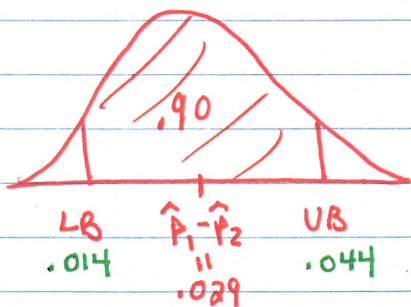∴ NO. A CI can only be compared to a 2-TAIL TEST.

Ⓑ CALCULATE AND INTERPRET THE DIFFERENCE OF PROPORTIONS.

∴ $\hat{p}_1 - \hat{p}_2 = .049 - .020 = .029$

IN REPEATED SAMPLES, THE PROPORTION OF PATIENTS SURVIVING TREATED WITH THE CARDIO PUMP IS 2.9% HIGHER THAN PATIENTS TREATED WITH CPR, IN THE LONG RUN

C+D USE CALC + ONLY SKETCH GRAPHS
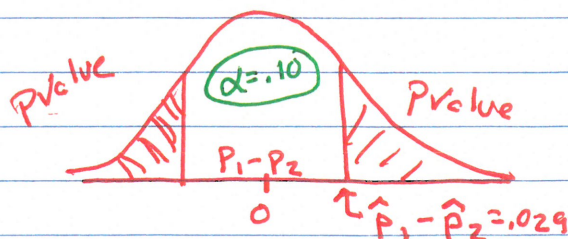
Ⓒ USE CALC TO FIND A 90% CI:

LB $\hat{p}_1 - \hat{p}_2$ UB
.014 $=$ .044
            .029

DECISION:

Reject H_0, because zero is NOT in CI

Ⓓ USE CALC TO DO A 2-TAIL TOH:

$H_0: p_1 = p_2$ $H_A: p_1 \neq p_2$

Pvalue   α=.10   Pvalue

$p_1 - p_2$

0        $\hat{p}_1 - \hat{p}_2 = .029$

Z=3.066   pvalue= .0022

DECISION:

pvalue < α=.10 Reject H_0

THESE ARE COMPARABLE

"FRAPPY"
{Free Response AP Problem...Yay!}

2006 Problem 4

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Patients with heart-attack symptoms arrive at and emergency room either by ambulance or self-transportation provided by themselves, family, or friends. When a patient arrives at the emergency room, the time of arrival is recorded. The time when the patient's diagnostic treatment begins is also recorded.

An administrator of a large hospital wanted to determine whether the mean wait time (time between arrival and diagnostic treatment) for patients with heart-attack symptoms differs according to the mode of transportation. A random sample of 150 patients with heart-attack symptoms who had reported to the emergency room was selected. For each patient, the mode of transportation and wait time were recorded. Summary statistics for each mode of transportation are show in the table below.

Mode of Transportation	Sample Size	Mean Wait Time (in minutes)	Standard Deviation of Wait Times (in minutes)
Ambulance	77	6.04	4.30
Self	73	8.30	5.16

Scoring:

(a) Use a 99 percent confidence interval to estimate the difference between the mean wait times for ambulance-transported patients and self-transported patients at this emergency room.

E   I

E   I

E   I

(b) Based only on this confidence interval, do you think the difference in the mean wait times is statistically significant? Justify your answer.

E   I

Total:___/4

② 2006 Q4 — 2 SAMPLE T INTERVAL FOR $\mu_1 - \mu_2$ |PART A|

Ⓐ μ_A = mean wait time ARRIVING BY AMBULANCE
μ_S = mean Wait time by self transportation
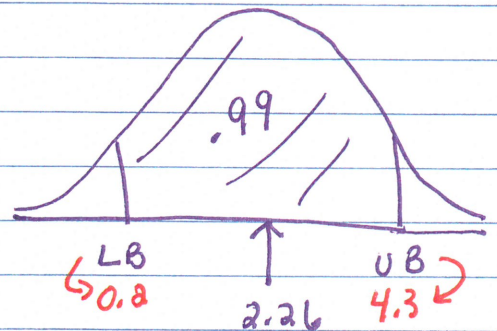
STATE: 2 SAMPLE t-INTERVAL FOR DIFFERENCE
OF MEANS.

CONDITIONS

Random — a random sample of 150 patients selected
INDEPENDENT — IT IS REASONABLE THERE ARE
MORE THAN 10(150) = 1,500 heart attack
PATIENTS TREATED BY THIS HOSPITAL
NORMAL — CLT APPLIES BECAUSE BOTH SAMPLES
WERE GREATER THAN 30 (n = 77, 73)

Ⓑ MECHANICS:

$\bar{X}_S - \bar{X}_A = 8.30 - 6.04 = 2.26$

$8.30 - 6.04 \pm t^* \sqrt{\dfrac{4.3^2}{77} + \dfrac{5.16^2}{73}}$

LB
0.2   2.26   UB
4.3

TECHNOLOGY
POOLED | NO | [.229, 4.29]
df = 140.4

BY HAND: $t^* = 2.65$ df = 73 - 1 = 72   [.199, 4.32]
write formula above and replace t^*

Ⓒ WE ARE 99% Confident that the true mean wait time difference
between ambulance and self transportation is between
0.2 and 4.3 MINUTES.

THE CONFIDENCE INTERVAL PROVIDES
EVIDENCE THE WAIT TIME IS SIGNIFICANT
BECAUSE ZERO (0) IS NOT IN OUR 99% CI.

WE WOULD REJECT H_0 AT THE $\alpha = .01$
FOR THE TEST OF HYPOTHESIS:

$$H_0: \mu_A = \mu_S$$
$$H_A: \mu_A \neq \mu_S$$

"FRAPPY"

2009B Problem 5

A bottle-filling machine is set to dispense 12.1 fluid ounces into juice bottles. To ensure that the machine is filling accurately, ever hour a worker randomly selects four bottles filled by the machine during the past hour and measures the contents. If there is convincing evidence that the mean amount of juice dispensed is different from 12.1 ounces or if there is convincing evidence that the standard deviation is greater than 0.05 ounce, the machine is shut down for recalibration. It can be assumed that the amount of juice that is dispensed into bottles is normally distributed. During one hour, the mean number of fluid ounces of four randomly selected bottles was 12.05 and the standard deviation was 0.085 ounce.

Scoring: Perform a test of significance to determine whether the mean amount of juice dispensed is different from 12.1 fluid ounces.

① **E** P I

① μ = true mean number of ounces in all juice bottles filled in the past Hour

$H_0: \mu = 12.1$
$H_A: \mu \neq 12.1$ $\alpha = .05$

② **E** P I

② TEST: 1 sample t-test for means (OR μ)

Conditions:

Random: 4 bottles were randomly selected this hour

Independent: It is reasonable more than $4(10) = 40$ bottles filled each hour.

③ **E** P I

NORMAL: assumed bottle filling is normally distributed

③ MECHANICS
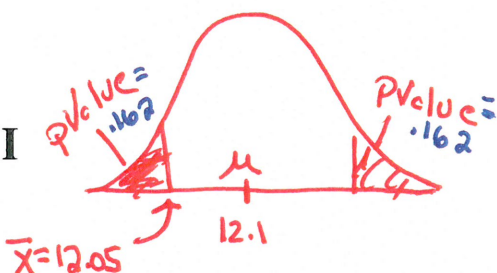
$n = 4 \longrightarrow \boxed{df = 3}$

$\bar{x} = 12.05$
$S_x = 0.085$

$t = \dfrac{12.05 - 12.1}{.085/\sqrt{4}} = -1.176$

PVALUE $= 2 \cdot P(t \leq -1.176) = .324$

④ **E** P I
ON Back

Pvalue = .162   Pvalue = .162

μ

12.1

$\bar{x} = 12.05$

Total: ___/4

(4)

Since the pvalue (.324) is GREATER THAN $\alpha = .05$, we fail to reject Ho.

There is not sufficient evidence to conclude the machine is filling the bottle with the 12.1 fluid ounces set by the machine.

2007B Problem 5

"FRAPPY"
{Free Response AP Problem...Yay!}

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

4

A serum cholesterol level above 250 milligrams per deciliter (mg/dl) of blood is a risk factor for cardiovascular disease in humans. At a medical center in St. Louis, a study to test the effectiveness of a new cholesterol-lowering drug was conducted. One hundred people with cholesterol levels between 250 mg/dl and 300 mg/dl were available for this study. Fifty people were assigned at random to each of two treatment groups. One group received the standard cholesterol-lowering medication and the other group received the new drug. After taking the drug for three weeks, the 50 subjects who received the standard treatment had a mean decrease in cholesterol level of 10 mg/dl with a standard deviation of 8 mg/dl, and the 50 subjects who received the new drug had a mean decrease of 18 mg/dl with a standard deviation of 12 mg/dl.

③ mechanics →

$n = 50$ each treatment

	Standard	New
	$\bar{x}_S = 10$	$\bar{x}_N = 18$
	$s_S = 8$	$s_N = 12$

Does the new drug appear to be more effective than the standard treatment in lowering mean cholesterol level? Give appropriate statistical evidence to support your conclusion.

①

$\mu_S =$ Mean decrease in cholesterol for standard drug

$\mu_N =$ mean decrease in cholesterol for new drug

$H_0: \mu_N = \mu_S$ $H_A: \mu_N > \mu_S$

$\alpha = .05$

Scoring:

① E P I

② 2 sample t-test for difference of means (or $\mu_1 - \mu_2$)

CONDITIONS

RANDOM: The 2 treatments were randomly assigned

INDEPENDENT: The treatments are independent (old vs new drug)

NORMAL: CLT applies. Both treatments were greater than 30.

② E P I

③ E P I

③ $t = \dfrac{18-10}{\sqrt{\dfrac{12^2}{50} + \dfrac{8^2}{50}}} = 3.92$

Pvalue $= P(t > 3.92) = .00014$

Conservative

$df = 50 - 1 = 49$
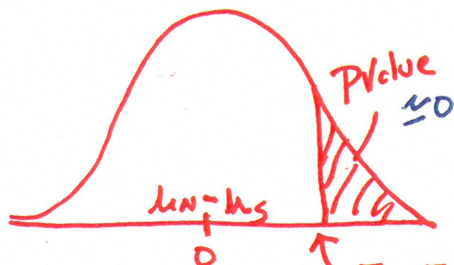
—— OR ——

Pvalue $= P(t > 3.92) = .000088$

Technology   Pooled → NO

$df = 85.4$

④ E P I

Pvalue ≈0

$\mu_N - \mu_S$ / 0
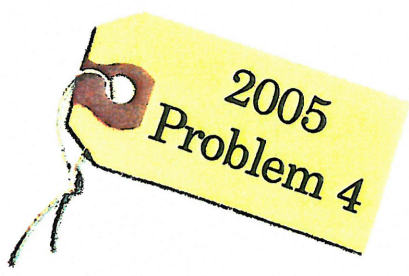
$\bar{x}_N - \bar{x}_S = 10 - 8 = 2$

Total:___/4

④ ___

4 CONCLUSION:

Since the pvalue (no) is less than $\alpha = .05$,
We reject H_0

There is convincing evidence that the mean
cholesterol reduction is greater for the new drug.

"FRAPPY"

{Free Response AP Problem...Yay!}

2005 Problem 4

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Some boxes of a certain brand of breakfast cereal include a voucher for a free video rental inside the box. The company that makes the cereal claims that a voucher can be found in 20 percent of the boxes. However, based on their experiences eating this cereal at home, a group of students believes that the proportion of boxes with vouchers is less than 0.2. This group of students purchased 65 boxes of the cereal to investigate the company's claim. The students found a total of 11 vouchers for free video rentals in the 65 boxes.

Suppose it is reasonable to assume that the 65 boxes purchased by the students are a random sample of all boxes of this cereal. Based on this sample, is there support for the students' belief that the proportion of boxes with vouchers is less than 0.2? Provide statistical evidence to support your answer.

Scoring:

A) E I

B) E I

C) E I

D) E I

[A] p = proportion of boxes of this brand of cereal with a voucher for free video rental

$H_0: p = .2$ $\alpha = .05$
$H_A: p < .2$

[B] 1 SAMPLE Z TEST FOR p

CONDITIONS:

RANDOM — RANDOM SAMPLE OF 65 BOXES

INDEPENDENT — REASONABLE THERE ARE MORE THAN $65(10) = 650$ BOXES OF THIS CEREAL

NORMAL — $65(.2) = 13 > 10$ ✓
$65(.8) = 52 > 10$ ✓

OVER →

Total:___/4

(C) $\hat{P} = \dfrac{11}{65} = .169$

$n = 65$

$Z = \dfrac{.169 - .2}{\sqrt{\dfrac{.2(.8)}{65}}} = -.62$

PVALUE
.2676

$\hat{P} = .169$ .20
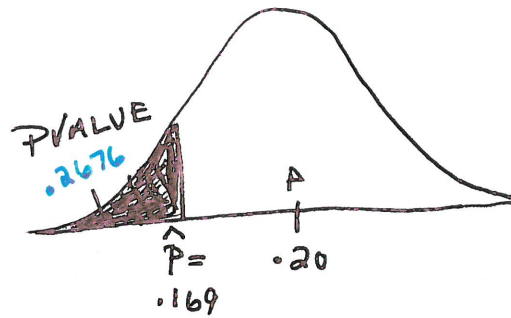
PVALUE $= P(Z < -.62) = .2676$

(D)   SINCE THE PVALUE (.2676) IS GREATER THAN $\alpha = .05$, WE FAIL TO REJECT H_o. WE DO NOT HAVE CONVINCING EVIDENCE TO SUPPORT THE STUDENT'S BELIEF THAT THE PROPORTION OF CEREAL BOXES WITH VIDEO COUPONS IS LESS THAN 20%

"FRAPPY"
{Free Response AP Problem...Yay!}

2007 Problem 4

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of *E. coli* bacteria contamination in beef. Two different methods (A and B) of determining the level of contamination were used on each of ten randomly selected specimens of a certain type of beef. The data obtained, in millimicrobes/liter of ground beef, for each of the methods are shown in the table below.

		Specimen									
		1	2	3	4	5	6	7	8	9	10
Method	A	22.7	23.6	24.0	27.1	27.4	27.8	34.4	35.2	40.4	46.8
	B	23.0	23.1	23.7	26.5	26.6	27.1	33.2	35.0	40.5	47.8

DIFF(A-B)   −.3   .5   .3   .6   .8   .7   1.2   .2   −.1   −1.0

Scoring:

Is there a significant difference in the mean amount of *E. coli* bacteria detected by the two methods for this type of beef? Provide a statistical justification to support your answer.

[A] E   I

[A] μ_D = mean difference (method A-B) in the level of E. COLI bacteria

H_0: $\mu_D = 0$
H_A: $\mu_D \neq 0$

$\boxed{\alpha = .05}$

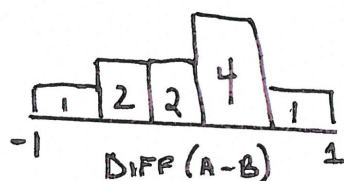[B] E   I

[B] PAIRED t-test

[C] E   I

CONDITIONS:

RANDOM: 10 Randomly selected specimens of beef

[D] E   I

INDEPENDENT: THE 10 PAIRS OF DATA ARE ASSUMED TO BE INDEPENDENT

NORMAL: HISTOGRAM OF DIFFERENCES IS SYMMETRIC WITH NO OUTLIERS
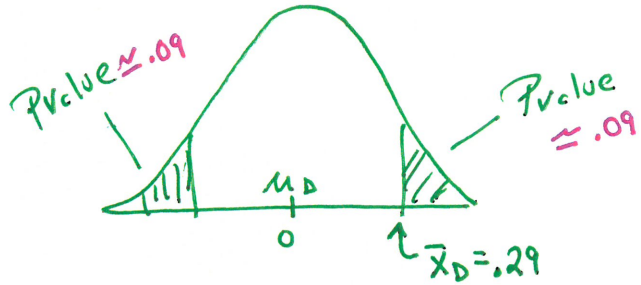

DIFF (A-B)

Total:___/4

C

$\bar{X}_D = .29$

$S_D = .6297$

$n = 10$

$df = 9$

$t = \dfrac{.29 - 0}{.6297/\sqrt{10}} = 1.46$

Pvalue $\approx .09$

Pvalue $\approx .09$

μ_D

0

$\bar{X}_D = .29$

Pvalue $= 2 \cdot P(t > 1.46) = .1793$

D SINCE THE PVALVE (.1793) IS GREATER THAN $\alpha = .05$, WE FAIL TO REJECT H_0. WE DO NOT HAVE CONVINCING EVIDENCE, THERE IS A DIFFERENCE IN THE 2 METHODS FOR MEASURING THE LEVEL OF E. COLI CONTAMINATION IN BEEF