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Section 9.3B
Inference for Means: Paired Data

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Objective

- **PERFORM significance tests for paired data are** called: **paired t procedures.**

- **Comparative studies (i.e. 2 observations on 1 individual or 1 observation on 2 similar individuals)**
  - are more convincing than single-sample investigations.
  - One-sample inference is less common than comparative inference.
  - **Study designs that involve making two observations on the same individual, or one observation on each of two similar individuals, result in paired data**.

- **Example of paired data**
  - By measuring the same quantitative variable twice, as in the job satisfaction study, we can make comparisons by analyzing the differences in each pair.
  - If the conditions for inference are met, we can **use one-sample *t* procedures to perform inference about the mean difference μ_d.**

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Paired T-Tests

Key Points

✓ If we somehow know σ, we can use a *z* test statistic and the standard Normal distribution to perform calculations.

✓ In practice, we typically do not know σ. Then, we use the **one-sample *t* statistic**

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$$

with *P*-values calculated from the *t* distribution with *n* - 1 degrees of freedom.

✓ Analyze **paired data** by first taking the difference within each pair to produce a single sample. Then use one-sample *t* procedures.

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- **Example**: Caffeine Withdrawal
- ***Carrying Out a Paired T- Test***

EXAMPLE: Researchers designed an experiment to study the effects of caffeine withdrawal. They recruited 11 volunteers who were diagnosed as being caffeine dependent to serve as subjects. Each subject was barred from coffee, colas, and other substances with caffeine for the duration of the experiment. During one two-day period, subjects took capsules containing their normal caffeine intake. During another two-day period, they took placebo capsules. The order in which subjects took caffeine and the placebo was randomized. At the end of each two-day period, a test for depression was given to all 11 subjects. Researchers wanted to know whether being deprived of caffeine would lead to an increase in depression.

Null Hypothesis (Ho)

① IF CAFFEINE DEPRIVATION HAS NO EFFECT ON DEPRESSION

② EACH SUBJECT RECEIVES BOTH TREATMENTS

③ THEREFORE Ho, we claim the mean difference in depression scores to be "0".

ALTERNATE HYPOTHESIS (HA)

Note: this is an experiment and treatments were randomly assigned

Ho Want to test Caffeine withdrawal. The higher the score indicates higher depression depriving caffeine

Results of a caffeine deprivation study

Subject	Depression (caffeine)	Depression (placebo)	Difference (placebo – caffeine)
1	5	16	
2	5	23	
3	4	5	
4	3	7	
5	8	14	
6	5	24	
7	0	6	
8	0	3	
9	2	15	
10	11	12	
11	1	0	

Put data into your calculator

L1=depression - caffeine

L2=depression - placebo

L3=L2-L1 (the difference placebo – caffeine)

Fill in the Difference column

Results of a caffeine deprivation study

Subject	Depression (caffeine)	Depression (placebo)	Difference (placebo – caffeine)
1	5	16	11
2	5	23	18
3	4	5	1
4	3	7	4
5	8	14	6
6	5	24	19
7	0	6	6
8	0	3	3
9	2	15	13
10	11	12	1
11	1	0	-1

- **Example**: Caffeine Withdrawal
- *Carrying Out a Paired T- Test*

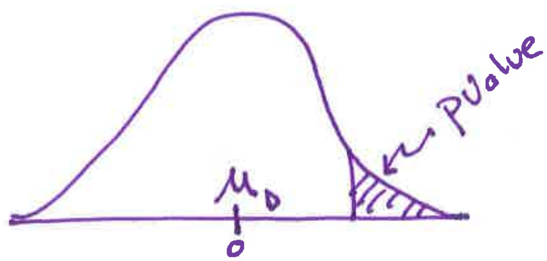
1) State Hypotheses and Sketch Graph

μ_d = the true mean difference (placebo-Caffeine) in depression score

$H_0: \mu_D = 0$ caffeine withdrawal has no effect on depression score

$H_A: \mu_D > 0$ depriving caffeine leads to an increase in depression

Set significance level: $\alpha = .05$



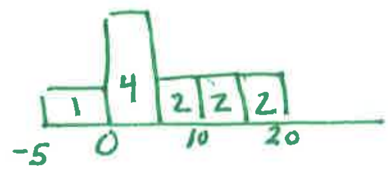
2) Check Conditions:

σ UNKNOWN → t-statistic

① Random In this experiment, subjects were randomly assign THE ORDER OF 2 TREATMENTS ① Caffeine capsule ② placebo

② INDEPENDENT WE ARE NOT SAMPLING SO WE DO NOT CHECK 10% CONDITION! We assume the experiment is conducted correctly and changes in depression scores are independent for each subject

③ NORMAL Since the sample is small (n=11) we need to look at the shape of the distribution. Reviewing a box-plot, we did NOT find any outliers. The histogram does not indicate strong skewness or outlier. The t-distribution is appropriate



3) Mechanics: Test statistic and P-value:

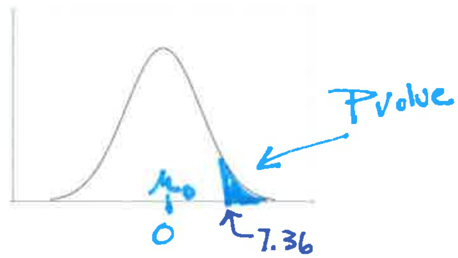
Name Test:: PAIRED t-test for difference of means (or μ_D)

Sampling Distribution – State statistic and redraw graph:

1 VAR STATS L3 →

$\bar{x}_d = 7.36$ $n = 11$

$s_d = 6.92$ $df = 10$



Calculate Test statistic

$$t = \frac{\bar{x}_D - \mu_D}{s_D/\sqrt{n}} = \frac{7.36 - 0}{6.92/\sqrt{11}} = 3.53$$

Calculate P-value (remember probability statement):

$P(t > 3.53) = .0027$ ← tcdf(3.53, E99, 10)

* ON AP EXAM must give df and probability stmt for full credit

Check with calculator: STAT> TESTS> T-Test

Stat > TESTS > 2: T-Test

Input: Data

$\mu_0 = 0$ FREQ: 1

List = L3 $> \mu_0$

$\mu > 0$
$t = 3.53$ $\bar{x} = 7.36$
$p = .0027$ $S_x = 6.917$
$n = 11$

✓ 1/2

4) Conclude:

Since the pvalue of 0.0027 is less than our chosen significance level of $\alpha = .05$, we reject the null hypothesis.

We have convincing evidence to conclude that depriving these caffeine dependent subjects of caffeine resulted in an average increase on depression score