AP Statistics – 9.3a	Name:
Goal: TOH and CI for Means (μ)	Date:

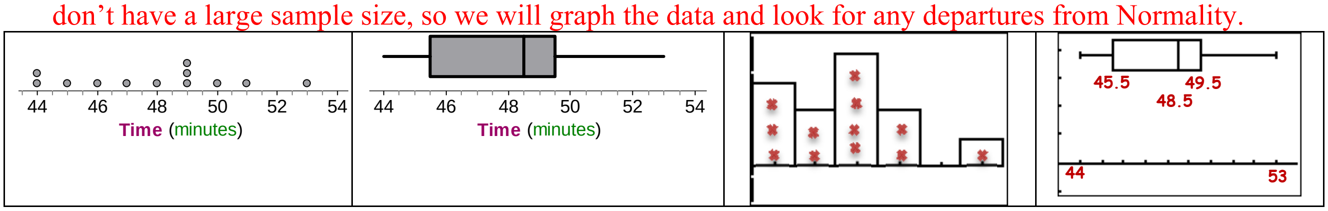
I. Left Sided Test of Significance for Means -- <u>Example #1</u> "*Less music?*"

A classic rock radio station claims to play an average of 50 minutes of music every hour. However, every time you turn to this station it seems like there is a commercial playing. To investigate their claim, you randomly select 12 different hours during the next week and record what the radio station plays in each of the 12 hours. Here are the number of minutes of music in each of these hours:

44 49 45 51 49 53 49 44 47 50 46 48

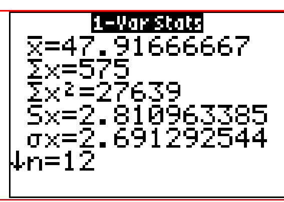
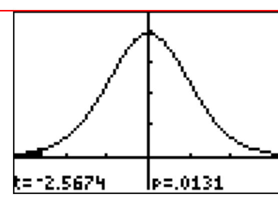
Carrying out a significance test of the company's claim that it plays an average of at least 50 minutes of music per hour

- **Parameter of Interest** μ = the true average number minutes of music played every hour.
- **Null Hypothesis** $H_0: \mu = 50$
- **Alternative Hypothesis** $H_a: \mu < 50$
- **Conditions of Test**
  - **<u>Random</u>**: A random sample of hours was selected.
  - **<u>Independent</u>**: There are more than 10(12) = 120 hours of music played during the week.
  - **<u>Population standard deviation</u>: UNKNOWN (t-test)**
  - **<u>Normal:</u>** We don't know if the population distribution of music times is approximately Normal and we don't have a large sample size, so we will graph the data and look for any departures from Normality.

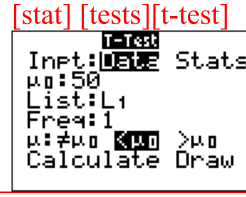
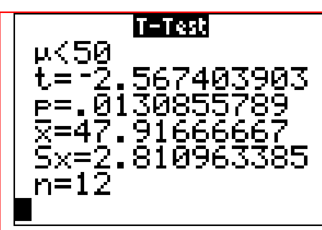


The graphs are roughly symmetric with no outliers, so it is reasonable to use t procedures for these data.

- **Level of Significance** α= 0.05 significance level
- **Choice of Test** one-sample t test for μ
- **Sampling Distribution (**Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean)

n=12 df=11 $\bar{x} = 47.9$ $s_x = 2.81$			

- **Test Statistic** (clearly show calculation)

$t = \dfrac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$	$t = \dfrac{47.9 - 50}{2.81/\sqrt{12}} = -2.59$	Calculator command [stat] [tests][t-test] 	

- **P-value** (Use correct probability notation.) *P-value =P(t < –2.59)= 0.0126* *tcdf(–e99,–2.59,11)*
- **Meaning of the P-value (Reject or Fail to reject null hypothesis) AND Conclusions (in context)**
  - Since the *P*-value(.013) is less than α=.05, **we reject the null hypothesis**.
  - **There is convincing evidence that the radio station plays less than 50min of music per hour.**

II. Right Sided Test of Significance for Means -- <u>Example #2</u> "*Construction zones*"

Every road has one at some point—construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

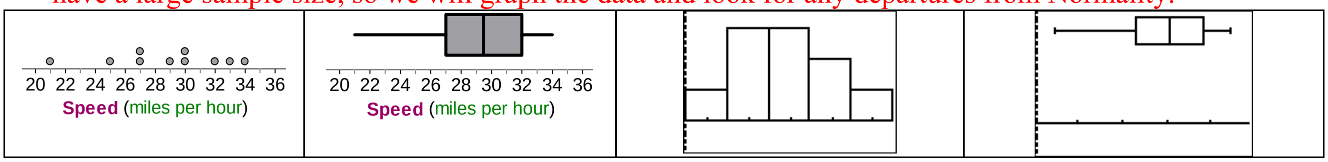
27 33 32 21 30 30 29 25 27 34

<u>Problem</u>:

(a) Can we conclude that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

(b) Given your conclusion in part (a), which kind of mistake—a Type I or a Type II error—could you have made? Explain what this mistake means in this context.

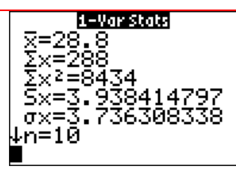
- **Parameter of Interest** μ =true mean speed of drivers in this 25 mph construction zone
- **Null Hypothesis** $H_0: \mu = 25$
- **Alternative Hypothesis** $H_a: \mu > 25$
- **Conditions of Test**
  - **<u>Random</u>**: A random sample of drivers were selected.
  - **<u>Independent</u>**: There are more than 10(10) = 100 drivers that go through this construction zone.
  - **<u>Population standard deviation</u>: UNKNOWN (t-test)**
  - **<u>Normal:</u>** We don't know if the population distribution of speeds is approximately Normal and we don't have a large sample size, so we will graph the data and look for any departures from Normality.

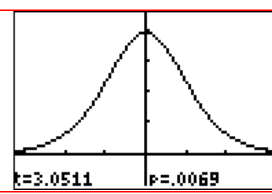


The graphs are only slightly skewed to the left with no outliers so it is reasonable to use *t* procedures for these data.

- **Level of Significance** α= 0.05 significance level
- **Choice of Test** one-sample *t* test for μ
- **Sampling Distribution (**Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean)

n=10
df=9
$\bar{x} = 28.8$ mph
$s_x = 3.94$ mph



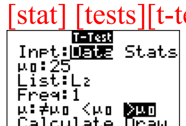


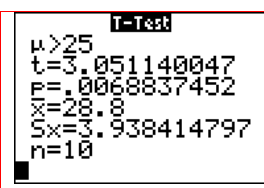
- **Test Statistic** (clearly show calculation)

Test statistic:

$$t = \frac{28.8 - 25}{3.94/\sqrt{10}} = 3.05$$

Calculator command
[stat] [tests][t-test]





- **P-value** (Use correct probability notation.) *P*-value =$P(t > 3.05) = 0.0069$ *tcdf(3.05,–e99,9)*

a) Meaning of the P-value (Reject or Fail to reject null hypothesis) AND Conclusions (in context)

Since the *P*-value is so small and less than is less than α=.05, we reject the null hypothesis. There is convincing evidence that the true average speed of drivers in this construction zone is greater than 25 mph.

b) Error Interpretation: Since we rejected the null hypothesis, it is possible that we made a Type I error. In other words, it is possible that we concluded that the average speed of drivers in this construction zone is greater than 25 mph when in reality it isn't.

III. 2-Sided Test of Significance for Means -- <u>Example #3</u> *"Don't break the ice"*

In the children's game Don't Break the Ice, small plastic ice cubes are squeezed into a square frame. Each child takes turns tapping out a cube of "ice" with a plastic hammer hoping that the remaining cubes don't collapse. For the game to work correctly, the cubes must be big enough so that they hold each other in place in the plastic frame but not so big that they are too difficult to tap out. The machine that produces the plastic ice cubes is designed to make cubes that are 29.5 millimeters (mm) wide, but the actual width varies a little. To make sure the machine is working well, a supervisor inspects a random sample of 50 cubes every hour and measures their width. The computer output below summarizes the data from a sample taken during one hour.

Collection 1

Width	29.4874 mm
	50
	0.0934676 mm
	0.0132183 mm
	29.2717 mm
	29.4225 mm
	29.4821 mm
	29.5544 mm
	29.7148 mm

S1 = mean ( )
S2 = count ( )
S3 = stdDev ( )
S4 = stdError ( )
S5 = min ( )
S6 = Q1 ( )
S7 = median ( )
S8 = Q3 ( )
S9 = max ( )

<u>Problem:</u>

(a) Interpret the standard deviation and the standard error provided by the computer output.

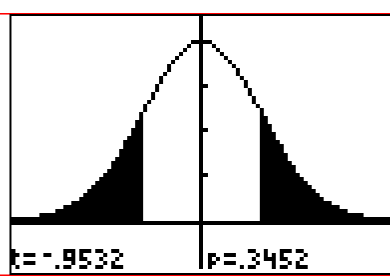
<u>Standard deviation:</u> The widths of the cubes are about 0.093 mm from the mean width, on average.

<u>Standard error:</u> In random samples of size 50, the sample mean will be about 0.013 mm from the true mean, on average.

(b) Do these data give convincing evidence that the mean width of cubes produced this hour is not 29.5 mm?

- **Parameter of Interest** μ = the true mean width of plastic ice cubes
- **Null Hypothesis** $H_0: \mu = 29.5$
- **Alternative Hypothesis** $H_a: \mu \neq 29.5$
- **Conditions of Test**
  - **<u>Random</u>**: A random sample of plastic ice cubes was selected.
  - **<u>Independent</u>**: It is reasonable to assume that there are more than 10(50) = 500 cubes produced by this machine each hour.
  - **<u>Population standard deviation</u>: UNKNOWN (t-test)**
  - **<u>Normal:</u>** We have a large sample size (n = 50 > 30), so it is OK to use t procedures.
- **Level of Significance** α= 0.05 significance level
- **Choice of Test** one-sample t test for μ
- **Sampling Distribution** (Sketch of the sampling distribution of the sample statistic under the null hypothesis, indicating the mean)

n=50
df=49
$\bar{x} = 29.4874$
$s_x = .09347$

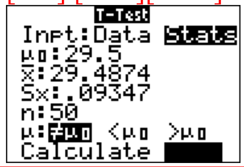


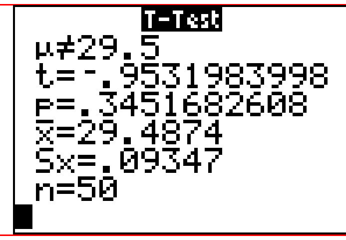
- **Test Statistic** (clearly show calculation)

Test statistic:

$$t = \frac{29.4874 - 29.5}{0.09347/\sqrt{50}} = -0.95$$

Calculator command
[stat] [tests][t-test]





- **P-value** (Use correct probability notation.)

  P-value = 2 * $P(t < -0.95)$= *.1734 * 2 = 0. 3468* *tcdf(–e99,–.95,49)*

  or

  ***P*-value = $P(t < -0.95)$ or $P(t > 0.95)$ = .1734 +.1734 = 0. 3468**

- **Meaning of the P-value (Reject or Fail to reject null hypothesis) AND Conclusions (in context)**
  - Since the *P*-value is so large and greater than is less than α=.05, **we fail to reject the null hypothesis**.
  - **There is not convincing evidence that the true width of the plastic ice cubes produced this hour is different from 29.5 mm.**

IV. Confidence intervals for Means -- <u>Example #3(continued)</u> *"Don't break the ice"*

Here is computer output for a 95% confidence interval for the true mean width of plastic ice cubes produced this hour.

```
Estimate of Collection 1                         Estimate Mean
Attribute (numeric): Width
Interval estimate for population mean of Width

Count:               50
Mean:                29.4874 mm
Std dev:             0.0934676 mm
Std error:           0.0132183 mm
Confidence level:    95.0 %
Estimate:            29.4874 mm +/- 0.0265632 mm
Range:               29.4609 mm to 29.514 mm
```

<u>Problem:</u>

a) Interpret the confidence interval. Would you make the same conclusion with the confidence interval as you did with the significance test in the previous example?

- We are 95% confident that the interval from 29.4609 mm to 29.514 mm captures the true mean width of plastic ice cubes produced this hour.
- Since the interval includes 29.5 as a plausible value for the true mean width, we do not have convincing evidence that the true mean is not 29.5 mm.
- This is the same conclusion we made in the significance test earlier.

b) Interpret the confidence level.

- If we were to take many random samples of 50 plastic ice cubes and make 95% confidence intervals for the true mean width of the cubes, then about 95% of the intervals we construct will include the true mean width.