6.7 Graph Linear Inequalities in Two Variables

Goal: Graph linear inequalities in two variables.

VOCABULARY

Linear inequality in two variables are the same as linear equations but have inequality symbols (≥, ≤, >, <).

Graph of an inequality in two variables:
Graph the linear equation and shade all the solutions for the inequality.

Example 1 Check solutions of a linear inequality
Tell whether the ordered pair is a solution of $3x - 4y > 9$.

a. Test (2, 0):

$3x - 4y > 9$  \hspace{1cm} Write inequality.

$3(2) - 4(0) > 9$  \hspace{1cm} Substitute 2 for $x$ and 0 for $y$.

$6 > 9$  \hspace{1cm} Simplify.

(2, 0) IS NOT a solution.

b. Test (2, -1):

$3x - 4y > 9$  \hspace{1cm} Write inequality.

$3(2) - 4(-1) > 9$  \hspace{1cm} Substitute 2 for $x$ and -1 for $y$.

$6 + 4 > 9$  \hspace{1cm} Simplify.

(2, -1) IS a solution.

c. Test the origin (0,0)

$3(0) - 4(0) > 9$  \hspace{1cm} The origin is not a solution.

$0 > 9$
GRAPHING A LINEAR INEQUALITY IN TWO VARIABLES

**Step 1** Graph the boundary line. Use a **dashed** line for < or >, and use a **solid** line for ≤ or ≥.

**Step 2** Test a point not on the line by checking whether the ordered pair is a solution of the inequality. Using (0, 0) is the easiest point to test.

**Step 3** Shade the **half plane** containing the point if the ordered pair is a solution of the inequality. Shade the **other half plane** if the ordered pair is not a solution.

**Example 2** Graph a linear inequality in two variables

Graph the inequality \( y < -\frac{1}{2}x + 4 \).

**Solution**

1. Graph the equation \( y = -\frac{1}{2}x + 4 \). The inequality is \(<\), so use a **dashed** line.

2. Test \((0, 0)\) in \( y < -\frac{1}{2}x + 4 \).

\[
0 < -\frac{1}{2}(0) + 4 \\
0 < 4 \text{ TRUE}
\]

3. Shade the half-plane that contains \((0, 0)\) because \((0, 0)\) is a solution of the inequality.

**Graph using Any Method**

1. Create a table

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>?</td>
</tr>
<tr>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>1</td>
<td>?</td>
</tr>
</tbody>
</table>

* Pick 3 easy pts:
  * \( x = -1, 0, 1 \)
  * \( y = \frac{1}{3}x + 5 \)
  * Use multiples of denominator (3):
    * \( x = -3, 0, 3 \)

2. Graph using \( y = mx + b \)

   * Plot y-intercept \((0, b)\)
   * Use slope to find additional points

   \( m = \frac{\text{Rise}}{\text{Run}} \)

3. Graph using \( x \) and \( y \) intercepts

   \( 5x - 2y = 10 \)
   
   \[ x = 2 \quad (2, 0) \]
   
   \[ y = 5 \quad (0, -5) \]

   **Solutions are all the points in this half-plane.**
Example 3  Graph a linear inequality in one variable

Graph the inequality $x \geq 4$.  

**Solution**

1. Graph the equation $x = 4$. The inequality is $\geq$, so use a **solid** line.

2. Test $(0, 0)$ in $x \geq 4$. You only substitute the $x$-coordinate because the inequality does not have the variable $y$.

   $0 \geq 4$ **FALSE**

3. Shade the half-plane that does not contain $(0, 0)$, because $(0, 0)$ is not a solution of the inequality.

**Checkpoint** Graph the inequality.

1. $2y + 4x > 8$

2. $y < 2$

**Homework**

- What side to shade? Pick a pt $(0, 0)$
  
  $2(0) + 4(0) > 8$

  $0 \geq 8$ **F**

  Shade the top half
Tell whether the ordered pair is a solution of the inequality.

1. $x + y > -9; (0, 0)$
   \[0 + 0 > -9\]
   \[0 > -9\]
   **Solution**

Graph the inequality.

10. $y - x < 6$

11. $x - y > -4$

12. $2y - x < 2$

13. $4y \leq 6x - 12$

14. $5y \leq 10x + 15$

15. $-6y + 6 \geq -18x$

19. $y > 7$

20. $x \leq -5$

14. Test $(0, 0)$
   \[0 \leq -12\] (F)

15. Test $(0, 0)$
   \[-6(0) + 6 \geq -18(0)\]
   \[6 \geq 0\] (F)

19. $y > 7$

20. $x \leq -5$
Tell whether the ordered pair is a solution of the inequality.

1. \( x + y > -9 \); (0, 0)
2. \( x - y \geq 8 \); (14, 9)
3. \( 2x - y > 4 \); (-6, -15)

Graph the inequality.

10. \( y - x < 6 \)
11. \( x - y > -4 \)
12. \( 2y - x < 2 \)
13. \( 4y \leq 6x - 12 \)
14. \( 5y \leq 10x + 15 \)
15. \( -6y + 6 \geq -18x \)
19. \( y > 7 \)
20. \( x \leq -5 \)