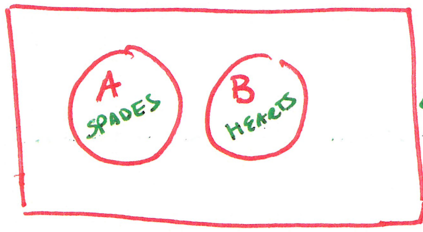
AP Statistics - 5.3 (part 2)	Name:
Conditional Probability and Independence Activity	Date: 2018 KEY

I. *Disjoint vs. Independence*

THINK ABOUT IT?: Can 2 mutually exclusive events ever be independent? Why?

PG 323



2 MUTUALLY EXCLUSIVE EVENTS CAN NEVER BE INDEPENDENT.

← MUTUALLY EXCLUSIVE EVENTS HAVE NO OUTCOMES IN COMMON. IF ONE EVENT OCCURS, THE OTHER EVENT IS GUARANTEED NOT TO OCCUR

II. *Conditional Probability Formula*

DEFINITION: Conditional Probability Formula (p324)

Use the General Multiplication Rule $P(A \cap B) = P(A) \cdot P(B|A)$ to find this formula:

$$\frac{P(A \cap B)}{P(A)} = \frac{\cancel{P(A)} \cdot P(B|A)}{\cancel{P(A)}}$$

$$\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$$

Look at your green AP sheet and find this formula. Hence, you do not need to memorize the general multiplication rule. Why? NO- USE YOUR ALGEBRA SKILLS

EXAMPLE: Conditional Probability Formula – Do problem 96 (page 331)

P(MAC) = .40
P(PC) = .60
P(UNDERGRAD) = .67
P(PC ∩ GRAD) = .23

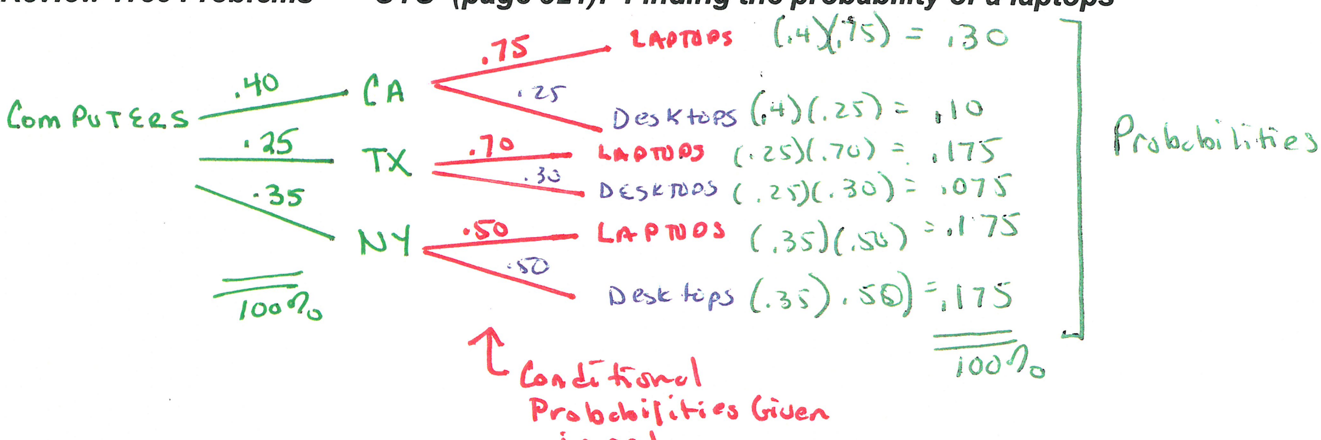
	GRAD	UNDERGRAD	
PC	.23		.60
MAC	.10		.40
	.33	.67	1.00

FIND P(GRAD | MAC) ⟹ $\frac{P(MAC \cap GRAD)}{P(MAC)} = \frac{.10}{.40} = .25$

IN CONTEXT: Approximately 25% of mac users are grad students.

III. *Review Tree Problems - CYU (page 321): Finding the probability of a laptops*

1



2 Probability that the computer is a laptop = P(Laptop) = .30 + .175 + .175 = .65

IN LONG HAND:

P(LAPTOP) = P(LAPTOP ∩ CA) + P(LAPTOP ∩ TX) + P(LAPTOP ∩ NY)

IV. Tree Diagrams can answer complex probability problems.

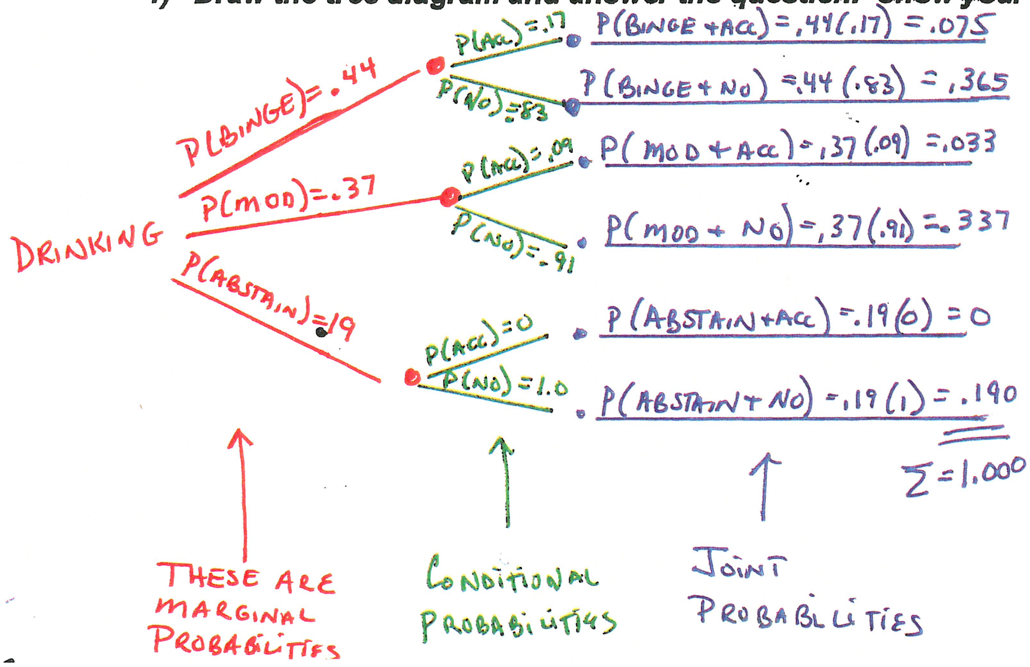
For men, binge drinking is defined as having five or more drinks in a row, and for women as having four or more drinks in a row. (The difference is because of the average difference in weight.) According to a study by the Harvard School of Public Health (H. Wechsler, G. W. Dowdall, A. Davenport, and W. DeJong, "Binge Drinking on Campus: Results of a National Study"), 44% of college students engage in binge drinking, 37% drink moderately, and 19% abstain entirely. Another study, published in the *American Journal of Health Behavior*, finds that among binge drinkers aged 21 to 34, 17% have been involved in an alcohol-related automobile accident, while among non-bingers of the same age, only 9% have been involved in such accidents. These are alcohol related accidents

What's the probability that a randomly selected college student will be a binge drinker who has had an alcohol-related car accident?

Given
P(BINGE) = .44
P(MOD) = .37
P(ABSTAIN) = .19
P(ACC | BINGE) = .17
P(ACC | MOD) = .09
P(ACC | ABSTAIN) = 0

because conditional probabilities were GIVEN

1) Draw the tree diagram and answer the question. Show your work.



To answer all these questions it would be easy to create a table (if you prefer)

	BINGE	MOD	ABSTAIN	
ACC	.075	.033	0	.108
NO ACC	.365	.337	.190	.892
	.44	.37	.190	1.00

P(BINGE ∩ ACCIDENT) = P(BINGE) • P(ACCIDENT | BINGE)
= (.44)(.17) = .0748 ~7.5%

Come directly from 2ND BRANCH

2) What is the probability an accident given a binge drinker? .17 P(ACC | BINGE)
3) What is the probability an accident given the student drinks moderately? .09 (P(ACC |
4) What is the probability an accident given the student abstains from drinking? 0

5) What is the probability of binge drinker? .44

6) What is the probability of having an accident?

P(ACCIDENT) = .0748 + .0333 + 0 = .1081 ~11%

7) What is the probability of not having an accident? P(NOT ACCIDENT) = 1 − .11 ~89%

8) What is the probability of binge drinking and having an accident? ~7.5% or .0748
9) What is the probability a student has an accident is a binge drinker?

$$P(\text{BINGE} \mid \text{ACCIDENT}) = \frac{P(\text{BINGE} \cap \text{ACCIDENT})}{P(\text{ACCIDENT})} = \frac{.0748}{.1081} = .69195$$

~69%

V. **"*Independence: A Special Multiplication Rule*"** [*TIP reference page 321*] **EXAMPLE: *Perfect Games***

In baseball, a perfect game is when a pitcher doesn't allow any hitters to reach base in all nine innings. Historically, pitchers throw a perfect inning—an inning where no hitters reach base—about 40% of the time. So, to throw a perfect game, a pitcher needs to have nine perfect innings in a row. **Problem:** What is the probability that a pitcher throws nine perfect innings in a row, assuming the pitcher's performance in an inning is independent of his performance in other innings?

GIVEN

P(PERFECT INNING) = .40

TOLD THE 9 INNINGS ARE INDEPENDENT [THEREFORE WE CAN MULTIPLY THE PROBABILITIES]

P(9 Perfect INNINGS*) = $(.4)^9$ = .00026 (0.026%)

* THINK...

P(INNING 1 PERFECT) · P(INN. 2 PERFECT) . . . P(INN. 9 PERFECT)

(.4) · (.4) · (.4) · (.4) · (.4) (.4) · (.4) (.4) (.4)

VI. ***Finding the probability of "at least one"*** [*TIP reference page 322*] **Example: First Trimester Screen**

The First Trimester Screen is a non-invasive test given during the first trimester of pregnancy to determine if there are specific chromosomal abnormalities in the fetus. According to a study published in the New England Journal of Medicine in November 2005, approximately 5% of normal pregnancies will receive a positive result. Among 100 women with normal pregnancies, what is the probability that there will be at least one false positive?

GIVEN

P(Normal pregnancy WITH POSITIVE TEST RESULT WITH ABNORMALITY) = .05

IT IS REASONABLE THAT THE WOMEN'S TESTS ARE INDEPENDENT.

P(FALSE POSITIVE) = .05 → P(NO FALSE POSITIVE) = .95

P(AT LEAST 1 POSITIVE TEST OUT OF 100 WOMEN) =

1 − P(NO POSITIVE RESULTS FOR 100 WOMEN) =

$1 - (.95)^{100} = 1 - .0059 = \boxed{.9941}$

AT LEAST THINK 1 − P(NONE)

CONCLUDE: THERE IS OVER A 99% PROBABILITY THAT AT LEAST 1 OF THE 100 WOMEN WITH NORMAL PREGNANCY WILL RECEIVE A FALSE POSITIVE TEST FOR DEFECT.

VII. *Compare using trees and tables* [*TIP reference page 326*] **EXAMPLE: *False Positives & Drug Testing***

Many employers require prospective employees to take a drug test. A positive result on this test indicates that the prospective employee uses illegal drugs. However, not all people who test positive actually use drugs. Suppose that 4% of prospective employees use drugs, the false positive rate is 5% and the false negative rate is 10%.

PROBLEM: What % OF PEOPLE WHO TEST POSITIVE ACTUALLY USE ILLEGAL DRUGS?

$$P(\text{TOOK DRUGS} \mid \text{POSITIVE TEST}) = \frac{P(\text{TOOK DRUGS AND POSITIVE})}{P(\text{TESTED POSITIVE})}$$

GIVEN INFO

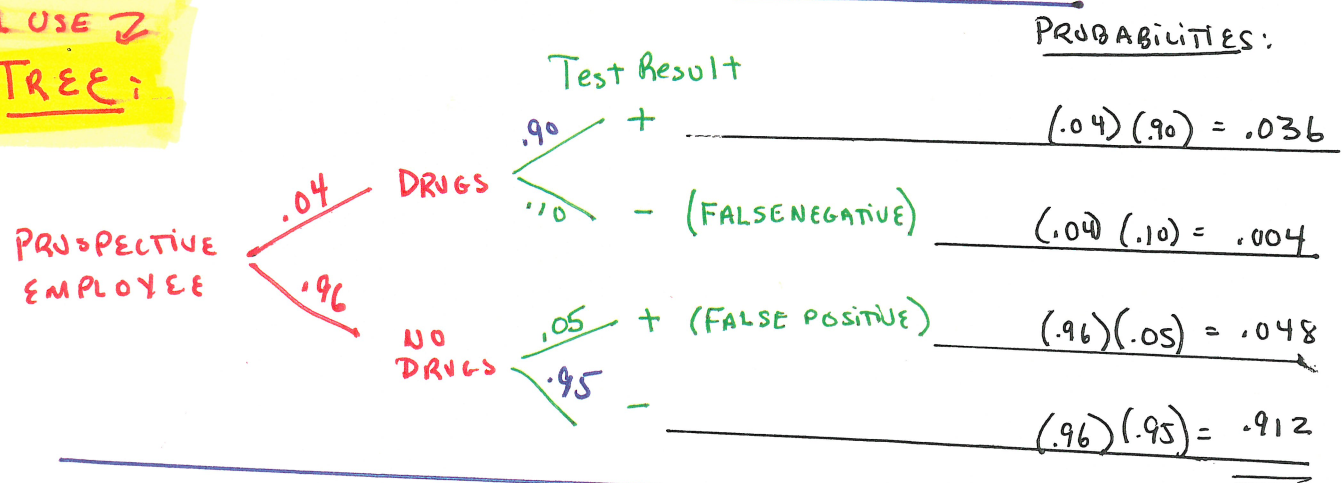
$P(\text{TOOK DRUGS}) = .04$

$P(\text{FALSE POSITIVE}) = .05 = P(\text{TEST POS} \mid \text{NO DRUGS})$

$P(\text{FALSE NEGATIVE}) = .10 = P(\text{TEST NEG} \mid \text{DRUGS})$

OPTION 1 USE

TREE:



$$P(\text{Took Drugs} \mid \text{Positive Test}) = \frac{P(\text{TOOK DRUGS AND POSITIVE})}{P(\text{Test Positive})} = \frac{.036}{.036 + .048} = \frac{.036}{.084} = .429$$

There is about 43% OF THE PROSPECTIVE EMPLOYEES WHO TEST POSITIVE POSITIVELY ACTUALLY TOOK DRUGS

OPTION 2 USE

TABLE:

MULTIPLY BY 1000

TEST	POSITIVE	NEGATIVE		
DRUGS	.90 36	FALSE NEGATIVE .10 4	.04	40
NO DRUGS	FALSE POSITIVE .05 48	.95 912	.96	960
	84	916	1.00	1000

$$P(D \mid +) = \frac{36}{84} = .429$$