10WP Notes Vertical Motion Word Problems

I. **Review Distance Models**: The formula used is: $D=R\cdot T$

⇒ D= distance R= rate T= time

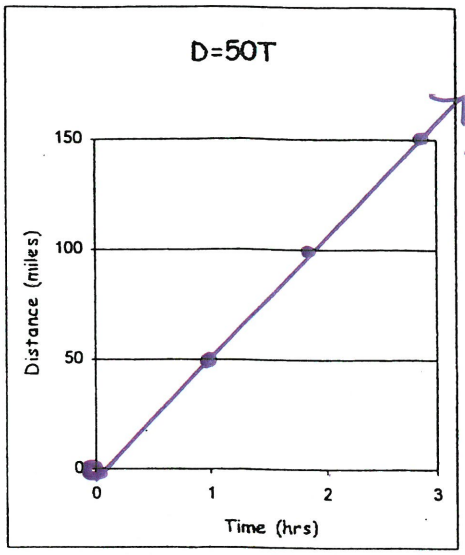
⇒ Rate is a constant and the relctionship is LINEAR

Example: A car travels at 50mph. How far will the car travel in 0, 1, 2, 3 hours? Complete the table and graph.

EQ: $D=50T$

Time (hrs)	0	1	2	3
Distance (miles)	0	50	100	150

KI: DISTANCE – FIND IT
RATE – 50mph
Time – 0, 1, 2, 3 hrs



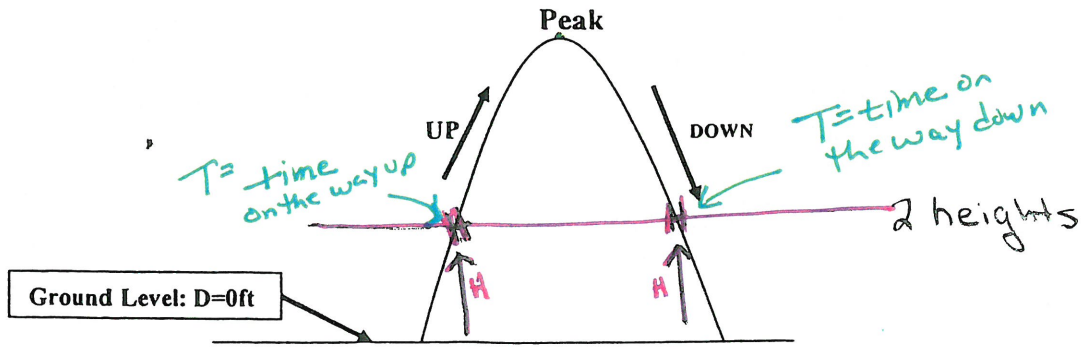
The relctionship is Linear

II. **Vertical motion models** describes the height of an object that is propelled into the air, but has no power to keep it self in the air.

- **Equation**: $H=-16T^2+VT+S$ (based on units in feet & seconds)
  - H= height of the object (in feet)
  - T= time the object has been in the air (in seconds)
  - V= INITAL VERTICAL velocity (in ft/second)
  - S= INITIAL HEIGHT (in ft)
  - -16 takes into account the effect of gravity but ignores other, less significant, factors such as air resistance.

Gravity used for ft/s

- Vertical motion problems do NOT have a constant rate and the shape of the graph is a parabola.



S=0ft

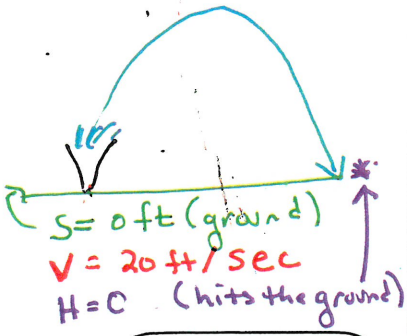
If the starting point is on the ground

9.4 Solve Polynomial Equations in Factored Form

Your Notes

KEY INFO

Looking for time (T)



Example 4 *Solve a multi-step problem*

Fountain A fountain sprays water into the air with an initial vertical velocity of 20 feet per second. After how many seconds does it land on the ground?

SOLVE BY FACTORING:

Step 1 Write a model for the water's height above ground.

$h = -16t^2 + vt + s$ (memorize) **Vertical motion model**

$h = -16t^2 + \underline{20}\,t + \underline{0}$ $v = \underline{20}$ and $s = \underline{0}$

Vertical motion EQUATION

$h = -16t^2 + \underline{20}$ **Simplify.**

Step 2 Substitute 0 ft for h. When the water lands, its height above the ground is 0 feet. Solve for t.

$\underline{0} = -16t^2 + \underline{20T}$ **Substitute 0 for h.**

$\underline{0} = \underline{-4T(4T - 5)}$ **Factor right side.**

$\underline{-4T = 0}$ or $\underline{4T - 5 = 0}$ **Zero-product property**

$\underline{T = 0}$ or $\underline{T = 5/4}$ **Solve for t.**

The water lands on the ground 1.25 seconds after it is sprayed.

The solution $t = 0$ means that before the water is sprayed, its height above the ground is 0 feet.

T=0 seconds is the time the water initially came out of the fountain.

Now Solve with the Quadratic Formula:

A = -16 B = 20 C = 0

$$x = \frac{-20 \pm \sqrt{400 - 4(-16)(0)}}{2(-16)} = \frac{-20 \pm \sqrt{400}}{-32}$$

$$x = \frac{-20 + 20}{-32} \qquad x = \frac{-20 - 20}{-32}$$

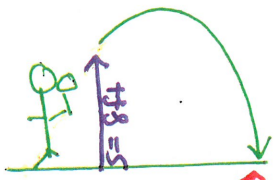
$$\boxed{x = 0} \qquad \boxed{x = 1.25}$$

9.6 Factor $ax^2 + bx + c$

Goal • Factor trinomials of the form $ax^2 + bx + c$.

Your Notes

KEY INFO



V = 62 ft/s

T = ?

S = 8 ft

H = 0 ft (when the ball hits the ground)

Example 4 *Write and solve a polynomial equation*

Tennis An athlete hits a tennis ball at an initial height of 8 feet and with an initial vertical velocity of 62 feet per second.

a. Write an equation that gives the height (in feet) of the ball as a function of the time (in seconds) since it left the racket.

b. After how many seconds does the ball hit the ground?

Solution

a. Use the VERTICAL MOTION MODEL to write an equation for the height h (in feet) of the ball.

$\boxed{h = -16t^2 + vt + s}$ ← Memorize

$h = -16t^2 + 62t + 8$ $\quad$ $v = 62$ and $s = 8$

EQUATION TO SOLVE

b. To find the number of seconds that pass before the ball lands, find the value of t for which the height of the ball is 0. Substitute 0 for h and solve the equation for t.

SOLVE BY FACTORING OR QUAD. FORMULA

$0 = -16t^2 + 62t + 8$ $\quad$ Substitute 0 for h.

$0 = -2(8T^2 - 31T - 4)$ $\quad$ Factor out -2.

$0 = -2(8T + 1)(T - 4)$ $\quad$ Factor the trinomial.

$8T + 1 = 0$ or $T - 4 = 0$ $\quad$ Zero-product property

impossible $\cancel{t = -1/8}$ or $t = 4$ $\quad$ Solve for t.

A negative solution does not make sense in this situation.

The tennis ball hits the ground after 4 sec.

Q.F.

A = -16
B = 62
C = 8

$$x = \frac{-62 \pm \sqrt{3844 - 4(-16)(8)}}{2(-16)} = \frac{-62 \pm \sqrt{4356}}{-32} = \frac{-62 \pm 66}{-32}$$

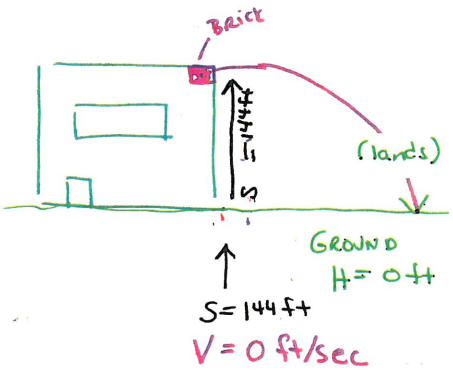
$$x = \frac{-62 + 66}{-32} \qquad \boxed{x = -.125}$$

$$x = \frac{-62 - 66}{-32} \qquad \boxed{x = 4}$$

9.7 Factor Special Products

Goal • Factor special products.

KEY INFORMATION:



Example 4 *Solve a vertical motion problem*

Falling Object A brick falls off of a building from a height of 144 feet. After how many seconds does the brick land on the ground?

Solve by Factoring:

Use the vertical motion model. The brick fell, so its initial vertical velocity is 0. Find the value of time t (in seconds) for which the height h (in feet) is 0.

$h = -16T^2 + VT + S$ ← Vertical motion model

$0 = -16T^2 + 0T + 144$ Substitute values.

$0 = -16T^2 + 144$

$0 = -16(T^2 - 9)$ Factor out -16.

$0 = -16(T-3)(T+3)$ Difference of two squares

$T - 3 = 0$ or $T + 3 = 0$ Zero-product property

$T = 3$ or $T = -3$ Solve for t.

The brick lands on the ground 3 seconds after it falls.

Now Solve with the Quadratic Formula:

$A = -16 \quad B = 0 \quad C = 144$

$$X = \frac{-0 \pm \sqrt{0 - 4(-16)(144)}}{2(-16)} = \frac{0 \pm \sqrt{9216}}{-32} = \frac{0 \pm 96}{-32}$$

$$X = \frac{0 + 96}{-32} \qquad X = \frac{0 - 96}{-32}$$

$$\boxed{X = -3} \qquad \boxed{X = 3}$$

Checkpoint

Memorize: $H = -16T^2 + VT + S$

S = starting height (ft)

t = time (sec)

V = initial velocity (ft/sec)

h = height on the Parabola (ft)

For the following word problem:

(a) Sketch and label the graph. Include units and label the variables.

(b) Write the model for height as a function of time using function notation.

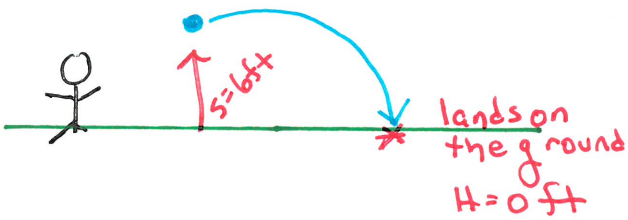
(c) Use the quadratic formula to solve. Clearly show your work!! Round solutions to "ONE DECIMAL". Circle your solutions.

(d) Answer question in a complete sentence.

A What If? → an athlete hits the tennis ball with an initial vertical velocity of 20 feet per second from a height of 6 feet. After how many seconds does the ball hit the ground?

KI

$V = 20$ ft/sec



THE BALL LANDS ON THE GROUND AT 1.5 SECONDS

$0 = -16T^2 + 20T + 6$

$A = -16 \quad B = 20 \quad C = 6$

$$X = \frac{-20 \pm \sqrt{400 - 4(-16)(6)}}{2(-16)}$$

$$X = \frac{-20 \pm \sqrt{784}}{-32}$$

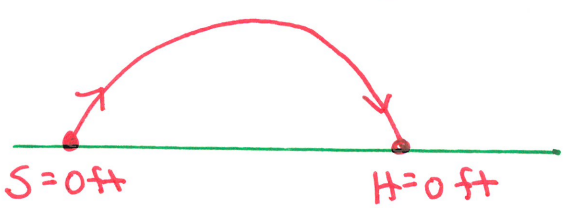
$$X = \frac{-20 + 28}{-32} \qquad X = \frac{-20 - 28}{-32}$$

$$\boxed{X = -.25} \qquad \boxed{X = 1.5}$$

B **Jump Rope** A child jumping rope leaves the ground at an initial vertical velocity of 8 feet per second. After how many seconds does the child land on the ground?

KI

$V = 8$ ft/sec



THE CHILD LANDS AT 0.5 seconds

$0 = -16T^2 + 8T + 0$

$A = -16 \quad B = 8 \quad C = 0$

$$X = \frac{-8 \pm \sqrt{64 - 4(-16)(0)}}{2(-16)}$$

$$X = \frac{-8 \pm \sqrt{64}}{-32}$$

$$X = \frac{-8 + 8}{-32} \qquad X = \frac{-8 - 8}{-32}$$

$$\boxed{X = 0} \qquad \boxed{X = .5}$$

C **Cliff Diving** A cliff diver jumps from a ledge 96 feet above the ocean with an initial upward velocity of 16 feet per second. How long will it take until the diver enters the water?

KI

$V = 16\ ft/sec$

$0 = -16T^2 + 16T + 96$

$A = -16 \quad B = 16 \quad C = 96$

$$X = \frac{-16 \pm \sqrt{256 - 4(-16)(96)}}{2(-16)}$$

$$X = \frac{-16 \pm \sqrt{6400}}{-32}$$

$$X = \frac{-16 + 80}{-32} \qquad X = \frac{-16 - 80}{-32}$$

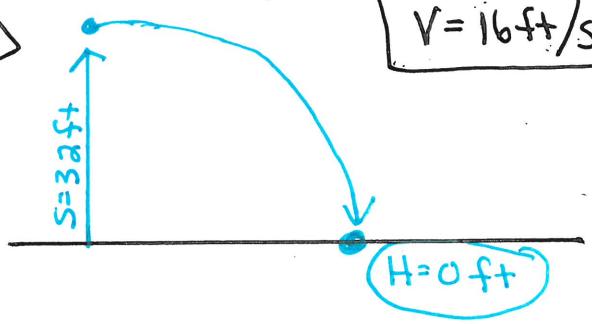
$X = -2 \qquad X = 3$

The diver hits the water at 3 seconds

D **Tennis Ball** For a science experiment, you toss a tennis ball from a height of 32 feet with an initial upward velocity of 16 feet per second. How long will it take the tennis ball to reach the ground?

KI

$V = 16\ ft/sec$



$0 = -16T^2 + 16T + 32$

$A = -16 \quad B = 16 \quad C = 32$

$$X = \frac{-16 \pm \sqrt{256 - 4(-16)(32)}}{2(-16)}$$

$$X = \frac{-16 \pm \sqrt{2304}}{-32}$$

$$X = \frac{-16 + 48}{-32} \qquad X = \frac{-16 - 48}{-32}$$

$\boxed{X = -1} \qquad \boxed{X = 2}$

The ball hits the ground at 2 Seconds