I. **Review Distance Models**: The formula used is: \[ D = R \times T \]

\[ \Rightarrow D = \text{distance} \quad R = \text{rate} \quad T = \text{time} \]

\[ \Rightarrow \text{Rate is a constant and the relationship is linear} \]

**Example**: A car travels at 50mph. How far will the car travel in 0, 1, 2, 3 hours? Complete the table and graph.

**EQ**: \[ D = 50T \]

<table>
<thead>
<tr>
<th>Time (hrs)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (miles)</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>150</td>
</tr>
</tbody>
</table>

\[KI: \text{DISTANCE - FIND IT} \]
\[ \text{RATE - 50mph} \]
\[ \text{TIME - 0, 1, 2, 3 hrs} \]

II. **Vertical motion models** describes the height of an object that is propelled into the air, but has no power to keep itself in the air.

- **Equation**: \[ H = -16T^2 + VT + S \] (based on units in feet & seconds)
  - \( H \): height of the object (in feet)
  - \( T \): time the object has been in the air (in seconds)
  - \( V \): initial vertical velocity (in ft/second)
  - \( S \): initial height (in ft)

\[ -16 \] takes into account the effect of gravity but ignores other, less significant, factors such as air resistance.

- Vertical motion problems do NOT have a constant rate and the shape of the graph is a parabola.

**Ground Level**: \( D = 0 \) ft
Example 4: Solve a multi-step problem

Fountain: A fountain sprays water into the air with an initial vertical velocity of 20 feet per second. After how many seconds does it land on the ground?

**Solve by factoring:**

Step 1: Write a model for the water's height above ground.

\[ h = -16t^2 + vt + s \]  
Vertical motion model

\[ v = 20 \] and \[ s = 0 \]  
Simplify.

\[ h = -16t^2 + 20t \]

Step 2: Substitute 0 ft for \( h \). When the water lands, its height above the ground is 0 feet. Solve for \( t \).

\[ 0 = -16t^2 + 20t \]  
Substitute 0 for \( h \).

\[ 0 = -16t(t - \frac{5}{4}) \]  
Factor right side.

\[ t = 0 \] or \[ t = \frac{5}{4} \]  
Zero-product property

The water lands on the ground \( \frac{5}{4} = 1.25 \) seconds after it is sprayed.

\[ t = 0 \text{ seconds is the time the water initially came out of the fountain.} \]
9.6 Factor \( ax^2 + bx + c \)

**Goal**  
- Factor trinomials of the form \( ax^2 + bx + c \).

---

**Example 4**  
**Write and solve a polynomial equation**

**Tennis**  
An athlete hits a tennis ball at an initial height of 8 feet and with an initial vertical velocity of 62 feet per second.

a. Write an equation that gives the height (in feet) of the ball as a function of the time (in seconds) since it left the racket.

b. After how many seconds does the ball hit the ground?

**Solution**

a. Use the **Vertical Motion Model** to write an equation for the height \( h \) (in feet) of the ball.

\[
h = -16t^2 + vt + s
\]

- **Memorize**

\[
h = -16t^2 + 62t + 8
\]

\( v = 62 \) and \( s = 8 \)  

**Equation to solve**

b. To find the number of seconds that pass before the ball lands, find the value of \( t \) for which the height of the ball is 0. Substitute 0 for \( h \) and solve the equation for \( t \).

\[0 = -16t^2 + 62t + 8\]

Substitute 0 for \( h \).  
Factor out -2.  
Factor the trinomial.  
Zero-product property  
Solve for \( t \).

A negative solution does not make sense in this situation.  
The tennis ball hits the ground after **4 sec**.

\[x = \frac{-62 \pm \sqrt{3844 - 4(-16)(8)}}{2(-16)} = \frac{-62 \pm \sqrt{4856}}{-32} = \frac{-62 \pm 66}{-32}
\]

\[x = \frac{-62 + 66}{-32} \quad x = \frac{-62 - 66}{-32}
\]

\[x = 4 \quad x = -1.25
\]
9.7 Factor Special Products

Example 4 Solve a vertical motion problem

Falling Object A brick falls off of a building from a height of 144 feet. After how many seconds does the brick land on the ground?

Solve by Factoring:

Use the vertical motion model. The brick fell, so its initial vertical velocity is 0. Find the value of time \( t \) (in seconds) for which the height \( h \) (in feet) is 0.

\[
\begin{align*}
\ h &= -16t^2 + v_0t + s \\
0 &= -16t^2 + 0t + 144 \\
0 &= -16(t^2 - 9) \\
0 &= -16(T - 3)(T + 3) \\
T - 3 &= 0 & \text{or} & & T + 3 &= 0 \\
T &= 3 & \text{or} & & T &= -3 \\
\end{align*}
\]

The brick lands on the ground 3 seconds after it falls.

Now Solve with the Quadratic Formula:

\[
A = -16 \quad B = 0 \quad C = 144
\]

\[
X = \frac{-0 \pm \sqrt{0 - 4(-16)(144)}}{2(-16)} = \frac{0 \pm \sqrt{9216}}{-32} = \frac{0 \pm 96}{-32}
\]

\[
X = \frac{0 + 96}{-32} \quad X = \frac{0 - 96}{-32}
\]

\[
X = -3 \quad X = 3
\]
**Checkpoint**

For the following word problem:
(a) Sketch and label the graph. Include units and label the variables.
(b) Write the model for height as a function of time using function notation.
(c) Use the quadratic formula to solve. Clearly show your work!!
Round solutions to "ONE DECIMAL". Circle your solutions.
(d) Answer question in a complete sentence.

**A What if?**

An athlete hits the tennis ball with an initial vertical velocity of 20 feet per second from a height of 6 feet. After how many seconds does the ball hit the ground?

\[ V = 20 \text{ ft/sec} \]

\[ S = 6 \text{ ft} \]

\[ H = 0 \text{ ft} \]

\[ V = 20 \text{ ft/sec} \]

\[ a = -16 \]

\[ b = 20 \]

\[ c = 6 \]

\[ X = -20 \pm \sqrt{400 - 4(-16)(6)} \]

\[ a = 2 \]

\[ b = -16 \]

\[ c = -32 \]

\[ X = -20 \pm \sqrt{784} \]

\[ a = -32 \]

\[ b = 28 \]

\[ c = -32 \]

\[ X = -20 \pm 28 \]

\[ X = -20 - 28 \]

\[ X = -32 \]

\[ X = 1.5 \]

**B Jump Rope**

A child jumping rope leaves the ground at an initial vertical velocity of 8 feet per second. After how many seconds does the child land on the ground?

\[ V = 8 \text{ ft/sec} \]

\[ S = 0 \text{ ft} \]

\[ H = 0 \text{ ft} \]

\[ V = 8 \text{ ft/sec} \]

\[ a = -16 \]

\[ b = 8 \]

\[ c = 0 \]

\[ X = -8 \pm \sqrt{64 - 4(-16)(0)} \]

\[ a = 2 \]

\[ b = -16 \]

\[ c = -32 \]

\[ X = -8 \pm \sqrt{64} \]

\[ a = -32 \]

\[ b = 8 \]

\[ c = -32 \]

\[ X = 0 \]

\[ X = 0.5 \]
Cliff Diving A cliff diver jumps from a ledge 96 feet above the ocean with an initial upward velocity of 16 feet per second. How long will it take until the diver enters the water?

\[ V = 16 \text{ ft/sec} \]

The diver hits the water at 3 seconds.

\[ 0 = -16T^2 + 16T + 96 \]

\[
A = -16 \quad B = 16 \quad C = 96
\]

\[
X = \frac{-16 \pm \sqrt{256 - 4(-16)(96)}}{2(-16)}
\]

\[
X = -16 \pm \sqrt{6400}
\]

\[
X = -16 \pm 80
\]

\[
X = -16 - 80
\]

\[
X = -32
\]

\[
X = 3
\]

Tennis Ball For a science experiment, you toss a tennis ball from a height of 32 feet with an initial upward velocity of 16 feet per second. How long will it take the tennis ball to reach the ground?

\[ V = 16 \text{ ft/sec} \]

The ball hits the ground at 2 seconds.

\[ 0 = -16T^2 + 16T + 32 \]

\[
A = -16 \quad B = 16 \quad C = 32
\]

\[
X = \frac{-16 \pm \sqrt{256 - 4(-16)(32)}}{2(-16)}
\]

\[
X = -16 \pm \sqrt{2304}
\]

\[
X = -16 \pm 48
\]

\[
X = -16 + 48
\]

\[
X = 32
\]

\[
X = 2
\]