QE: $Ax^2+Bx+C=0$ ← to solve

QF: $y=Ax^2+Bx+C$ ← to graph

# 10.3 Solve Quadratic Equations by Graphing

**Goal** • Solve quadratic equations by graphing.

**Your Notes**

STANDARD FORM OF A QE

**VOCABULARY**

Quadratic equation → $Ax^2+Bx+C=0$ where $A\neq 0$, $A, B, C$ real #'s

SOLUTIONS TO QE's: are the x-intercepts
- with the coordinates $(x, 0)$
- NOTICE THE Y-COORDINATE IS ALWAYS ZERO

**Example 1** *Solve a quadratic equation having two solutions*

**Solve $-x^2 + 2x = -8$ by graphing.**

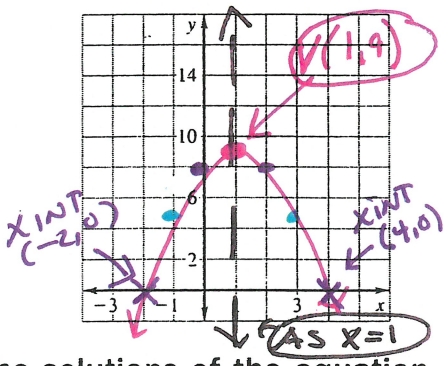
**Step 1** Write the equation in STANDARD FORM.

$-x^2 + 2x = -8$ Write original equation.

QE: $-x^2 + 2x + 8 = 0$ (replace 0 with y) Add 8 to each side.

**Step 2** Graph the function $y = -x^2 + 2x + 8$.

The x-intercepts are $(-2, 0)$ and $(4, 0)$




$A=-1$ $B=2$ $C=8$

AS: $x=\frac{-B}{2A}=\frac{-2}{2(-1)}$ $x=1$

Vertex: $(1, 9)$

$y=-(1)^2+2(1)+8=9$

| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| y | 0 | 5 | 8 | 9 | 8 | 5 | 0 |

The solutions of the equation $-x^2 + 2x = -8$ are $-2$ and $4$. ($x=-2, 4$)

***CHECK*** You can check $-2$ and $4$ in the original equation. ($x=-2$)

C: $-x^2 + 2x = -8$

$-(-2)^2 + 2(-2) \stackrel{?}{=} -8$

$-4 + -4 = -8$

$-8 = -8$ ✓

($x=4$)

C: $-x^2 + 2x = -8$

$-(4)^2 + 2(4) \stackrel{?}{=} -8$

$-16 + 8 = -8$

$-8 = -8$ ✓

Graphing Steps
1. put into STD Form
2. Find A, B, C
3. Find A.S. $x=\frac{-B}{2A}$
4. Find the Vertex
5. Create a table of 5 values with the vertex in the middle
6. Graph – U shape

Review Factoring to Solve QE

$-x^2+2x+8=0$

$-1(x^2-2x-8)=0$

$-1(x-4)(x+2)=0$

$x-4=0$ → $x=4$

$x+2=0$ → $x=-2$

The solutions $x=-2, 4$ have the ordered pairs $(-2,0)$ and $(4,0)$

**Your Notes**

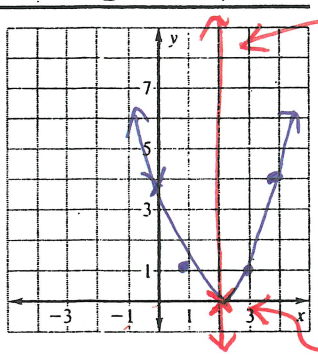
**Example 2** *Solve a quadratic equation having one solution*

**Solve $x^2 - 4x = -4$ by graphing.**

**Step 1** Write the equation in standard form.

$x^2 - 4x = -4$ Write original equation.

$x^2 - 4x + 4 = 0$ Add 4 to each side.




X=-2

Vertex (2,0) and X int (2,0)

**Step 2** GRAPH the function $y = x^2 - 4x + 4$.
The x-intercept is 2.

The solution of the equation $x^2 - 4x = -4$ is X=2

QE: $x^2 - 4x + 4 = 0$ $A = 1$ $B = -4$ $C = 4$

AS: $x = \frac{-B}{2A} = \frac{4}{2(1)}$ $x = 2$

$y = x^2 - 4x + 4$ to Graph

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 4 | 1 | 0 | 1 | 4 |

V

STEP 3 Check solution in the orig EQ

$C: (2)^2 - 4(2) = -4$

$4 - 8 = -4$

$-4 = -4$ ✓

FACTOR TO FIND Zero's

$\rightarrow x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$(x-2)^2 = 0$

To Solve

$x - 2 = 0$

$x = 2$

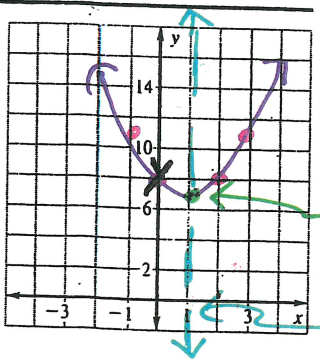
**Example 3** *Solve a quadratic equation having no solution*

**Solve $x^2 + 8 = 2x$ by graphing.**

**Step 1** Write the equation in standard form.

$x^2 + 8 = 2x$ Write original equation.

$x^2 - 2x + 8 = 0$ Subtract $2x$ from each side.




V (1, 7)

AS $x = 1$

$A=1$ $B=-2$ $C=8$

AS: $x = \frac{-B}{2A} = \frac{2}{2(1)}$

$x = 1$

| x | y |
|---|---|
| -1 | 11 |
| 0 | 8 |
| V 1 | 7 |
| 2 | 8 |
| 3 | 11 |

**Step 2** Graph the function $y = x^2 - 2x + 8$.

The graph has No x-intercepts.

The equation $x^2 + 8 = 2x$ has NO SOLUTION

$x =$ NO SOLUTION

FACTOR: $x^2 - 2x + 8 = 0$ → Cannot Factor PRIME

**Checkpoint** **Complete the following exercise.**

**Solve the quadratic function by graphing (FUNC.e.4)**

**For the graph include a table with 5 points; and clearly label y-intercept, axis of symmetry, vertex and x-intercept(s).**

**State solution(X=___) and circle.**
**Then Check all solutions in the original equations.**

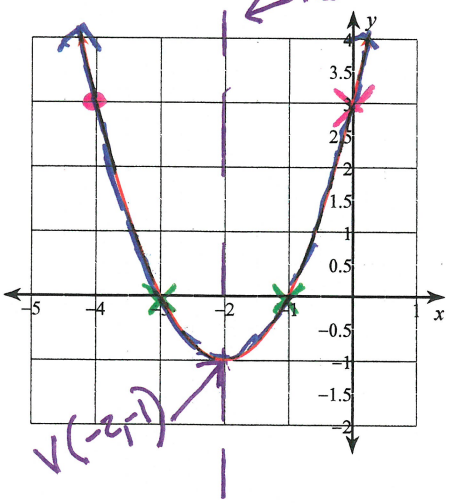
1) $f(x) = x^2 + 4x + 3$

$A=1$

$B=4$

$C=3$ ← yint

AS $x=-2$




AS: $x = \frac{-B}{2A} = \frac{-4}{2(1)} = -2$

$x = -2$

| x | -4 | -3 | -2 | -1 | 0 |
|---|---|---|---|---|---|
| y | 3 | 0 | -1 | 0 | 3 |

X INTS

SOLUTIONS: $x = -1, -3$

C: $(-1)^2 + 4(-1) + 3 = 0$

$0 = 0$ ✓

C: $(-3)^2 + 4(-3) + 3 = 0$

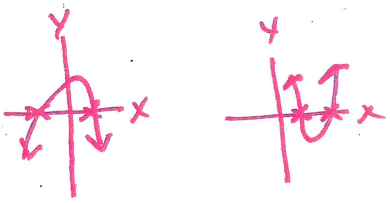
$0 = 0$ ✓

**Your Notes**

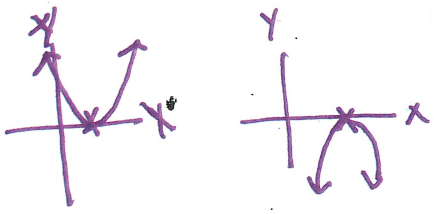
## NUMBER OF SOLUTIONS OF A QUADRATIC EQUATION

1. A quadratic equation has two solutions if the graph of its related function has 2 x intercepts.
2. A quadratic equation has one solution if the graph of its related function has 1 x intercept
3. A quadratic equation has no solution if the graph of its related function has No x intercepts

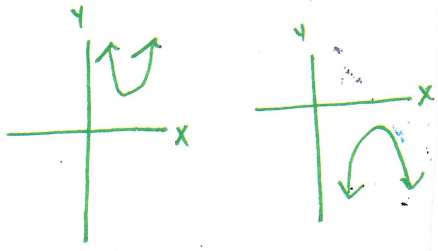
2 SOLUTIONS




1 SOLUTIONS




NO SOLUTION




Vocabulary

These mean the same:

- Solutions
- x intercepts (x, 0)
- Zero's → $Ax^2+Bx+C=0$
- roots

**Homework**

$f(x) = y$

"f of x"

FUNCTION NOTATION

## RELATING SOLUTIONS OF EQUATIONS, *x*-INTERCEPTS OF GRAPHS, AND ZEROS OF FUNCTIONS

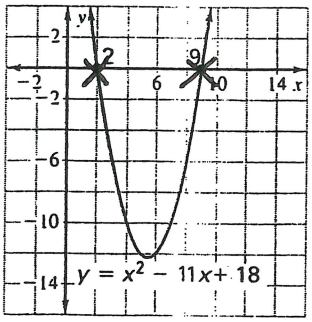
### Solutions of an Equation

Read the Graph

The solutions of the equation $x^2 - 11x + 18$ are 2 and 9.

### *x*-Intercepts of a Graph

The *x*-intercepts of the graph of $y = x^2 - 11x + 18$ occur where $y =$ 0, so the *x*-intercepts are 2 and 9, as shown. (2,0) (9,0)




### Zeros of a Function

The zeros of the function $f(x) = x^2 - 11x + 18$ are the values of *x* for which $f(x) =$ 0, so the zeros are 2 and 9.

FACTOR

$0 = x^2 - 11x + 18$

$0 = (x-9)(x-2)$

$x = 2, 9$