**9.5a Factor Quadratic Equations When the Leading Coefficient \(a=1\)**

**VOCABULARY:**
- **Standard Form of a Quadratic Equation** \(Ax^2 + Bx + C = 0\);
  
  Where \(a, b, c\) are real number; and \(A \neq 0\)
- **Factoring a quadratic trinomials** when \(a=1\) into a product of 2 binomial factors

**Example:** \(x^2 + 5x + 6 = (x+2)(x+3)\) because \(2 + 3 = 5\) and \(2 \cdot 3 = 6\).

**Example 1** *Factor when \(b\) and \(c\) are positive*

**Steps to Factor:** \(x^2 + 10x + 16 = \) \(x+2\) \(x+8\) \(\text{or}\) \(x+8\) \(x+2\)

1. Identify \(a\), \(b\), and \(c\). \(a=1\) \(b=10\) \(c=16\)
2. Write 2 sets of \((\)\)'s. One for each factor.
3. The first term in both factors is “\(x\)”. Why? \(X \cdot X = X^2\) \(\text{that is the 1st Term}\)
4. What must the signs have to be for each factor? both signs are + \(b/c\) \(b+c\) are positive
5. What are the factors of 16? Put them under the 16.
6. Find the 2 factors \(2 + 8 = 10\) AND \(2 \cdot 8 = 16\)
7. CHECK by Multiplying the factors \(x^2 + 10x + 16 = x^2 + 8x + 2x + 16 = x^2 + 10x + 16\)

**CHECK POINT: Factor and Check by mentally multiplying**

<table>
<thead>
<tr>
<th>2) (x^2 + 9x + 8)</th>
<th>3) (x^2 + 12x + 20)</th>
<th>4) (x^2 + 9x + 18)</th>
<th>5) (x^2 + 13x + 40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{8}) (\frac{1}{4})</td>
<td>(\frac{1}{2}) (\frac{10}{5})</td>
<td>(\frac{1}{18}) (\frac{2}{9})</td>
<td>(\frac{1}{40}) (\frac{2}{20})</td>
</tr>
<tr>
<td>(\frac{1}{18}) (\frac{1}{30}) (\checkmark)</td>
<td>(\frac{1}{2}) (\frac{10}{5}) (\checkmark)</td>
<td>(\frac{1}{2}) (\frac{9}{3}) (\checkmark)</td>
<td>(\frac{1}{4}) (\frac{10}{5}) (\checkmark)</td>
</tr>
</tbody>
</table>
Example 6  Factor when \( b \) is negative and \( c \) is positive

Steps to Factor: \[ x^2 - 5x + 6 = \frac{(x - 2)(x - 3)}{1 2 3} \text{ or } \frac{(x - 3)(x - 2)}{1 2 3} \]

1) Identify \( a \), \( b \), and \( c \). \( a = 1 \) \( b = -5 \) and \( c = 6 \)
2) What must the signs have to be for each factor? Both Negative b/c \( -b \) and \( +c \)
3) What are the factors of 6? Put them under the 6.
4) Find the 2 factors \(-2\) \( + (-3) = -5 \) AND \(-2 \cdot -3 = 6 \)
5) CHECK by Multiplying the factors

CHECK POINT: Factor and Check by mentally multiplying

7) \[ x^2 - 10x + 21 = \frac{(x - 3)(x - 7)}{1 2 3 7} \text{ remember mental mult. to check!} \]
8) \[ x^2 - 10x + 16 = \frac{(x - 2)(x - 8)}{1 2 8} \]

Example 9  Factor when \( c \) is negative

Steps to Factor: \[ x^2 + 3x - 10 = \frac{(x + 5)(x - 2)}{1 2 5} \text{ or } \frac{(x - 2)(x + 5)}{1 2 5} \]

1) Identify \( a \), \( b \), and \( c \). \( a = 1 \) \( b = 3 \) and \( c = -10 \)
2) What must the signs have to be for each factor? OPPOSITE SIGNS (+, -) \( b/c \) \( c \) is negative
3) What are the factors of 10? Put them under the 10.
4) Find the 2 factors \(-2\) \( + 5 = 3 \) \( \sqrt{\text{AND}} \) \(-2 \cdot 5 = -10 \)
5) CHECK by Multiplying the factors

CHECK POINT: Factor and Check by mentally multiplying

| 9) \[ x^2 - 5x - 50 = \frac{(x + 5)(x - 10)}{1 2 5 10} \text{ or } \frac{(x - 5)(x + 10)}{1 2 5 10} \] | 10) \[ x^2 + 2x - 24 = \frac{(x + 6)(x - 4)}{1 2 1 2 3 4} \text{ or } \frac{(x - 6)(x + 4)}{1 2 1 2 3 4} \] |
| 12) \[ x^2 + 4x - 21 = \frac{(x + 7)(x - 3)}{1 2 7 3} \] | 13) \[ x^2 - 4x - 32 = \frac{(x + 4)(x - 8)}{1 2 4 8} \text{ or } \frac{(x - 4)(x + 8)}{1 2 4 8} \] |
Example 1  Steps to Solve Quadratic Equations by Factor:

\[ x^2 + 7x = 18 \]

1) Put in standard form
\[ Ax^2 + Bx + C = 0 \]

2) Factor

\[ (x + 9)(x - 2) = 0 \]

3) Set each factor to “0” and solve

- \[ x + 9 = 0 \]
- \[ x - 2 = 0 \]

Notice: there are 2 solutions

4) Check each solution in the original equation

\[ C: (-9)^2 + 7(-9) = 18 \]
\[ 81 - 63 = 18 \]
\[ 18 = 18 \]

\[ C: 2^2 + 7(2) = 18 \]
\[ 4 + 14 = 18 \]
\[ 18 = 18 \]
### CHECK POINT: Solve by Factoring and Check

2) \( x^2 + x = 12 \)

\[
\begin{array}{c}
\frac{-12}{12} \\
\frac{-12}{12} = 0 \\
\end{array}
\]

\( x^2 + x - 12 = 0 \)

\[
\frac{12}{2} \\
\frac{12}{12} = 0 \\
\frac{2}{2} = 1 \\
\frac{2}{4} = 3 \\
\end{array}
\]

\((x+4)(x-3) = 0\)

\[
\begin{array}{c}
x + 4 = 0 \\
x = -4 \\
\end{array}
\]

\[
\begin{array}{c}
x - 3 = 0 \\
x = 3 \\
\end{array}
\]

\(C: (4)^2 + (-4) = 12\)

\(16 - 4 = 12\)

\(12 = 12\)

3) \( x^2 - 14x = -40 \)

\[
\begin{array}{c}
+40 \\
+40 \\
\end{array}
\]

\[
\frac{10^2 - 14 \cdot 10 + 40}{50} = 0 \\
\frac{100 - 140 + 40}{50} = 0 \\
\frac{-40}{50} = 0 \\
\end{array}
\]

\((x-4)(x-10) = 0\)

\[
\begin{array}{c}
x - 4 = 0 \\
x = 4 \\
\end{array}
\]

\[
\begin{array}{c}
x - 10 = 0 \\
x = 10 \\
\end{array}
\]

\(C: 10^2 - 14(10) + 40 = 0\)

\(100 - 140 + 40 = 0\)

\(-40 = -40\)

4) \( x^2 + 12x = -36 \)

\[
\begin{array}{c}
+36 \\
+36 \\
\end{array}
\]

\[
\frac{x^2 + 12x + 36}{12} = 0 \\
\frac{1}{2} \\
\frac{18}{2} = 9 \\
\frac{12}{2} = 6 \\
\frac{9}{6} = 1.5 \\
\end{array}
\]

\(C: (-4)^2 + (-4) = 12\)

\(\frac{12}{12} = 0\)

5) \( x^2 + 3x + 10 = 0 \)

\[
\begin{array}{c}
-10 \\
-10 \\
\end{array}
\]

\[
\frac{x^2 + 3x}{10} = 0 \\
\frac{x}{10} = 0 \\
\frac{x+3}{10} = 0 \\
\frac{x-3}{10} = 0 \\
\end{array}
\]

\(x(x+3) = 0\)

\[
\begin{array}{c}
x = 0 \\
\end{array}
\]

\[
\begin{array}{c}
x + 3 = 0 \\
x = -3 \\
\end{array}
\]

\(C: (-3)^2 + 3(-3) + 10 = 10\)

\(9 - 9 + 10 = 10\)

\(10 = 10\)

6) \( x^2 - 80 = 20 \)  
 **(CHALLENGE PROBLEM ☞)**

\[
\begin{array}{c}
-20 \\
-20 \\
\end{array}
\]

\[
\frac{x^2 - 100}{x} = 0 \\
\frac{100}{25} = 4 \\
\frac{25}{25} = 1 \\
\frac{5}{20} = 0.25 \\
\frac{10}{10} = 1 \\
\end{array}
\]

\(a = 1, b = 0, c = -100\)

\* So what are signs of factors?

\((x + \_)(x - \_)= 0\)

\[
\begin{array}{c}
(x + 10)(x - 10) = 0 \\
X + 10 = 0 \\
X - 10 = 0 \\
X = -10 \\
X = 10 \\
\end{array}
\]

\(C: 100 - 80 = 20\)

\(20 = 20\)