

3.4 Equation with variables on both sides Goal • Solve equations with variables on both sides.

VOCABULARY

Fill in “ EVALUATE ” expressions; and “ SOLVE ” equations.

Equations have 3 types of solutions

1. $X = 1$ SOLUTION
2. $X = \text{NO SOLUTION}$ OR $X = \emptyset$
3. $X = \text{ALL REAL NUMBERS (IDENTITY)}$ $X = \mathbb{R}$

EXAMPLE 1: Solve and Check

<p>Solve</p> $4x - 7 = \frac{1}{3}(9x - 15)$ $4x - 7 = 3x - 5$ $\begin{array}{r} 4x - 7 = 3x - 5 \\ -3x \quad -3x \\ \hline x - 7 = -5 \\ +7 \quad +7 \\ \hline x = 2 \end{array}$	<p>CHECK:</p> $C: 4(2) - 7 = \frac{1}{3}(9 \cdot 2 - 15)$ $8 - 7 = \frac{1}{3}(3)$ $1 = 1 \checkmark$ <p>SUBSTITUTE THE SOLUTION INTO THE ORIGINAL EQ!!</p>
---	---

EXAMPLE 2: Special Case #1

<p>Solve:</p> $\frac{1}{2}(6x + 18) = 3(x + 3)$ $3x + 9 = 3x + 9$ $\begin{array}{r} 3x + 9 = 3x + 9 \\ -3x \quad -3x \\ \hline 9 = 9 \checkmark \text{ TRUE} \end{array}$	<p>How can you check this solution?</p> <p>ANY NUMBER YOU PLUG IN WILL WORK. Mentally Try:</p> $C: x = 0 \rightarrow 9 = 9 \checkmark$ $C: x = 1 \rightarrow 12 = 12 \checkmark$
--	---

What is this special case? $X = \text{ALL REAL NUMBERS}$

How do you identify the solution? WHEN THE VARIABLES DROP OUT AND THE STATEMENT (CONSTANTS) IS TRUE THEN $X = \mathbb{R}$

EXAMPLE 3: Special Case #2

<p>Solve:</p> $5(3x - 2) = 3(5x + 2)$ $15x - 10 = 15x + 6$ $\begin{array}{r} 15x - 10 = 15x + 6 \\ -15x \quad -15x \\ \hline -10 \neq 6 \text{ FALSE} \end{array}$	<p>How can you check this solution?</p> <p>① TRY $x = 0$ IT SHOULD NOT CHECK $C: x = 0 \rightarrow -10 \neq 6$</p> <p>② DOUBLE CHECK STEPS TO MAKE SURE NO ERROR MADE</p>
---	--

What is this special case? $X = \text{NO SOLUTION}$

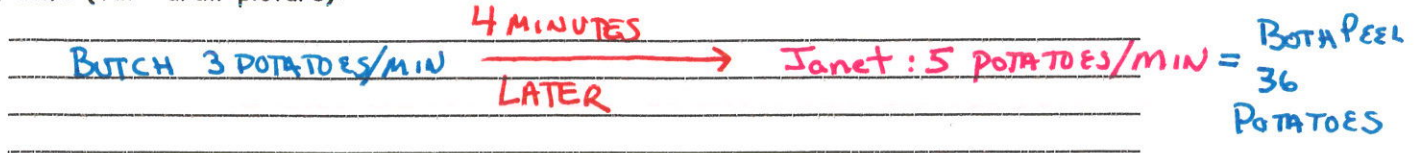
How do you identify the solution? WHEN THE VARIABLES DROP OUT AND THE STATEMENT IS FALSE THEN $X = \emptyset$

Honors Algebra 1 Notes...

EXAMPLE 4: Potato Peeling Problem Butch peels potatoes at the rate of 3 potatoes per minute. Four minutes later Janet joins him and peels at the rate of 5 potatoes per minutes. They peel a total number of 36 potatoes. How many minutes did Butch peel potatoes?

Word Problem Template (use this format to to solve all word problems assigned)

Key Info (TIP: draw picture):



Define Variable - remember units: $X = \# \text{ MINUTES BUTCH PEELS } 3 \text{ POTATOES}$

TIP to help you write the equation (mentally think about these expressions):

- Write an expression for the number of minutes Janet has peeled potatoes = $X - 4$
- Write an expression for the number of potatoes Butch has peeled = $3X$
- Write an expression for the number of potatoes Janet has peeled = $5(X - 4)$

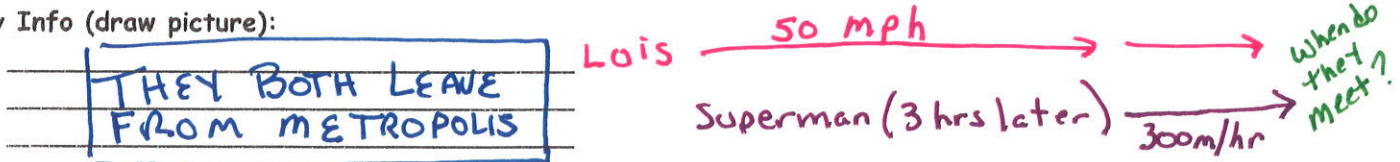
<p>Now write the equation → Define Equation: $3X + 5(X - 4) = 36$</p> <p>Solve: $3X + 5X - 20 = 36$ $8X - 20 = 36$ $+20 \quad +20$ <hr/> $8X = 56$ $\frac{8X}{8} = \frac{56}{8}$ $X = 7$</p>	<p>Check - $3(7) + 5(7 - 4) = 36$ $21 + 15 = 36$ $36 = 36 \checkmark$</p> <p>Does the solution make sense?</p> <ul style="list-style-type: none"> How many potatoes did Butch Peel? 21 POTATOES How many potatoes did Janet Peel? 15 POTATOES <p>36 Total</p>
--	---

Answer (in words) BUTCH PEELED FOR 7 MINUTES.

EXAMPLE 5: Lois and Superman Lois Lane leaves Metropolis driving 50m/hr. Three hours later Superman leaves Metropolis to catch her, flying 300m/hr. When does Superman catch up with Lois?

Word Problem Template (use this format to to solve all word problems assigned)

Key Info (draw picture):



Define Variable - remember units: $X = \text{Lois's } \# \text{ of hrs traveling}$ (Can define the var. in terms of Lois or Superman)

- Write an expression for Superman = $X - 3$ Remember $D = R \cdot T$

<p>Now write the equation → Define Equation: $50X = 300(X - 3)$</p> <p>Solve: $50X = 300X - 900$ $+900 \quad -50X \quad -50X \quad +900$ <hr/> $900 = 250X$ $\frac{900}{250} = \frac{250X}{250}$ $X = 3.6$</p>	<p>Check - $50(3.6) = 300(3.6 - 3)$ $180 = 180 \checkmark$</p> <p>Does the solution make sense? 180 miles away.</p> <ul style="list-style-type: none"> How far were they from Metropolis? 180 miles away.
--	---

Answer (in words) Superman catches Lois in 3.6 hrs

3.5 Write Ratios and Proportions Goal • Find ratios and write and solve proportions.

VOCABULARY

Ratio THE USE OF DIVISION TO COMPARE 2 QUANTITIES.

1. A ratio uses Division to compare two quantities.
2. The ratio of two quantities, a and b , where b is not equal to 0, can be written in three ways:

$$\frac{a}{b} \qquad a:b \qquad a \text{ to } b$$

3. Each ratio is read "the ratio of a to b "

4. Ratios should be written in simplest form. **Example: Simplify** $\frac{60}{40} = \frac{6}{4} = \frac{3}{2}$
 Reduce + keep as an improper fraction

EXAMPLE 1: Write a ratio

Cell Phone Use A person makes 6 long distance calls and 15 local calls in 1 month.

a) Find the ratio of long distance calls to local calls.

$$\frac{\text{long distance calls}}{\text{local calls}} = \frac{6}{15} = \frac{2}{5}$$

simplify

b) Find the ratio of long distance calls to all calls.

$$\frac{\text{LONG} \rightarrow \text{long distance calls}}{\text{LONG+Local} \rightarrow \text{ALL calls}} = \frac{6}{6+15} = \frac{6}{21} = \frac{2}{7}$$

simplify

EXAMPLE 2: Shawn and Myra are selling tickets to their school's talent show. Shawn sold 36 tickets, and Myra sold 44 tickets. Find the specified ratio. [show your work]

a) The number of tickets Shawn sold to the number of tickets Myra sold.

Write in words \rightarrow $\frac{\text{Shawn}}{\text{Myra}} = \frac{36}{44} = \frac{9}{11}$

b) The number of tickets Myra sold to the number of tickets Shawn and Myra sold

$$\frac{\text{Myra}}{\text{Shawn+Myra}} = \frac{44}{36+44} = \frac{44}{80} = \frac{11}{20}$$

Honors Algebra 1 Notes...

VOCABULARY

Proportion IS 2 RATIOS THAT ARE EQUAL

EXAMPLE 3: Solve a proportion

Solve the proportion

$$\frac{y}{15} = \frac{3}{5}$$

Do you remember? Cross multiply and divide.

- 1) CROSS MULTIPLY $5 \cdot y = 3 \cdot 15$
- 2) DIVIDE $\frac{5y}{5} = \frac{45}{5}$

$$y = 9$$

3) DON'T FORGET TO CHECK!!!!

$$C: \frac{9}{15} = \frac{3}{5}$$

$.6 = .6 \checkmark$

EXAMPLE 4: Solve a proportion word problem

A restaurant owner uses 3 cloves of garlic for every 5 pints of sauce. The restaurant uses 210 pints of sauce during the day. Find the number of cloves of garlic the restaurant uses to make the sauce.

STEP 1 Write a proportion involving two ratios that compare the amount of garlic to the pints of sauce; define the variable you solving for.

KEY INFO

3 CLOVES to 5 PINTS SAUCE
Total 210 pints sauce

$$\frac{\text{GARLIC}}{\text{SAUCE}} = \frac{3}{5} = \frac{x}{210}$$

$x = \# \text{ garlic cloves}$

STEP 2 Solve the proportion.



$$\frac{630}{5} = \frac{5x}{5}$$
$$x = 126$$

STEP 3 Check; does your answer make sense?

$$\frac{3}{5} = \frac{126}{210}$$
$$.6 = .6 \checkmark$$

STEP 4 Write your answer in a sentence

➤ The restaurant uses 126 cloves of garlic to make the sauce.

~~Done!!!!~~

Honors Algebra 1 Notes...

3.6 Solve Proportions Using Cross Products Goal • Solve proportions using cross products.

VOCABULARY CROSS PRODUCTS PROPERTY:

Words The cross products of a proportion are equal.

Algebra If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$ → then $ad = bc$
 ↖ cross product

Example

$$\frac{5}{8} = \frac{10}{12} \rightarrow \begin{array}{l} 6 \cdot 10 = 60 \\ 5 \cdot 12 = 60 \end{array}$$

If the cross products are equal, then it proves it is a true proportion.

EXAMPLE 1: Determining if ratios are proportions

<p>a) Is this a proportion $\frac{14}{35} = \frac{6}{15}$ $(14)(15) = (6)(35)$ $210 = 210 \checkmark$ TRUE</p>	<p><u>Explain:</u> PROPORTION BECAUSE CROSS PRODUCTS ARE EQUAL.</p>
<p>b) Is this a proportion $\frac{18}{17} = \frac{27}{24}$ $(18)(24) = (17)(27)$ $432 \neq 459$ FALSE</p>	<p><u>Explain:</u> NOT A PROPORTION BECAUSE CROSS PRODUCTS ARE NOT EQUAL.</p>

EXAMPLE 2: Solve a proportion using cross products

<p>Solve</p> $\frac{5}{y} = \frac{15}{75}$ $\frac{15y}{15} = \frac{375}{15}$ $y = 25$	<p><u>CHECK:</u></p> $C: \frac{5}{25} = \frac{15}{75}$ $.2 = .2 \checkmark$
--	---

EXAMPLE 3: Solve a proportion using cross products

<p>Solve</p> $\frac{6}{x} = \frac{3}{x-2}$ $3x = 6(x-2)$ $3x = 6x - 12$ $\begin{array}{r} -6x \quad -6x \\ \hline -3x = -12 \\ \hline -3 \quad -3 \\ \hline x = 4 \end{array}$	<p><u>CHECK:</u></p> $C: \frac{6}{4} = \frac{3}{4-2}$ $1.5 = \frac{3}{2}$ $1.5 = 1.5 \checkmark$
---	--

Honors Algebra 1 Notes...

EXAMPLE 4: Solve a proportion word problem

Plant Food To feed your plants, you need to mix 3 tablespoons of plant food with 16 ounces of water. If it takes 80 ounces of water to feed all of your plants, how many tablespoons of plant food are needed?

STEP 1 Write a proportion involving two ratios that compare the amount of plant food with the amount of water amount of plant food amount of water; define the variable you solving for.

KEY INFO

3T plant food to 16oz water
Total 80oz Water for all plants

$X = \# \text{ of T of plant food}$

Proportion $\frac{\text{Food}}{\text{Water}} = \frac{3\text{T food}}{16\text{oz Water}} = \frac{X}{80\text{oz}}$

STEP 2 Solve the proportion.

$$\frac{3}{16} = \frac{X}{80}$$
$$\frac{16X}{16} = \frac{240}{16}$$
$$X = 15$$

STEP 3 Check; does your answer make sense?

C: $\frac{3}{16} = \frac{15}{80}$
 $.1875 = .1875 \checkmark$

STEP 4 Write your answer in a sentence

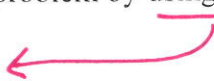
➤ You need 15 tablespoons of plant food for 80 ounces of water.

3.7 Solve Percent Problems Goal • Solve percent problems.

VOCABULARY

Method 1: You can represent percent problem by using the proportion

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100}$$



TYPES OF PERCENT PROBLEMS

EXAMPLE 1: Find a percent using a proportion. What percent of 50 is 33?

STEP 1 Write a proportion:

$$\frac{P}{100} = \frac{33}{50}$$

STEP 2 Solve the proportion:

$$\begin{aligned} 50P &= 3300 \\ \frac{50P}{50} &= \frac{3300}{50} \\ P &= 66\% \end{aligned}$$

$$\begin{aligned} C: \frac{66}{100} &= \frac{33}{50} \\ .66 &= .66 \checkmark \end{aligned}$$

STEP 3 Check:

EXAMPLE 2: Find a number using a proportion. 27 is 25% of what number?

STEP 1 Write a proportion:

$$\frac{25}{100} = \frac{27}{N}$$

STEP 2 Solve the proportion:

$$\begin{aligned} 25N &= 2700 \\ \frac{25N}{25} &= \frac{2700}{25} \\ N &= 108 \end{aligned}$$

$$\begin{aligned} C: \frac{25}{100} &= \frac{27}{108} \\ .25 &= .25 \checkmark \end{aligned}$$

STEP 3 Check:

EXAMPLE 3: Find a number using a proportion. What number is 32.5% of 60? **N=19.5**

STEP 1 Write a proportion:

$$\frac{32.5}{100} = \frac{N}{60}$$

STEP 2 Solve the proportion:

$$\begin{aligned} 100N &= 1950 \\ \frac{100N}{100} &= \frac{1950}{100} \\ N &= 19.5 \end{aligned}$$

$$\begin{aligned} C: \frac{32.5}{100} &= \frac{19.5}{60} \\ .325 &= .325 \checkmark \end{aligned}$$

STEP 3 Check:

Honors Algebra 1 Notes...

VOCABULARY

Method 2: You can represent percent problem using an equation by translating the words into an algebraic equation.

- IS means EQUALS =
- OF means MULTIPLY
- And Change %'s to decimals

TYPES OF PERCENT PROBLEMS

EXAMPLE 1: Find a percent using an equation. What percent of 252 is 84? P

STEP 1 Write equation:
$$\frac{P}{252} \cdot 252 = 84$$

STEP 2 Solve the equation:

$$P = .333\bar{3} \rightarrow P = 33.33\%$$

Change Decimal \rightarrow %

STEP 3 Check:

C: $(.3333)(252) = 84$
 $83.99 \approx 84$

APPROXIMATE

ALWAYS
ROUND TO
2 DECIMALS
UNLESS
OTHERWISE
TOLD!

EXAMPLE 2: Find a number using an equation. 360 is 225% of what number? N **N=160**

STEP 1 Write a equation:

$$\frac{360}{2.25} = \frac{2.25 \cdot N}{2.25}$$

STEP 2 Solve the equation:

$$N = 160$$

STEP 3 Check:

C: $360 = 2.25(160)$
 $360 = 360 \checkmark$

EXAMPLE 2: Find a number using an equation. What number is 3.5% of 90? N **N=3.15**

STEP 1 Write a equation:

$$N = .035 \cdot 90$$
$$N = 3.15$$

STEP 2 Solve the equation:

STEP 3 Check:

C: $3.15 = (.035)(90)$
 $3.15 = 3.15 \checkmark$

3.8 Rewrite Equations and Formulas Goal • Write equations in function form and rewrite formulas.

VOCABULARY

Literal equation IS AN EQUATION WITH MULTIPLE VARIABLES THAT REPRESENT KNOWN VALUES.

EXAMPLE 1: Solve a literal equation. Typically we are given a formula and need to manipulate the formula to find the variable of interest.

PROBLEM: The interest I on an investment of P dollars at an interest rate r for t years is given by the formula $I = Prt$. Find the time it takes to earn \$100 interest on \$1000 at a rate of 5.0%.

STEP 1: Solve the formula for the time t .

$$I = \frac{P}{PR} \cdot \frac{r}{PR} \cdot t \quad \text{ISOLATE } t$$

$$\boxed{t = \frac{I}{PR}} \quad \text{New formula}$$

STEP 2: Use the rewritten formula to find the time it takes to earn \$100 interest on \$1000 at a rate of 5.0%.

$$t = \frac{I}{PR}$$

$$t = \frac{100}{(1000)(.05)}$$

$$t = \frac{100}{50} = 2$$

$$\boxed{t = 2 \text{ YEARS}}$$

$\left\{ \begin{array}{l} I = \text{INTEREST} = \$100 \\ P = \text{INVESTMENT} = \$1,000 \\ R = \text{interest Rate} = 5\% = .05 \end{array} \right.$
 substitute these values INTO THE NEW FORMULA.

EXAMPLE 2: Find the investment P if $I = \$400$, $r = 4\%$, and $t = 4$ years. Write a formula for investment P .

Formula $\frac{I}{RT} = \frac{P}{RT} \cdot \frac{r}{RT} \cdot t$ Isolate "P"

$$P = \frac{I}{RT}$$

$$P = \frac{400}{(.04)(4)}$$

$\left\{ \begin{array}{l} I = \$400 \\ r = 4\% = .04 \\ t = 4 \end{array} \right.$

$$P = \frac{400}{.16}$$

$$\boxed{P = \$2,500 \text{ INVESTED}}$$

Honors Algebra 1 Notes...

VOCABULARY

Function form IS THE SAME AS $Y=MX+B$ TO GRAPH LINES.

GIVEN AN EQUATION WITH X AND Y, REWRITE IN FUNCTION FORM IN TERMS OF X. YOU ISOLATE Y
↳ "y = _____"

EXAMPLE 3: Rewrite an equation in function form

PROBLEM: Write $2x - 2y = 10$ in function form.

$$\begin{array}{r} 2x - 2y = 10 \\ \underline{-2x} \quad \underline{-2x} \end{array}$$

← ISOLATE Y

$$\begin{array}{r} -2y = -2x + 10 \\ \underline{-2} \quad \underline{-2} \quad \underline{-2} \end{array}$$

$$\boxed{y = x - 5}$$

← EQUATION IN FUNCTION FORM

EXAMPLE 4: Write the equation in function form.

$$\begin{array}{r} 9 + 6x = 3 - 6y - 9x \\ \underline{+9x} \quad \underline{+9x} \end{array}$$

← ISOLATE Y

← X IS ON THE OPPOSITE SIDE OF Y

$$\begin{array}{r} 9 + 15x = -6y + 3 \\ \underline{-3} \quad \underline{-3} \end{array}$$
$$\begin{array}{r} -6y = 15x + 6 \\ \underline{-6} \quad \underline{-6} \quad \underline{-6} \end{array}$$

$$\boxed{y = -\frac{5}{2}x - 1}$$

KEEP AS AN IMPROPER FRACTION
IN THE NEXT CHAPTER WE LEARN
THIS IS THE SLOPE OF THE LINE.