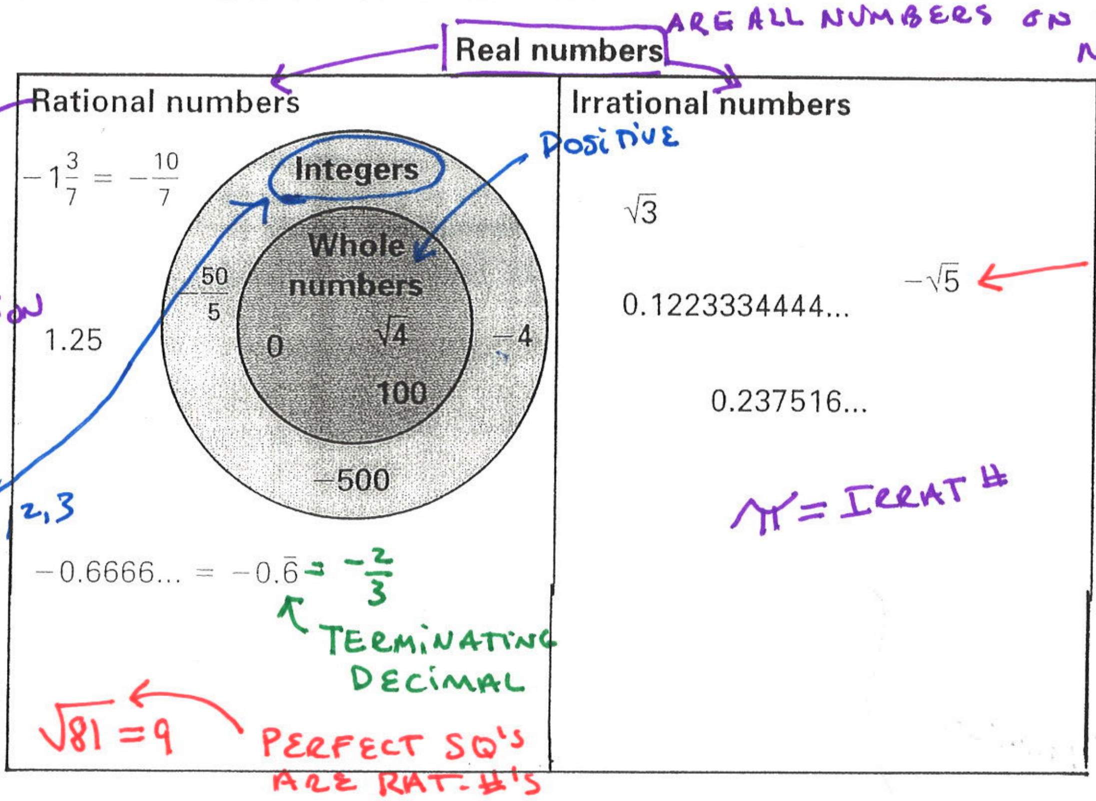
1.1 Lesson Opener

DATE: ______________________

REAL NUMBERS AND NUMBER OPERATIONS

The Venn diagram below shows subsets of the real numbers, the numbers used most often in algebra.



1 Simplify $\frac{6}{3}$. Is $\frac{6}{3}$ an integer? Is it a whole number?

2 All terminating decimals (such as 1.25) and all repeating decimals (such as $0.\overline{6}$) can be written as the ratio of two integers. How can you write 1.25 and $0.\overline{6}$ as ratios?

3 Place each number in the correct region of the diagram.

293 W, I, R $\quad -47\frac{5}{9}$ RAT. $\quad$ 0.184 RAT. $\quad \frac{6}{3} = 2$ W, I, RAT

$\sqrt{7}$ IRRAT $\quad -0.\overline{42}$ RAT $\quad \pi$ IRR $\quad -\frac{7}{1}$ -7 I, RAT $\quad -\sqrt{9}$ -3 I, RAT $\quad 4\frac{2}{3}$ RAT

LESSON 1.1

NAME ____________ DATE ____________

Practice with Examples

For use with pages 3–10

GOAL Use a number line to graph and order real numbers and identify properties of and use operations with real numbers

VOCABULARY

The **graph** of a real number is the point on a real number line that corresponds to the number. On a number line, the numbers increase from left to right, and the point labeled 0 is the origin.

The number that corresponds to a point on a number line is the **coordinate** of the point.

The **opposite,** or *additive inverse,* of any number a is $-a$.

The **reciprocal,** or *multiplicative inverse,* of any nonzero number a is $\frac{1}{a}$.

EXAMPLE 1

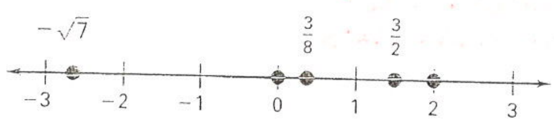
Graphing and Ordering Real Numbers

EXAMPLE

Graph and write the numbers in increasing order: $-\sqrt{7}, 0, \frac{3}{2}, 2, \frac{3}{8}$.

SOLUTION

$-\sqrt{7} \approx -2.6, \frac{3}{2} = 1.5, \frac{3}{8} \approx 0.4$ Rewrite each number in decimal form.



Plot the points on the real number line.

$-\sqrt{7}, 0, \frac{3}{8}, \frac{3}{2}, 2$ Write the numbers from least to greatest.

Exercises for Example 1

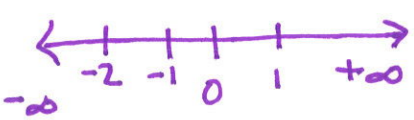
Write the numbers in increasing order.

1. $1, \frac{1}{3}, \sqrt{2}$ — $\frac{1}{3}, 1, \sqrt{2}$

2. $\frac{3}{5}, -1, 1$ — $-1, \frac{3}{5}, 1$

3. $\sqrt{5}, \frac{2}{3}, 3.25$ (2.2, .66) — $\frac{2}{3}, \sqrt{5}, 3.25$

4. $-4, 1, -1$ — $-4, -1, 1$



5. $0, -2, \frac{1}{3}$ — $-2, 0, \frac{1}{3}$

6. $-\sqrt{2}, -15, 5.7$ — $-15, -\sqrt{2}, 5.7$

7. $\frac{5}{2}, -5, -10$ (→2.5) — $-10, -5, \frac{5}{2}$

8. $3, -5, 0$ — $-5, 0, 3$

LESSON 1.1 CONTINUED

DATE ____________

Practice with Examples

For use with pages 3–10

Need to KNOW THESE PROPERTIES

EXAMPLE 2 Identifying Properties of Real Numbers

Identify the property shown.

a. $5(10 + 2) = 5 \cdot 10 + 5 \cdot 2$

b. $(6 \cdot 4)5 = 6(4 \cdot 5)$

SOLUTION

a. Distributive property

b. Associative property of multiplication

Exercises for Example 2

Identify the property shown.

9. $5 + 3 = 3 + 5$

$5 \cdot 3 = 3 \cdot 5$

$4 \cdot x = x \cdot 4$

COMMUNATIVE PROPERTY

10. $7 + (-7) = 0$

OPPOSITES

ADDITION INVERSE PROPERTY

11. $-2 + 0 = -2$ ← ADDITION

$-2 \cdot 1 = -2$ ← MULT

IDENTITY PROPERTY

12. $2(x + 1) = 2 \cdot x + 2 \cdot 1$

$5(x - 6) = 5 \cdot x + 5(-6)$

DISTRIBUTIVE PROPERTY

13. $8 \cdot \frac{1}{8} = 1$ ← reciprocal

MULT INVERSE PROPERTY

14. $(5 + 7) + 3 = 5 + (7 + 3)$

$(5 \cdot 7) \cdot 3 = 5 \cdot (7 \cdot 3)$

* NOTICE #'S ARE IN THE SAME ORDER

* ( )'S CHANGE

ASSOCIATIVE PROPERTY

Chapter 1

LESSON 1.1 CONTINUED

NAME ____________ DATE ____________

Practice with Examples

For use with pages 3–10

UNIT ANALYSIS

EXAMPLE 3 Operations with Real Numbers

Example

At rest, the average person's heart beats 65 times per minute. During aerobic exercise, this rate increases by 40%.

a. How many times does the average person's heart beat per hour?

b. How many times will the average person's heart beat per minute during aerobic exercise?

SOLUTION

M/H

a. $\left(\frac{65 \text{ beats}}{1 \text{ minute}}\right)\left(\frac{60 \text{ minutes}}{1 \text{ hour}}\right) = 3900$ beats per hour

✓65·60

b. To find 40% of 65, multiply.

$40\% \times 65 = 0.4 \times 65$ Rewrite 40% as 0.4.

$= 26$ Simplify.

140% ... 1.4 ... 91

During aerobic exercise, the average person's heart would beat $65 + 26 = 91$ times per minute.

2 OR 91 beat/MIN

Exercises for Example 3

In Exercises 15 and 16, use the following information.

At Indianapolis Motor Speedway, one lap is 2.5 miles in length. The average speed of an Indy racing car is 190 miles per hour.

15. Find the length of one lap in yards.

KI: 1 lap = 2.5 miles
Rate 190 mph

$\frac{2.5 \text{ miles}}{1 \text{ lap}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ yds}}{3 \text{ ft}} = 4,400$ Yards per lap

CALC: 2.5 · 5280 ÷ 3

16. How many seconds would it take to complete one lap?

(ROUND TO seconds)

$\frac{2.5 \text{ miles}}{1 \text{ lap}} \cdot \frac{1 \text{ hr}}{190 \text{ miles}} \cdot \frac{60 \text{ MIN}}{1 \text{ hr}} \cdot \frac{60 \text{ SEC}}{1 \text{ MIN}} = 47 \frac{\text{Seconds}}{\text{lap}}$

CALC 2.5 · 60 · 60 [enter] ÷ 190 = 47.3

Chapter 1

NAME ____________________ DATE __________

Practice with Examples

For use with pages 11–17

GOAL Evaluate algebraic expressions and simplify algebraic expressions by combining like terms

VOCABULARY

A **variable** is a letter that is used to represent one or more numbers.

An **algebraic expression** is an expression involving variables.

Like terms are expressions that have the same variable part. **Constant terms** such as -4 and 2 are also like terms.

The **base** of an exponent is the number or variable that is used as a factor in repeated multiplication. For example, in the expression 4^b, 4 is the base.

An **exponent** is the number or variable that represents the number of times the base is used as a factor. For example, in the expression 4^b, b is the exponent.

A **power** is the result of repeated multiplication. For example, in the expression $4^2 = 16$, 16 is the second power of 4.

Any number used to replace a variable is a **value of the variable**.

When the variables in an algebraic expression are replaced by numbers, the result is called the **value of the expression**.

Terms are the parts that are added in an expression, such as 5 and $-x$ in the expression $5 - x$.

A **coefficient** is the number multiplied by a variable in a term.

Two algebraic expressions are **equivalent** if they have the same value for all values of their variable(s).

EXAMPLE 1 Using Order of Operations

$2(3 + 18 \div 3^2 - 7) = 2(3 + 18 \div 9 - 7)$	Evaluate the power.
$= 2(3 + 2 - 7)$	Divide.
$= 2(-2)$	Add within parentheses.
$= -4$	Multiply.

Exercises for Example 1

Evaluate the expression.

1. $(-1 + 3) - 4^2$

2. $14 - 12 \div 3$

3. $(-5)^3$

4. $5 - (-2 + 4)^2$

5. $36 \div (-3)^2 - 1$

6. -5^2

LESSON **1.2** CONTINUED

NAME ______________________ DATE __________

Practice with Examples

For use with pages 11–17

EXAMPLE 2 Evaluating an Algebraic Expression

Evaluate $2t^2 - 3$ when $t = 4$.

SOLUTION

$2t^2 - 3$	$= 2(4)^2 - 3$	Substitute 4 for t.
	$= 2(16) - 3$	Evaluate the power.
	$= 32 - 3$	Multiply.
	$= 29$	Subtract.

EVALUATING EXPONENTS

$(-2)^3 = -2 \cdot -2 \cdot -2 = -8$

ODD EXPONENT - RESULT IS (–)

$(-2)^4 = -2 \cdot -2 \cdot -2 \cdot -2 = 16$

EVEN EXPONENT - RESULT IS (+)

$-2^4 = -16$ NO ( )'S → Result is (–)

Chapter 1

Exercises for Example 2

① Remember use ( )'s when substituting

Evaluate the expression.

7. $x^2(4 - x)$ when $x = 2$

$(2)^2(4-2) = 4 \cdot 2 \rightarrow$ 8

① Show substitution ② Simplify ③ Circle

8. $x - (x + 5)$ when $x = 20$

$20 - (20+5) =$ –5

9. $x^2 + 5$ when $x = -3$

$(-3)^2 + 5 = 9 + 5$ → 14

10. $3x^3 + 4$ when $x = -2$

$3(-2)^3 + 4 =$

$3(-8) + 4 =$

$-24 + 4 = -20$

–20

11. $4x - 3y + 2$ when $x = 4$ and $y = -3$

$4(4) - 3(-3) + 2 =$

$16 + 9 + 2 =$ 27

12. $9(m - n)^2$ when $m = 4$ and $n = 1$

$9(4-1)^2 = 9(3)^2 = 9 \cdot 9 =$ 81

LESSON 1.2 CONTINUED

NAME ______________________ DATE __________

Practice with Examples

For use with pages 11–17

EXAMPLE 3 Simplifying by Combining Like Terms

Simplify $6(x - y) - 4(x - y)$.

SOLUTION

$6(x - y) - 4(x - y)$	$= 6x - 6y - 4x + 4y$	Distributive property
	$= (6x - 4x) + (-6y + 4y)$	Group like terms.
	$= 2x - 2y$	Combine like terms.

Chapter 1

Exercises for Example 3

Simplify the expression.

13. $7x - (9x + 5)$

$7x - 9x - 5 =$

$-2x - 5$

14. $2(n^2 + n) - 5(n^2 - 4n)$

$2n^2 + 2n - 5n^2 + 20n$

$-3n^2 + 22n$

IMPORTANT HOW TO ORDER EXPRESSIONS

ORDER TERMS: H → L EXPONENTS; constant last

ORDER TERMS: VARIABLES ABC order; constant last

15. $-6x^2 + 4x - x^2 + 15x$

$-7x^2 + 19x$

16. $7x - 2y + 3 - 9y + 4 + 5x$

$12x - 11y + 7$

NAME ______________________________ DATE __________

Practice with Examples

For use with pages 19–24

GOAL Solve linear equations and use linear equations to answer questions about real-life situations

Solve Equations: To find the value of the variable.

Vocabulary

An **equation** is a statement in which two expressions are equal.

A **linear equation** in one variable is an expression that can be written in the form $ax = b$ where a and b are constants and $a \neq 0$.

A number is a **solution** of an equation if the statement is true when the number is substituted for the variable.

Two equations are **equivalent** if they have the same solutions.

EXAMPLE 1 Variable on One Side

Solve $-19 = -2y + 5$.

Solution

$-19 = -2y + 5$ Write original equation.

$-24 = -2y$ To isolate y, subtract 5 from each side.

$12 = y$ Divide each side by -2.

Exercises for Example 1

SOLVE AND CHECK

Solve the equation.

1. $3 = -x - 2$ $x = -5$

   $+2 \quad +2$; $5 = -x$; $\frac{5}{-1} = \frac{-x}{-1}$; $x = -5$

2. $-18 = y + 6$ $y = -24$

   $-6 \quad -6$; $y = -24$

3. $9 - z = 5$ $z = 4$

   $-9 \quad -9$; $-z = -4$; $\frac{-z}{-1} = \frac{-4}{-1}$; $z = 4$

Always check in original eq!

$3 = -(-5) - 2$

$3 = 3$ ✓

Use calc to check!!

4. $6 + 6x = -12$ $x = -3$

   $-6 \quad -6$; $6x = -18$; $\frac{6x}{6} = \frac{-18}{6}$; $x = -3$

5. $2x - 5 = 1$ $x = 3$

   $+5 \quad +5$; $2x = 6$; $\frac{2x}{2} = \frac{6}{2}$; $x = 3$

6. $-\frac{x}{3} = 2$ $x = -6$

   $-3\left(-\frac{x}{3}\right) = (2)(-3)$; $x = -6$

   Multiply by the reciprocal

NAME ______________________ DATE __________

Practice with Examples

For use with pages 19–24

STEP 1: GET VARIABLE ON THE SAME SIDE
STEP 2: UNDO +, −
STEP 3: UNDO ×, ÷
STEP 4: ALLWAYS CHECK IN ORIGINAL EQ! USE CALC!

EXAMPLE 2 Variable on Both Sides

Solve $4x - 2x = 15 - 3x$.

Solution

$4x - 2x = 15 - 3x$	Write original equation.
$2x = 15 - 3x$	Combine like terms.
$5x = 15$	To collect the variable terms, add $3x$ to each side.
$x = 3$	Divide each side by 5.

Exercises for Example 2

Solve the equation.

7. $15 - 3a = -4a + 16$ A=1
   +4A +4A
   15+A = 16
   −15 −15
   A = 1
   C: 12=12 ✓

(B) 8. $-3m + 6 = 24m + 6$ M=0
   +3m +3M
   6 = 27m+6
   −6 −6
   0 = 27m
   27 27
   m=0
   C: 6=6 ✓

11. $x - 4 = 2x + 7$ X=−11
   −X −X
   −4 = X+7
   −7 −7
   X=−11
   C: −15=−15 ✓

12. $4x = 24 + 16x$ X=−2
   −16X −16X
   −12X = 24
   −12 −12
   X = −2
   C: −8=−8 ✓

(A) 9. $4s - 6 = 7s + 3$ S=−3
   −4s −4s
   −6 = 3S+3
   −3 −3
   3S= −9
   3 3
   S=−3
   C: −18=−18 ✓

(C) 10. $8t - t + 1 = 10 - 2t$ T=1
   ALWAYS SIMPLIFY IF POSSIBLE
   7T+1 = −2T+10
   +2T +2T
   9T+1 = 10
   −1 −1
   9T=9
   9 9
   T=1
   C: 8=8 ✓

EXAMPLE 3 *Using the Distributive Property*

Solve $15(4 - y) = 5(10 + 2y)$.

SOLUTION

$15(4 - y) = 5(10 + 2y)$	Write original equation.
$60 - 15y = 50 + 10y$	Distributive property
$60 = 50 + 25y$	To collect the variable terms, add 15*y* to each side.
$10 = 25y$	Subtract 50 from each side.
$\frac{2}{5} = y$	Divide each side by 25.

WHEN POSSIBLE, SIMPLIFY BOTH SIDES. (NOTICE PINK IS COMPLETELY SIMPLIFIED)

Then follow same steps in Example #2 (prior page)

Exercises for Example 3

Solve the equation.

13. $5(x - 3) + 12 = -2(x - 2)$ $\boxed{x=1}$

$5x-15+12 = -2x+4$

$5x-3 = -2x+4$

$+2x \qquad +2x$

$7x-3 = 4$

$+3 \quad +3$

$7x = 7$

$\frac{7x}{7} = \frac{7}{7}$

$x=1$

C: 2=2 ✓

14. $-4(k - 2) + 3(k + 1) = 7$ $\boxed{k=4}$

$-4k+8+3k+3=7$

$-k+11=7$

$-11 \quad -11$

$-k = -4$

$\frac{-k}{-1} = \frac{-4}{-1}$

$k=4$

C: 7=7 ✓

15. $-2x = 2(x + 1)$ $\boxed{x=-1/2}$

$-2x = 2x+2$

$-2x \quad -2x$

$-4x = 2$

$\frac{-4x}{-4} = \frac{2}{-4}$

$x = -1/2$ or $-.5$

C: 1=1 ✓

16. $3x - 9 = 2(x - 5)$ $\boxed{x=-1}$

$3x-9 = 2x-10$

$-2x \quad -2x$

$x-9 = -10$

$+9 \quad +9$

$x = -1$

C: -12=-12 ✓

EXAMPLE 4 Solving an Equation with Fractions

Solve $\frac{2}{3}x + \frac{3}{5} = \frac{4}{15}$.

SOLUTION

$\frac{2}{3}x + \frac{3}{5} = \frac{4}{15}$	Write original equation.
$15\left(\frac{2}{3}x + \frac{3}{5}\right) = 15\left(\frac{4}{15}\right)$	Multiply each side by the LCD, 15.
$10x + 9 = 4$	Distributive property
$10x = -5$	To isolate x, subtract 9 from each side.
$x = -\frac{1}{2}$	Divide each side by 10.

ADD $\frac{1}{5}(25x - 10) = 18$

DISTRIBUTE 1/5 → $5x - 2 = 18$

$+2 \quad +2$

$5x = 20$

$\frac{5x}{5} = \frac{20}{5}$

$x = 4$

C: 18=18 ✓

Exercises for Example 4

Solve the equation.

SKIP **17.** $6n = \frac{2}{3}(5n - 2)$ $N = -1/2$

18. $\frac{3}{4}x + 1 = 4$ $X = 4$

$-1 \quad -1$

mult by reciprocal → $\left(\frac{4}{3}\right)\frac{3}{4}x = 3\left(\frac{4}{3}\right)$

$x = 4$

C: 4=4 ✓

SKIP **19.** $\frac{1}{2}x - \frac{2}{3} = 4x$ $x = -4/21$ $x \approx .19$

SKIP **20.** $\frac{3}{5}x = \frac{2}{3}x + 1$ $x = -15$