4.2b Notes

There are 5 METHODS TO SOLVE Quadratic FUNCTIONS

Method #1 - Graphing

SOLUTIONS ARE THE X-INTERCEPTS (x,0)

ALSO CALLED ROOTS

EXAMPLE ***Finding the Zeros of a Quadratic Function*** using the TI Calc.

Find the zeros of $y = x^2 - 6x$. → Graph the FUNCTION — STEP 1 IS TO LOOK AT THE TABLE FOR ZERO's (x, 0)

STEP 2

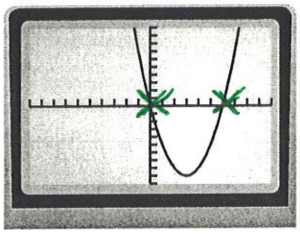
TI Calc - sketch each graph and find the x-intercepts

1) *x-intercepts always have a y-coordinate of 0.* ↗ Solutions are Integers

2) *use [2nd][calc] --> 2:zero* → USE FOR X-INT WITH Decimals

See #2

- Use [trace] to estimate x intercepts
- [2ND] [calc] → 2:ZERO
- FOR EACH X INT Mark the LEFT + RIGHT BOUND (← → arrows)
- 3 ENTERS → ZERO X= Y=0



V

X	Y
0	0
3	-9
6	0

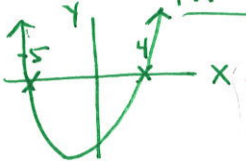
Solution to this QF is $\boxed{x = 0, 6}$

Exercises for Example ●

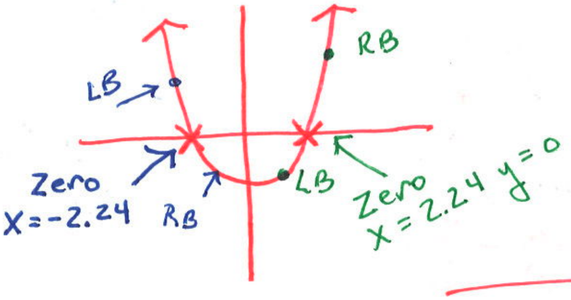
Sketch each graph and find the zeros

1) $y = x^2 + x - 20$ $\boxed{x = -5, 4}$

X	Y
-5	0
4	0

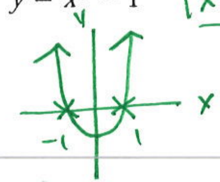


2) $y = x^2 \cancel{+} 5$ $\boxed{x = 2.24, -2.24}$

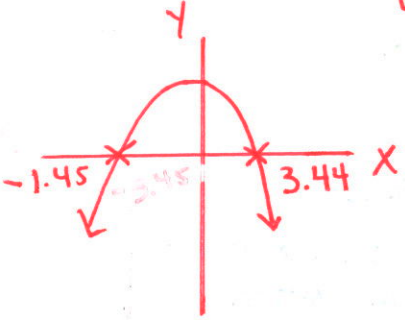


3) $y = x^2 - 1$ $\boxed{x = \pm 1}$

X	Y
-1	0
1	0

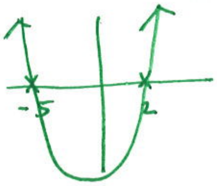


4) $y = -x^2 + 2x + 5$ $\boxed{x = -1.45, 3.44}$



5) $y = x^2 + 3x - 10$ $\boxed{x = -5, 2}$

X	Y
-5	0
2	0



5 Methods to Solve Quadratic Functions

1) **Graphing**
2) **Solve by finding square roots**
3) **Quadratic Formula**
4) **Completing the Square**
5) **Factoring**

$\quad\quad\quad A \quad\quad B \quad\quad C$

EX $y = -2x^2 + 5x + 6$

A=-2 opendown ∩

B=5

C=6 ← y intercept (0,6)

① Graph and clearly mark 5-6 points

② FIND + Label the Vertex

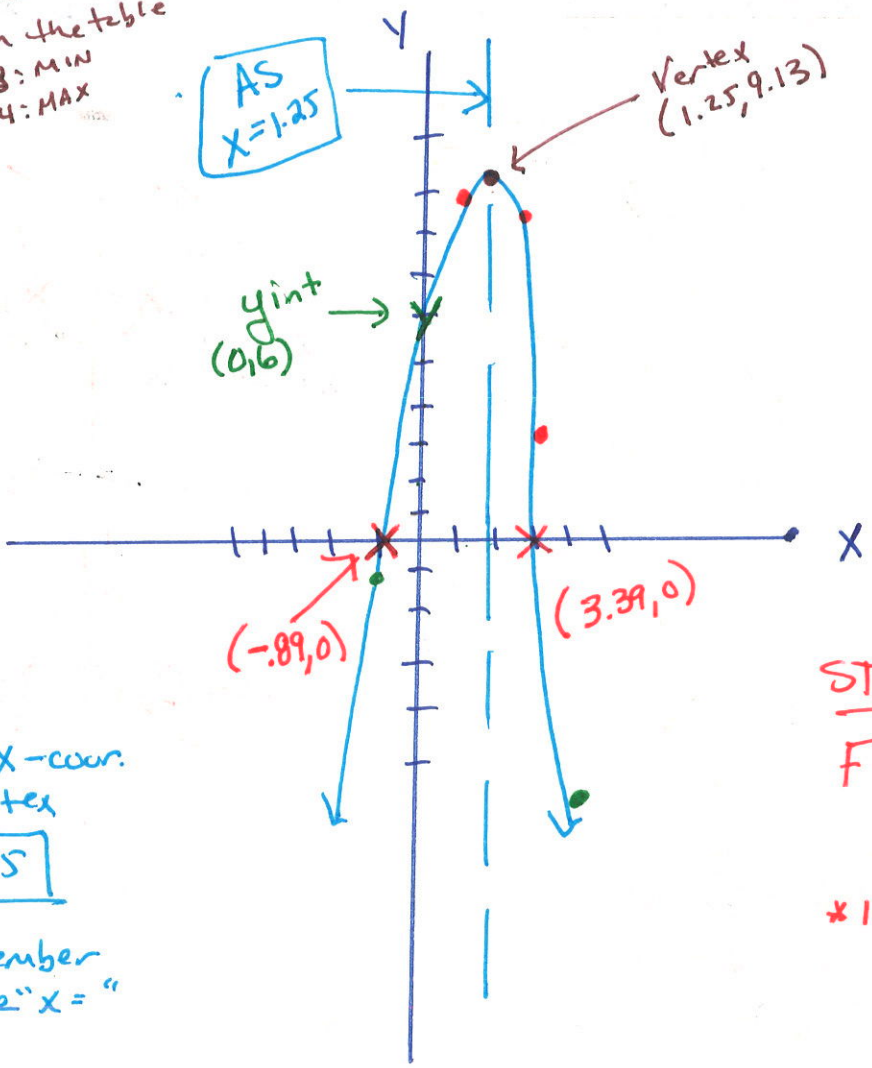
③ FIND + Label the A.S.

④ FIND THE Solutions (AKA X intercepts, ZEROS, ROOTS) AND Label the Xint's

STEP I: FIND VERTEX

- Can't find in the table
- (2ND) Calc 3: MIN
  4: MAX

	X	Y
	0	6
	1	9
V	1.25	9.13
	2	8
	3	3
ADD	4	-6
	-1	-1



STEP II

FIND A.S

It is the X-coor. of the Vertex

AS X=1.25

remember the "X="

STEP III

FIND XINT (X,0)

* IN THE TABLE we have NO Xint that are INTEGERS

* USE (2ND) (CALC) 2: ZERO

SOLUTIONS X = -.89, 3.39

Method #2

Solving Quadratic Equations by Finding Square Roots

Goals
- Solve quadratic equations.
- Use quadratic equations to solve real-life problems.

Your Notes

VOCABULARY

Square root ex) $\sqrt{25} = \textcircled{5}$ | ex) $-\sqrt{25} = \textcircled{-5}$

Radical sign

$\sqrt{\quad}$

↑ SQ ROOTS

Rules to simplify radicals:
① NO PERFECT SQUARES UNDER THE RADICAL
② NO FRACTIONS UNDER THE RADICAL
③ NO RADICALS IN THE DENOMINATOR

Rationalizing the denominator is the process to eliminate a radical ($\sqrt{\quad}$) in the denominator (See example 1D)

PROPERTIES OF SQUARE ROOTS ($a > 0$, $b > 0$)

Product Property: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\boxed{\sqrt{a}}}{\boxed{\sqrt{b}}}$

List the perfect squares from 1 to 100

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Your Notes

Example 1 Using Properties of Square Roots

Simplify the expression.

What is the largest perfect SQUARE FACTOR OF 27

simplify $\sqrt{9}=3$

a. $\sqrt{27} = \sqrt{9} \cdot \sqrt{3} = \boxed{3\sqrt{3}}$

b. $\sqrt{6} \cdot \sqrt{15} = \sqrt{6 \cdot 15} = \sqrt{90} = \sqrt{9}\sqrt{10} = \boxed{3\sqrt{10}}$

c. $\sqrt{\dfrac{5}{36}} = \dfrac{\boxed{\sqrt{5}}}{\boxed{\sqrt{36}}} = \boxed{\dfrac{\sqrt{5}}{6}}$

Rationalize the denominator

d. $\sqrt{\dfrac{13}{3}} = \dfrac{\boxed{\sqrt{13}}}{\boxed{\sqrt{3}}} \cdot \dfrac{\boxed{\sqrt{3}}}{\boxed{\sqrt{3}}} = \boxed{\dfrac{\sqrt{39}}{3}}$

Cannot have a $\sqrt{\ }$ in the den.

PSQ Leftover

(e) $\sqrt{75} = \sqrt{25}\sqrt{3} = \boxed{5\sqrt{3}}$

(f) $\sqrt{\dfrac{81}{121}} = \dfrac{\sqrt{81}}{\sqrt{121}} = \boxed{\dfrac{9}{11}}$

QUESTIONS FOR 1D

- What is $\dfrac{\sqrt{3}}{\sqrt{3}} = 1$
- What is $\sqrt{3} \cdot \sqrt{3} = 3$

Checkpoint Simplify the expression.

1. $\sqrt{5} \cdot \sqrt{8}$

$\sqrt{40} =$

largest perf SQ

$\sqrt{4}\ \sqrt{10} =$

$\boxed{2\sqrt{10}}$

2. $\sqrt{\dfrac{3}{5}} = \dfrac{\sqrt{3}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \boxed{\dfrac{\sqrt{15}}{5}}$

GOAL **Solve quadratic equations by finding square roots and use quadratic equations to solve real-life problems**

VOCABULARY

If $b^2 = a$, then b is a **square root** of a. A positive number a has two square roots, $\sqrt{a}$ and $-\sqrt{a}$. The symbol $\sqrt{}$ is a **radical sign,** a is the **radicand,** and $\sqrt{a}$ is a **radical.**

Rationalizing the denominator is the process of eliminating square roots in the denominator of a fraction.

HW

EXAMPLE 1 *Using Properties of Square Roots*

Simplify the expression.

a. $\sqrt{99} = \sqrt{9}\sqrt{11} = 3\sqrt{11}$

b. $\sqrt{6} \cdot \sqrt{8} = \sqrt{48} = \sqrt{16}\sqrt{3} = 4\sqrt{3}$

c. $\sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{\sqrt{25}} = \frac{\sqrt{3}}{5}$

d. $\sqrt{\frac{36}{5}} = \frac{\sqrt{36}}{\sqrt{5}} = \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$

Exercises for Example 1

Simplify the expression.

1. $\sqrt{60}$

$\sqrt{4}\sqrt{15} =$

$2\sqrt{15}$

2. $\sqrt{2} \cdot \sqrt{18}$

$\sqrt{36}$

$= 6$

3. $\sqrt{\frac{81}{121}} = \frac{\sqrt{81}}{\sqrt{121}} = \frac{9}{11}$

EXAMPLE 2 *Solving a Quadratic Equation*

Leave answers in simple radical form.

Solve $\frac{x^2}{6} - 4 = 10$.

$+4 \quad +4$

$6 \cdot \left(\frac{x^2}{6}\right) = (14) \cdot 6$

$\sqrt{x^2} = \sqrt{84}$

$x = \pm\sqrt{84}$

$x = \pm\sqrt{4}\sqrt{21}$

$x = \pm 2\sqrt{21}$

C: $\frac{(-2\sqrt{21})^2}{6} - 4 = 10$

$10 = 10$ ✓

C: $\frac{(2\sqrt{21})^2}{6} - 4 = 10$

$10 = 10$ ✓

Method 2: Solve QF by taking SQUARE ROOTS.

STEP 1 ISOLATE x^2

STEP 2 Take SQ ROOT OF BOTH SIDES AND Remember ±

STEP 3 put in simple radical form

STEP 4 Check

Exercises for Example 2 Leave answers in simple radical form.

Solve the equation.

4. $4x^2 - 5 = -1$

$+5 \quad +5$

$\cancel{4}x^2 = 4$

$\overline{4} \quad \overline{4}$

$\sqrt{x^2} = \sqrt{1}$

$\boxed{x = \pm 1}$ = $\boxed{x = 1, -1}$

5. $12 - 2y^2 = 4$

$-12 \quad -12$

$-2y^2 = -8$

$\overline{-2} \quad \overline{-2}$

$\sqrt{y^2} = \sqrt{4}$

$\boxed{y = \pm 2}$

6. $\frac{p^2}{4} - 3 = 33$

$+3 \quad +3$

$4\left(\frac{p^2}{4}\right) = (36) \cdot 4$

$\sqrt{p^2} = \sqrt{144}$

$\boxed{p = \pm 12}$

EXAMPLE 3 ***Solving a Quadratic Equation*** Leave answers in simple radical form.

Solve $5(x - 7)^2 = 135.$

$\overline{5} \quad \overline{5}$

$\sqrt{(x-7)^2} = \sqrt{27}$

$x - 7 = \pm\sqrt{27}$

$+7 \quad +7$

$x = 7 \pm \sqrt{27}$

TO SOLVE BINOMIAL SQUARES:

① ISOLATE THE BINOMIAL $(\quad)^2$

② TAKE SQ ROOT OF BOTH SIDES

* Remember ±

③ SIMPLIFY ④ Check in original EQ

Ⓐ SIMPLIFY IN Radical Form

$x = 7 \pm \sqrt{9}\sqrt{3}$

$\boxed{x = 7 \pm 3\sqrt{3}}$

Ⓑ Round to 2 decimals

$x = 7 + \sqrt{27} = \boxed{12.20}$

$x = 7 - \sqrt{27} = \boxed{1.80}$

Exercises for Example 3 Leave answers in simple radical form.

Solve the equation.

HW

7. $\sqrt{(y + 3)^2} = \sqrt{9}$

$y + 3 = \pm\sqrt{9}$

$-3 \quad -3$

$y = -3 \pm 3$

$y = -3 + 3$ $\boxed{y = 0}$ ✓

$y = -3 - 3$ $\boxed{y = -6}$ ✓

9. $-2(x - 3)^2 = -120$

$\overline{-2} \quad \overline{-2}$

$\sqrt{(x-3)^2} = \sqrt{60}$

$x - 3 = \pm\sqrt{60}$

$+3 \quad +3$

$x = 3 \pm \sqrt{60}$

$x = 3 \pm \sqrt{4}\sqrt{15}$

$\boxed{x = 3 \pm 2\sqrt{15}}$

$\boxed{x \approx -4.75, 10.75}$

10. $\sqrt{(r - 8)^2} = \sqrt{50}$

$r - 8 = \pm\sqrt{50}$

$+8 \quad +8$

$r = 8 \pm \sqrt{50}$

$r = 8 \pm \sqrt{25}\sqrt{2}$

$\boxed{r = 8 \pm 5\sqrt{2}}$

$\boxed{r \approx 0.93, 15.07}$

11. $5(x - 3)^2 = 500$

$\overline{5} \quad \overline{5}$

$\sqrt{(x-3)^2} = \sqrt{100}$

$x - 3 = \pm\sqrt{100}$

$+3 \quad +3$

$x = 3 \pm 10$

$x = 3 + 10 = \boxed{13}$

$x = 3 - 10 = \boxed{-7}$

$\boxed{x = -7, 13}$

ALSO Round solutions to 2 decimals

Method #2
Solve Quadratic Functions by finding square roots with "Complex & Imaginery Numbers"

real portion

imaginary portion

VOCABULARY

The **imaginary unit *i*** is defined as $i = \sqrt{-1}$.

A **complex number** written in **standard form** is a number $a + bi$, where a and b are real numbers.

If $b \neq 0$, then $a + bi$ is an **imaginary number.**

If $a = 0$ and $b \neq 0$, then $a + bi$ is a **pure imaginary number.**

Sum of complex numbers:

add the real portions

then add the coef. of the imag. portions

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

Difference of complex numbers:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

In the **complex plane,** the horizontal axis is called the **real axis** and the vertical axis is called the **imaginary axis.**

The expressions $a + bi$ and $a - bi$ are called **complex conjugates.** The product of complex conjugates is always a real number.

EXAMPLE 1 ***Solving a Quadratic Equation*** Leave answers in simple radical form.

Solve $2x^2 - 12 = -44$

$+12 \quad +12$

$\frac{\cancel{2}x^2}{\cancel{2}} = \frac{-32}{2}$

$\sqrt{x^2} = \sqrt{-16}$

$x = \pm\sqrt{-16} \rightarrow \boxed{x = \pm 4i}$

STEP I : ISOLATE X^2

STEP II : TAKE SQ ROOT OF BOTH SIDES

STEP III : Simplify in radical form

Exercises for Example 1 Leave answers in simple radical form.

Solve the equation.

1. $\sqrt{x^2} = \sqrt{-16}$

$\boxed{x = \pm 4i}$

2. $\frac{\cancel{5}y^2}{\cancel{5}} = \frac{-200}{5}$

$\sqrt{y^2} = \sqrt{-40}$

$y = \pm i\sqrt{4}\sqrt{10}$

$\boxed{y = \pm 2i\sqrt{10}}$

3. $r^2 - 100 = -121$

$+100 \quad +100$

$\sqrt{r^2} = \sqrt{-21}$

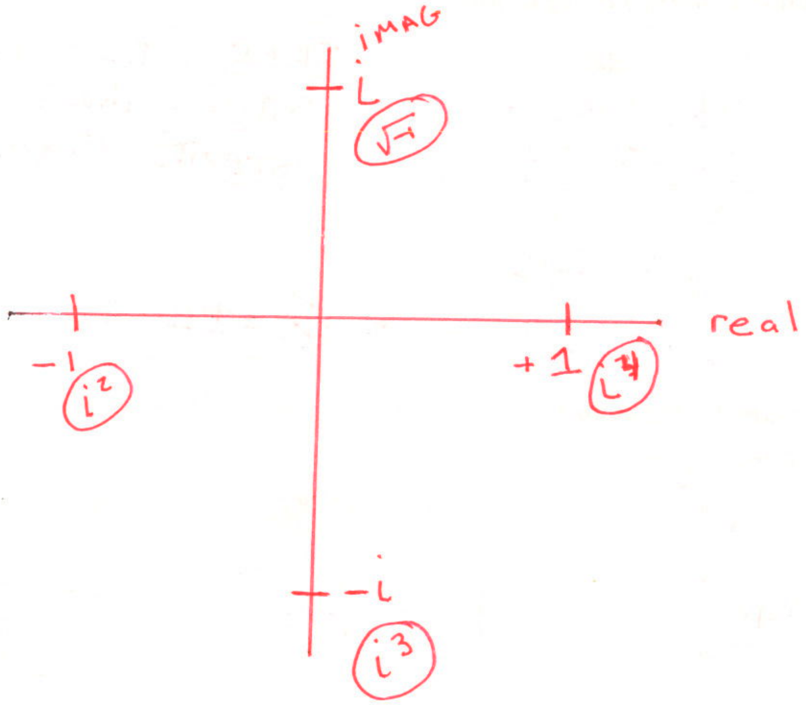
$\boxed{r = \pm i\sqrt{21}}$

What you need to know about "*i*"?

- **What is √-1 ?** $\sqrt{-1} = \boxed{i}$
- **What is √-4 ?** $\sqrt{-4} = \sqrt{4}\sqrt{-1} = \boxed{2i}$
- **What is √-7 ?** $\sqrt{-7} = \sqrt{7}\sqrt{-1} = \boxed{i\sqrt{7}}$ OR $\boxed{\sqrt{7}\cdot i}$
- **What is *i*?** $i = \sqrt{-1}$
- **What is i^2?** $i^2 = \sqrt{-1}\cdot\sqrt{-1} = \boxed{-1}$
- **What is i^3?** $i^3 = (\sqrt{-1}\cdot\sqrt{-1})\cdot\underbrace{\sqrt{-1}} = -1i = \boxed{-i}$
- **What is i^4?** $i^4 = \underbrace{\sqrt{-1}\cdot\sqrt{-1}}_{(-1)}\cdot\underbrace{\sqrt{-1}\cdot\sqrt{-1}}_{(-1)} =$

  $(-1)\cdot(-1) = \boxed{1}$

GRAPH OF THE COMPLEX NUMBER PLANE



• (10 - 2i)

EXAMPLE 2 Adding and Subtracting Complex Numbers

−1

Write $(6 + 3i) - (-4 - 2i) - 7i$ as a complex number in standard form.

Rewrite as an addition problem
Then add real portions
then add imag. portions

$(6+3i) + (4+2i) + (-7i) = \boxed{10 - 2i}$

COMPLEX NUMBER

Exercises for Example 2

Write the expression as a complex number in standard form.

7. $(5 + 4i) + (7 + 2i)$

$\boxed{12+6i}$

8. $(-6 + 3i) + (5 + i)$

$\boxed{-1+4i}$

9. $i - (5 - 6i)$ −1

$i - 5 + 6i =$

$-5+7i$

10. $(12 - 8i) - (6 - 6i)$ −1

$12 - 8i - 6 + 6i =$

$\boxed{6-2i}$

11. $(6 - 7i) + (-3 - i)$

$3-8i$

12. $12 - (8 - 10i)$

$12 - 8 + 10i$

$\boxed{4+10i}$

EXAMPLE 3 Multiplying Complex Numbers

Write $(7 - 3i)(1 - 4i)$ as a complex number in standard form.

$7(1) + 7(-4i) - 3i(1) - 3i(-4i)$

$7 - 28i - 3i + 12i^2$ Use FOIL.

$7 - 31i + 12(-1) =$ Simplify and use $i^2 = -1$.

$-5 - 31i$ Standard form $a+bi$

Exercises for Example 3

Write as a complex number in standard form.

13. $-2i(5 + i)$

$-10i - 2i^2$

$-10i - 2(-1)$ ← mental step

$2 - 10i$

14. $4i(3 - 5i)$ (−1)

$12i - 20\cancel{i^2}$

$20+12i$

15. $(2 + 3i)(2 - 3i)$ (−1)

$4 - 6i + 6i - 9\cancel{i^2}$

$4 + 9 =$

13

16. $(4 - i)(-2 + 6i)$

$-8 + 24i + 2i - 6\cancel{i^2}$ (−1)

$-8 + 6 + 26i$

$\boxed{-2+26i}$

17. $(5 + 3i)^2$ EXPAND

$(5+3i)(5+3i)$ −9

$25 + 15i + 15i + \cancel{9i^2}$

$\boxed{16 + 30i}$

HW

Method #3: The Quadratic Formula and the Discriminant

Goals
- Solve equations using the quadratic formula.
- Use the quadratic formula in real-life situations.

Your Notes

Standard Q.E.: $Ax^2 + Bx + C = 0$

Discriminant of a quadratic equation $= \boxed{B^2 - 4AC}$

IT TELLS THE NUMBER OF SOLUTIONS OR X-INTERCEPTS

THE QUADRATIC FORMULA A= B= C=

Let *a*, *b*, and *c* be real numbers such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

TAKE THE OPPOSITE

Give answers in BOTH
1) simple radical form
2) approximate form
(round at the END to 2 decimals)

Example 1 ***Quadratic Equation with Two Real Solutions***

Solve $\overset{A}{3x^2} - \overset{B}{3x} - \overset{C}{5} = 0$.

a= 3 b= -3 c= -5

QF $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ → SIMPLIFY

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-5)}}{2(3)} = \frac{3 \pm \sqrt{9 - 4(3)(-5)}}{6}$$

$$x = \frac{3 \pm \sqrt{69}}{6}$$

SPLIT

The solutions are

$x = \dfrac{3 + \sqrt{69}}{6}$ and $x = \dfrac{3 - \sqrt{69}}{6}$

Radical form IS EXACT!

ROUND TO 2 Decimals

$x \approx 1.88$ $x \approx -.88$ ← Approximate Solutions

Your Notes

SOLVING Q.E. with the Q.F.:

STD FORM QE:

$Ax^2+Bx+C=0$

⇓

QF

Take the opposite

$$X=\frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

Example 2 Quadratic Equation with One Real Solution

Solve $x^2 + 4x + 11 = 7$.

$-7 \quad -7$

First, put in standard form.

$x^2+4x+4=0$

$a = 1$, $b = 4$, $c = 4$

$$X=\frac{-4\pm\sqrt{16-4(1)4}}{2(1)}$$ Quadratic formula

$$X=\frac{-4\pm\sqrt{0}}{2}=\frac{-4}{2}$$ Simplify.

$\boxed{x=-2}$

The solution is ___.

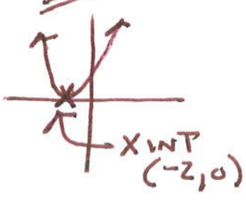
Check Substitute -2 for x in the original equation.

$(-2)^2 + 4(-2) + 11 \overset{?}{=} 7$

$4 - 8 + 11 = 7$

$7 = 7$ ✓

Check #2



Example 3 Quadratic Equation with Two Imaginary Solutions

Solve $x^2 - 4x = -8$.

$+8 \quad +8$

First, put in standard form.

$x^2-4x+8=0$

$a = 1$, $b = -4$, $c = 8$

$$X=\frac{4\pm\sqrt{16-4(1)(8)}}{2(1)}$$ Quadratic formula

$$X=\frac{4\pm\sqrt{-16}}{2}$$ Simplify. → $\sqrt{-1}\sqrt{16}=4i$

$$X=\frac{4\pm4i}{2}$$ Write using the imaginary unit *i*.

$X=2\pm2i$ Simplify.

The solutions are $\boxed{X=2+2i, 2-2i}$ ← 2 imaginary solutions

Check Substitute an imaginary solution into the original equation. C: $\boxed{X=2+2i}$

$(2+2i)^2 - 4(2+2i) \overset{?}{=} -8$

$(2+2i)(2+2i)-8-8i=-8$

$4+4i+4i+4i^2 \quad -8-8i=-8$ (i^2: -1)

$4+8i-4-8-8i=-8$

$-8=-8$ ✓

Graph

Since the graph does not cross the x axis.....

There are NO real X-INTERCEPTS.

$\boxed{i^2=-1}$

Check $x=2-2i$

C: $(2-2i)^2-4(2-2i)=-8$

$(2-2i)(2-2i)-8+8i=-8$

$4-4i-4i+4i^2-8+8i=-8$

$4-8i-4-8+8i=-8$

$-8=-8$ ✓

Show work on the next page (OVER)

Your Notes

Checkpoint Solve the quadratic equation.

1. $x^2 + 3x = 10$

on back

2. $x^2 + 7 = 8x - 9$

on back

3. $x^2 - 6x + 3 = -7$

on back

THE DISCRIMINANT = B^2-4AC (under $\sqrt{}$)

QUADRATIC FORMULA

$$X = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

NUMBER AND TYPE OF SOLUTIONS OF A QUADRATIC EQUATION

Consider the quadratic equation $ax^2 + bx + c = 0$.

- If $b^2 - 4ac > 0$, then the equation has +D 2 REAL SOLUTIONS.
- If $b^2 - 4ac = 0$, then the equation has D=0 1 REAL SOLUTION.
- If $b^2 - 4ac < 0$, then the equation has −D 2 IMAGINERY SOLUTIONS.

Example 4 *Using the Discriminant*

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

a. $\overset{A}{x^2} + \overset{B}{2x} - \overset{C}{3} = 0$

b. $\overset{A}{x^2} + \overset{B}{2x} + \overset{C}{1} = 0$

c. $\overset{A}{x^2} + \overset{B}{2x} + \overset{C}{5} = 0$

Solution

Discriminant

$\boxed{b^2 - 4ac}$.

Solution(s)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4) $D= 4-4(1)(-3)=16$ (2 real solutions)

5) $D= 4-4(1)(1)=0$ (1 real solution)

6) $D= 4-4(1)(5)=-16$ (2 imag solutions)

SOLUTIONS: Show work

Ⓐ $X=\frac{-2\pm\sqrt{16}}{2(1)}$

$X=\frac{-2\pm4}{2}$

$\boxed{X=1,-3}$

Ⓑ $X=\frac{-2\pm\sqrt{0}}{2(1)}$

$X=\frac{-2\pm0}{2}$

$\boxed{X=-1}$

Ⓒ $X=\frac{-2\pm\sqrt{-16}}{2(1)}$

$X=\frac{-2\pm4i}{2}$

$\boxed{X=-1\pm2i}$

Check point:

① $x^2 + 3x = \cancel{10}$

$\quad\quad\quad -10 \quad \cancel{-10}$

$x^2 + 3x - 10 = 0$

A=1
B=3
C=-10

$$x = \frac{-3 \pm \sqrt{9-4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

SPLIT +/-

+

−

$$x = \frac{-3+7}{2}$$

$$x = \frac{4}{2}$$

$$\boxed{x=2}$$

$$x = \frac{-3-7}{2}$$

$$x = \frac{-10}{2}$$

$$\boxed{x=-5}$$

② $x^2 + 7 = 8x \,/\, -9$

$\quad -8x \quad \cancel{-8x}$

$x^2 - 8x + 7 = \cancel{-9}$

$\quad\quad +9 \quad \cancel{+9}$

$x^2 - 8x + 16 = 0$

A=1
B=-8
C=16

$$x = \frac{8 \pm \sqrt{64-4(1)(16)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{0}}{2}$$

$$x = \frac{8}{2}$$

$$\boxed{x=4}$$

③ $x^2 - 6x + 3 = \cancel{-7}$

$\quad\quad +7 \quad \cancel{+7}$

$x^2 - 6x + 10 = 0$

A=1
B=-6
C=10

$$x = \frac{6 \pm \sqrt{36-4(1)(10)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{-4}}{2}$$

$$x = \frac{6 \pm 2i}{2}$$

$$\boxed{x = 3 \pm i}$$

Method #4:
SOLVE QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

GOAL **Solve quadratic equations by completing the square, and use completing the square to write quadratic functions in vertex form**

VOCABULARY

Completing the square is the process of rewriting an expression of the form $x^2 + bx$ as the square of the binomial. To complete the square for $x^2 + bx$, you need to add $\left(\frac{b}{2}\right)^2$. This leads to the rule

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2.$$

EXAMPLES OF PERFECT BINOMIAL SQUARES

↓

FACTOR:

Ⓐ $X^2+10X+25$

$(X+5)(X+5)$

$\boxed{(X+5)^2}$

Ⓑ $X^2-20X+100$

$(X-10)(X-10)$

$\boxed{(X-10)^2}$

EXAMPLE 1 *Completing the Square*

Find the value of c that makes $x^2 + 1.6x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

$X^2+1.6X+C$ ← FIND C

* TO DO THIS THE COEF. OF X MUST BE 1

$1.6/2=(.8)^2$

$C=.64$

① STEP 1 TAKE 1/2 OF B

② SQUARE IT

BINOMIAL SQUARE $(X+.64)^2$

Exercises for Example 1

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 + 6x + c$ $=(6/2)^2$

X^2+6X+9

$\boxed{(X+3)^2}$

2. $x^2 - 12x + c$ $\left(-\frac{12}{2}\right)=(-6)^2$

X^2-12X+ 36

$\boxed{(X-6)^2}$

3. $x^2 + 2x + c$

$C=\frac{1}{2}(2)=1^2=1$

X^2+2X+ 1

$\boxed{(X+1)^2}$

EXAMPLE 2 Solving a Quadratic Equation if the Coefficient of x^2 is 1

Solve $x^2 - 2x - \boxed{3} = 0$ by completing the square.

① $+3 \quad +3$

$x^2 - 2x + \underline{\ 1\ } = 3 + 1$

To isolate the terms containing x, add $\boxed{3}$ to each side.

Add $\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$ to each side.

② $\sqrt{(x-1)^2} = \sqrt{4}$

Write the left side as a binomial squared.

③ $x - 1 = \pm 2$

Take sqare roots of each side.

④ $+1 \quad +1$

$x = 1 \pm 2$

To solve for x, add 1 to each side.

$x = 1 + 2$ $\quad \boxed{x = 3}$

$x = 1 - 2$ $\quad \boxed{x = -1}$

MULT $(x-1)^2$

$(x-1)(x-1) =$

$x^2 - x - x + 1 =$

$x^2 - 2x + 1$

MISSING C

Exercises for Example 2

Solve the equation by completing the square.

4. $x^2 - x = 1$

$x^2 - x + \underline{1/4} = 1 + 1/4$

$\sqrt{(x - 1/2)^2} = \sqrt{1.25}$

$x - \cancel{.5} = \pm\sqrt{1.25}$

$+\cancel{.5} \quad +.5$

$x = .5 \pm \sqrt{1.25}$

$x = .5 - \sqrt{1.25}$

$\boxed{x = -0.62}$

$x = .5 + \sqrt{1.25}$

$\boxed{x = 1.62}$

5. $x^2 + 6x + \cancel{5} = 0$

$-\cancel{5} \quad -5$

$x^2 + 6x + \underline{\ 9\ } = -5 + 9$

$\sqrt{(x + 3)^2} = \sqrt{4}$

$x + \cancel{3} = \pm 2$

$-\cancel{3} \quad -3$

$x = -3 \pm 2$

$x = -3 + 2$

$\boxed{x = -1}$

$x = -3 - 2$

$\boxed{x = -5}$

EXAMPLE 3 Solving a Quadratic Equation if the Coefficient of x^2 is Not 1

Solve $4x^2 - 6x + 1 = 0$ by completing the square.

SKIP

SOLUTION

$4x^2 - 6x + 1 = 0$

Divide by 4 to make the coefficient of x^2 be 1.

Write the left side in the form $x^2 + bx$.

Add $\left(\frac{-3}{4}\right)^2 = \frac{9}{16}$ to each side.

Write the left side as the square of a binomial.

Take square roots of each side.

To solve for x, add $\frac{3}{4}$ to each side.

Method #5

Solving Quadratic Equations by Factoring

Goals
- Factor quadratic expressions and solve quadratic equations by factoring.
- Find zeros of quadratic functions.

VOCABULARY

Binomial	2 TERMS	EX: $2X+1$
Trinomial	3 TERMS	EX: $X^2-9X+14$
Factoring		
Monomial	1 TERM	EX: 5 OR X OR $-5X$

Terms v. Factors

Seperated by + and - signs

seperated by mult signs

Greatest Common Factor:

FIND THE GCF

.

.

.1st step in factoring is ALWAYS!

Example 1 *Factoring a Trinomial of the Form $x^2 + bx + c$*

Factor $x^2 - 9x + 14$.

A=1

1 14

2 7

Solution

1) Standard Quadratic Function is $f(x)=AX^2+BX+C$

2) When the LEADING COEFFICIENT (A) is 1;

Ask yourself ....

"What are 2 factors of 14 (C) that sum to -9 (B)?"

$x^2 - 9x + 14 = \boxed{(X-2)(X-7)}$.

↑ ↑

Check by mentally distributing

Your Notes

HW

Example 2 Factoring a Trinomial of the Form $ax^2 + bx + c$

Factor $2x^2 + 13x + 6$.

1 2   1 6   2 3

Solution

1) The LEADING COEFFICIENT (A) is NOT 1
2) Use Guess and Check:
    - * Find the factors of 2 (A)
    - * Find the factors of 6 (C)
    - * Muliply the factors of 2 and 6 so they sum to 13 (B)

12x

x

← MUST ADD TO 13X

The correct factorization is

$2x^2 + 13x + 6 = \boxed{(2x + 1)(x + 6)}$

↑   ?   7

SPECIAL FACTORING PATTERNS

perfect SQ   MINUS   perfect SQ

Difference of Two Squares	**Example**
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = \boxed{(X+3)(X-3)}$
Perfect Square Trinomial	**Example**
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 12x + 36 = \boxed{(X+6)^2(x+6)}$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 8x + 16 = \boxed{(X-4)^2(x-4)}$

or → $\boxed{(x-4)^2}$

Example 3 Factoring with Special Patterns

Factor the quadratic expression.

a. $9x^2 - 16 = \boxed{(3x+4)(3x-4)}$ **Difference of two squares**

PSQ   Minus   PSQ

b. $16y^2 + 40y + 25$ **Perfect square trinomial**

$= \boxed{(4y+5)(4y+5)}$ OR $\boxed{(4y+5)^2}$

$=$

c. $64x^2 - 32x + 4$ **Perfect square trinomial**

$= \boxed{(8x-2)(8x-2)}$ ← Distribute

$=$ $64x^2 - 16x - 16 + 4$

OR

$\boxed{(8x-2)^2}$

Your Notes

STEP 1

ALWAYS FACTOR OUT THE GREATEST COMMON FACTOR

Example 4 *Factoring Monomials First*

Factor the quadratic expression.

a. $12x^2 - 3 = 3(4x^2 - 1) = \boxed{3(2x+1)(2x-1)}$

b. $3u^2 - 9u + 6 = 3(u^2 - 3u + 2) = \boxed{3(u-2)(u-1)}$

c. $7v^2 - 42v = \boxed{7v(v-6)} =$

d. $2x^2 + 8x + 2 = \boxed{2(x^2+4x+1)} =$

✓ *Checkpoint* Factor the expression.

1. $\frac{6c^2}{6} - \frac{48c}{6} - \frac{54}{6}$

$6(c^2 - 8c - 9) =$

$\boxed{6(c+1)(c-9)}$

2. $81x^2 - 1$

Perfect SQ

MINUS

perfect SQ

$\boxed{(9x+1)(9x-1)}$

3. $49h^2 + 42h + 9$

$\boxed{(7H+3)(7H+3)}$

4. $\frac{16x^2}{4} - \frac{4}{4}$

$4(4x^2 - 1) =$

$\boxed{4(2x+1)(2x-1)}$

ZERO PRODUCT PROPERTY

Let A and B be real numbers or algebraic expressions. If $AB = 0$, then $A = \underline{0}$ or $B = \underline{0}$.

A and B are FACTORS

Example 5 *Solving Quadratic Equations*

Solve $4x^2 + 13x + 11 = -3x - 5$.

$\boxed{Ax^2 + Bx + C = 0}$

Solution

$4x^2 + 13x + 11 = -3\cancel{x} - \cancel{5}$ **Write original equation.**

$+3x + 5 \quad +3x + 5$

Write in standard form.

$4x^2 + 16x + 16 = 0$

Factor Completely

$4(x^2 + 4x + 4) = 0$ **1st Factor GCF** (GCF = 4)

$4(x+2)(x+2) = 0$

$4(x+2)^2 = 0$ **Use zero product property.**

SET EVERY FACTOR = 0

NO VAR.

$4 = 0$ $\quad x + 2 = 0 \quad \boxed{x = -2}$ **Solve for *x*.**

The solution is $\underline{-2}$. Check this in the original equation.

C: $4(-2)^2 + 13(-2) + 11 = -3(-2) - 5$

$1 = 1$ ✓

Checkpoint Solve the quadratic equation. BY FACTORING

FACTOR →

SET = 0 →

Solve →

USE CALC → To check Both Solutions in oriG EQ.

5. $x^2 + 15x + 26 = 0$

1 26
2 13

$(x+2)(x+13)=0$

$x+2=0$
$-2 \quad -2$
$\boxed{x=-2}$ ✓

$x+13=0$
$-13 \quad -13$
$\boxed{x=-13}$ ✓

6. $2x^2 + x + 3 = -5x + 19 + x^2$

$2x^2 + x + 3 = \cancel{-5x} + \cancel{19} + \cancel{x^2}$
$-x^2 + 5x - 19 \quad +5x - 19 - x^2$

$x^2 + 6x - 16 = 0$ ← FACTOR

$(x+8)(x-2)=0$

$x+8=0$
$\boxed{x=-8}$
C: 123=123✓

$x-2=0$
$\boxed{x=2}$
C: 13=13✓

Example 6 *Finding the Zeros of a Quadratic Function*

Find the zeros of $y = x^2 + 4x + 3$.

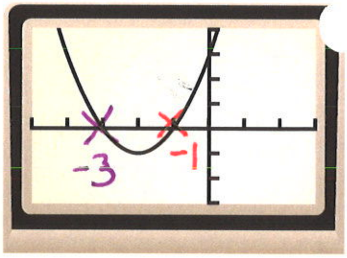
Use factoring to write the function in intercept form.

$y = x^2 + 4x + 3$

$\boxed{y = (x+3)(x+1)}$

The zeros of the function are -3 and -1.

Check Graph $y = x^2 + 4x + 3$. The graph passes through (-3, 0) and (-1, 0), so the zeros are -3 and -1.



NOTE
These all mean the same:
- Solutions
- Zero's
- X intercepts
- roots

Checkpoint Complete the following exercise.

7. Find the zeros of $y = 3x^2 - x - 2$.

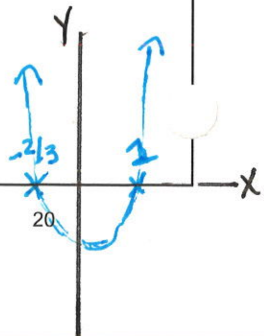
FACTOR

$0 = (3x+2)(x-1)$

$3x+2=0$
$-2 \quad -2$
$\frac{\cancel{3}x}{\cancel{3}} = \frac{-2}{3}$
$\boxed{x = -2/3}$

$x-1=0$
$\boxed{x=1}$

Check by Graphing:
STD FORM
$y = 3x^2 - x - 2$
Intercept form
$y = (3x+2)(x-1)$



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