6.5 Solve Absolute Value Equations

Goal: Solve absolute value equations.

**VOCABULARY**

Absolute value equation contains an absolute value expression. Ex: \(|2x + 5| = 25\)

Remember: Absolute Value is the distance from zero.

Mentally Solve: \(|x| = 5\) \(x = 5, -5\)

**SOLVING AN ABSOLUTE VALUE EQUATION**

The equation \(|ax + b| = c\) where \(c \geq 0\) is equivalent to the statement \(ax + b = +c\) or \(ax + b = -c\).

**Example 1** Solve an absolute value equation

Solve \(|x - 9| = 2\).

1. **Write original equation.**
2. **Rewrite as two equations.**
3. **Solve both equations.**

Check \(|x - 9| = 2\):

- **Original equation.**
- **Substitute for \(x\).**
- **Simplify, Solution checks.**

The solutions are 7 and 11. Check your solution.
Example 2  Rewrite an absolute value equation

Solve \(4 |2x + 8| + 6 = 30\).

**Solution**

First, rewrite the equation in the form \(\frac{4 |2x + 8|}{\quad} = \frac{24}{\text{4}}\).

Write original equation.

Subtract 6 from each side.

Divide each side by 4.

Next, solve the absolute value equation.

Write absolute value equation.

Rewrite as two equations.

Then solve both eq's.

**STEP 3:** Check in original eq:

\[
\begin{align*}
C: 4 |2(-1) + 8| + 6 &= 30 \\
\quad &\quad \text{30} = 30 \\
C: 4 |(x-7)+8| + 6 &= 30 \\
\quad &\quad \text{30} = 30
\end{align*}
\]

**Checkpoint** Solve the equation.

1. \( |x + 6| = 11 \)

\[
\begin{align*}
\frac{x + 6}{\quad} &= \frac{11}{\text{11}} \\
\quad &\quad \text{11} = 11 \\
11 &= 11 \checkmark
\end{align*}
\]

\[
\begin{align*}
\frac{x + 6}{\quad} &= \frac{-11}{\text{-11}} \\
\quad &\quad \text{-11} = -11 \checkmark
\end{align*}
\]

2. \( 3 |5x - 10| + 6 = 21 \)

\[
\begin{align*}
\frac{5x - 10}{\quad} &= \frac{15}{\text{15}} \\
\quad &\quad \text{15} = 15 \\
\frac{5x}{\quad} &= \frac{5}{\text{5}} \\
\frac{x}{\quad} &= \frac{3}{\text{3}} \\
3 &= 3 \checkmark
\end{align*}
\]

\[
\begin{align*}
\frac{5x - 10}{\quad} &= \frac{-5}{\text{-5}} \\
\quad &\quad \text{-5} = -5 \\
\frac{5x}{\quad} &= \frac{5}{\text{5}} \\
\frac{x}{\quad} &= \frac{1}{\text{1}} \\
1 &= 1 \checkmark
\end{align*}
\]

C: \( 3 |(3)-10| + 6 = 21 \)

\[
\begin{align*}
3 + 51 + 6 &= 21 \\
21 &= 21 \checkmark
\end{align*}
\]

\[
\begin{align*}
3 + 51 + 6 &= 21 \\
21 &= 21 \checkmark
\end{align*}
\]
Example 3: Decide if an equation has no solutions

Solve $|7x - 3| + 8 = 5$, if possible.

\[
\begin{align*}
|7x - 3| + 8 &= 5 \\
|7x - 3| &= -3
\end{align*}
\]

The absolute value of a number is never **negative**. So, there are no solutions.

\[x = \text{no solution}\]

Summary

1. You must always isolate the \(| | \) symbols

2. **General Rules**

   \[
   |x| = +N \quad (2 \text{ solutions})
   \]

   \[
   |x| = 0 \quad (1 \text{ solution})
   \]

   \[
   |x| = -N \quad (\text{no solution})
   \]

3. \[|\text{expression}| = N \quad +/\quad -\]

\[\text{expression}^* = +N \quad \text{expression}^* = -N\]

*Notice the split equations do not have the \(| | \) symbol.*