# 6.5 Solve Absolute Value Equations

**Goal** · Solve absolute value equations.

**Your Notes**
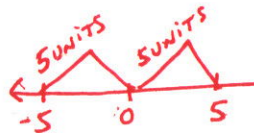
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**VOCABULARY**

Absolute value equation *CONTAIN AN ABSOLUTE VALUE EXPRESSION.* Ex: $|2x+5| = 25$

*Remember:* Absolute Value is the distance from ZERO.

Mentally Solve $|x| = 5$

$x = 5, -5$   Graph:

$C: |5| = 5$
$5 = 5 ✓$

$C: |-5| = 5$
$5 = 5 ✓$

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**SOLVING AN ABSOLUTE VALUE EQUATION**

The equation $|ax + b| = c$ where $c \geq 0$ is equivalent to the statement $ax+b = +c$ or $ax+b = -c$.

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**Example 1**   Solve an absolute value equation

Solve $|x - 9| = 2$.

$x - 9 = +2$         $x - 9 = -2$         Write original equation.

$+9 \quad +9$         $+9 \quad +9$         ① Rewrite as two equations.

$x = 11$              $x = 7$              ② SOLVE BOTH Add _9_ to each side.

→ LEFT SIDE — Remove $||$

→ RIGHT SIDE SPLIT WITH $+, -$ NUMBER

The solutions are __7__ and __11__. Check your solution.

**CHECK** $x = 7$         $x = 11$

$|x - 9| = 2$         $|x - 9| = 2$         Write original equation.

$|\underline{7} - 9| = 2$         $|\underline{11} - 9| = 2$         Substitute for x.

$|\underline{-2}| = 2$         $|\underline{2}| = 2$         Subtract.

$2 = 2 ✓$ ✓         $2 = 2$ ✓         Simplify. Solution checks.

*STUDENTS DO Checkpoint #1 Next page*

To solve absolute value EQ's you **MUST** isolate the | | symbol !!!!

**Your Notes**

**Example 2** *Rewrite an absolute value equation*

Solve $4|2x + 8| + 6 = 30$.

**Solution**

**STEP 1:** ISOLATE

First, rewrite the equation in the form $|\quad| = \#$.

$4|2x + 8| + 6 = 30$

$\underline{\qquad -6 \quad -6 \qquad}$

$\dfrac{4|2x+8|}{4} = \dfrac{24}{4}$

Write original equation.

Subtract $6$ from each side.

Divide each side by $4$.

**STEP 2** Next, solve the absolute value equation.

$|2x + 8| = 6$

$+$ ⟋ ⟍ $-$

$2x + 8 = 6 \qquad\qquad 2x + 8 = -6$

$\underline{-8 \quad -8} \qquad\qquad \underline{-8 \qquad -8}$

$\dfrac{2x}{2} = \dfrac{-2}{2} \qquad\qquad \dfrac{2x}{2} = \dfrac{-14}{2}$

$\boxed{x = -1} \qquad\qquad \boxed{x = -7}$

Write absolute value equation.

Rewrite as two equations.

Then solve both eq's

> ✗ Remember to check your solutions in the original equation for accuracy.

**STEP 3:** Check in original EQ:

* MUST SHOW checks on HW!

$\begin{cases} C: 4|2(-1)+8|+6 = 30 \\ \qquad\qquad 30 = 30 ✓ \\ C: 4|(2)(-7)+8|+6 = 30 \\ \qquad\qquad 30 = 30 ✓ \end{cases}$

✓ **Checkpoint** Solve the equation.

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1. $|x + 6| = 11$

$+$ ⟋ ⟍ $-$

$x + 6 = 11 \qquad\qquad x + 6 = -11$

$\underline{-6 \quad -6} \qquad\qquad \underline{-6 \quad -6}$

$\boxed{x = 5} \qquad\qquad \boxed{x = -17}$

$C: |5 + 6| = 11$

$|11| = 11$

$11 = 11 ✓$

$C: |-17 + 6| = 11$

$|-11| = 11$

$11 = 11 ✓$

2. $3|5x - 10| + 6 = 21$

$\underline{\qquad -6 \quad -6}$

$\dfrac{3 \cdot |5x - 10|}{3} = \dfrac{15}{3}$

$|5x - 10| = 5$

$+$ ⟋ ⟍ $-$

$5x - 10 = +5 \qquad\qquad 5x - 10 = -5$

$\underline{+10 \quad +10} \qquad\qquad \underline{+10 \quad +10}$

$\dfrac{5x}{5} = \dfrac{15}{5} \qquad\qquad \dfrac{5x}{5} = \dfrac{+5}{5}$

$\boxed{x = 3} \qquad\qquad \boxed{x = 1}$

$C: 3|5(3) - 10| + 6 = 21$

$3|+5| + 6 = 21$

$21 = 21 ✓$

$C: 3|5(1) - 10| + 6 = 21$

$21 = 21 ✓$

Add Example 4:

$$5|2x+4| - 10 = -10$$
$$\phantom{5|2x+4|} +10 \quad +10$$
$$\frac{5|2x+4|}{5} = \frac{0}{5}$$

$$|2x+4| = 0 \quad \xrightarrow{\text{NOTICE "0" Cannot be split}}$$

$$2x+4 = 0$$
$$\phantom{2x} -4 \quad -4$$
$$\frac{2x}{2} = \frac{4}{2}$$
$$\boxed{X = 2}$$

$$C: \; 5|2(-2)+4| - 10 = -10$$
$$-10 = -10 \checkmark$$

**Your Notes**

| Example 3 | Decide if an equation has no solutions |
|---|---|

Solve $|7x - 3| + 8 = 5$, if possible.

$$\phantom{|7x-3|} -8 \quad -8$$
$$|7x-3| = -3 \quad \longleftarrow \text{NOTICE NEG } \# \; \text{STOP}$$

The absolute value of a number is never _NEGATIVE_. So, there are no solutions.

$$\boxed{X = \text{NO SOLUTION}}$$

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## SUMMARY

① You must always isolate the
$|\;|$ symbols

② GENERAL RULES

$$|\;\;| = +N \quad (2 \text{ solutions})$$
$$|\;\;| = 0 \quad (1 \text{ solution})$$
$$|\;\;| = -N \quad (\text{NO SOLUTION}$$

⓪ $|\text{EXPRESSION}| = N$

$+ \diagup \qquad \diagdown -$

$\text{EXPRESSION}^* = +N \qquad\qquad \text{EXPRESSION}^* = -N$

\* NOTICE THE SPLIT EQUATIONS DO
NOT HAVE THE $|\;|$ SYMBOL.