

## 11.1 M&M Activity - Chi-Square Goodness of Fit Test



# Distribution of m&m<sup>s</sup> Candies

When bags of M&M candies are produced by the Mars, Incorporated the machines that make the candies are set to produce a certain percentage of each color of M&M. It depends on what type (plain, peanut, almond, etc.) of M&M to the blend of colors. The color blends were selected by conducting consumer preference tests, which indicate the assortment of colors that pleased the greatest number of people and created the most attractive overall effect.

For Milk Chocolate M&M's this distribution of colors is:

Blue	Brown	Green	Orange	Red	Yellow
.24	.13	.16	.20	.13	.14

The company claims each batch (production) of M&M's is blended to these ratios and mixed. However, the individual packages are filled by weight and not by count. Therefore, it is possible to have an unusual color distribution.

What was the size of your bag of M&M's (circle): (SRS from a 42oz bag) (Two 1.69 oz. individual bag combined)

### Observed Counts

Each group will be given a bag of M&M's. First record the number of M&M's of each color. These are known as our **observed counts (O)**. Record the observed counts in the table below:

Color	Blue	Brown	Green	Orange	Red	Yellow	Sample Size(n)
<b>Observed Count</b>	<b>35</b>	<b>8</b>	<b>10</b>	<b>32</b>	<b>18</b>	<b>14</b>	<b>117</b>

### Expected Counts

Now we need to calculate how many candies would we have expected if we assume the proportions given by the company are true? These are known as our **expected counts (E)**. Record the expected counts for our class in the table below:

Color	Blue	Brown	Green	Orange	Red	Yellow	Total=n
<b>Expected Count</b> (round 2 decimals)	<b>.24*117=28.08</b>	<b>15.21</b>	<b>18.72</b>	<b>23.40</b>	<b>15.21</b>	<b>16.38</b>	<b>117.00</b>

We want to know "Does your bag of M&M's follow the proportions suggested by the company?"

We are faced with a problem where we want to know if our sample follows a particular distribution. This type of question is answered using a **Chi-squared Goodness of Fit test**.

### State Null Hypothesis

Our null hypothesis for this type of test is

**$H_0$ :  $p_{\text{blue}} = .24$ ,  $p_{\text{brown}} = .13$ ,  $p_{\text{green}} = .16$ ,  $p_{\text{orange}} = .20$ ,  $p_{\text{red}} = .13$ ,  $p_{\text{yellow}} = .14$**

**That is that each color has the distribution specified by the company.**

### State Alternative Hypothesis

Our alternative hypothesis for this type of test is

**$H_A$ : at least one of the proportions specified by the company is incorrect**

### Assumptions/Conditions

There are a few things that must be checked to perform this test

- A RANDOM** sample was obtained from a 42oz bag of M&M and the bag is a random sample of all M&M's produced. Or we must assume that the \_\_oz bag of M&M's was a random same of all plain M&M produced.
- The **SAMPLE IS LARGE ENOUGH** to perform test (every expected count is greater than 5). You must state all expected counts: 15.21, 18.72, 23.4, 15.21, and 16.38.
- INDEPENDENT**: since we are sampling without replacement, it is reasonable our sample is 1/10 th of the population of all plain M&M's.

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**Test Statistic:** State the test by Name "Chi-Square Goodness of Fit Test" or by Formula:

$$\chi_T^2 = \sum \left[ \frac{(O - E)^2}{E} \right] \quad df = k - 1$$

State the significance level: **if it is not given use  $\alpha = .05$ .**

☀ **FILL OUT THIS TABLE:** To calculate this test statistic it is easier to do in a table (or use lists in your calculator).

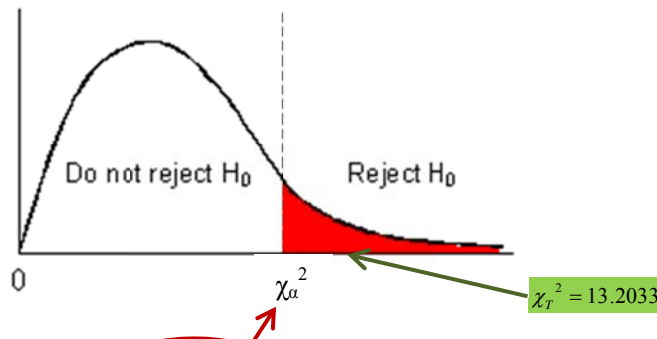
Color	Company %'s	Observed (O)	Expected (E) (round 2 decimals)	O-E (round 2 decimals)	(O-E) <sup>2</sup> (round 3 decimals)	(O-E) <sup>2</sup> /E (round 3 decimals)
	Company %'s	Company	Observed	Expected	O-E	(O-E) <sup>2</sup>
			(O)	(E)		E
Blue	24%	35	28.08	6.92	47.8864	1.7054
Orange	20%	32	23.40	8.60	73.9600	3.1607
Green	16%	10	18.72	-8.72	76.0384	4.0619
Yellow	14%	14	16.38	-2.38	5.6644	0.3458
Red	13%	18	15.21	2.79	7.7841	0.5118
Brown	13%	8	15.21	-7.21	51.9841	3.4178
	100%	117.00	117.00	0.00	263.3174	13.2033

$$\chi_T^2 = 13.2033 \quad df = k - 1 = 6 - 1$$

### Chi-square Table or P-value

Since we performing our test at the 5% significance level, we compare the Chi-square value for  $\alpha = .05$  to our our test statistic ( $\chi_T^2$ ). The **degrees of freedom** for this test is (k-1) where k is the number of categories in the experient. In our case k = 6 (blue, brown, green, orange, red, and yellow) therefore our degrees of freedom is 5.

For a **Goodness of Fit test** we always use the value with the significance level probability in the upper tail.



☀ The rejection region is any value large than  $\chi_{.05}^2 = 11.07$  (NO calculator command, must use Table C. Describe how you located this value: **Find the intersection of  $p = \alpha = .05$  and  $df = 5$ , the value is 11.07**)

☀ The p-value of this test is  $P(\chi_T^2 > 13.2033) = .0215$  Give the calculator command: **chi2cdf(13.2033, e99, 5)**

### Test Conclusion:

- ☀ What is the conclusion from your test? **Since the p-value is about .02 and it is less than our predetermined significance level  $\alpha = .05$ , we reject  $H_0$  in favor of  $H_a$ .**
- ☀ What did you conclude about the distribution of our M&M's? **We have convincing evidence to doubt the companies reported distribution of colors.**

### Follow Up Analysis:

- ☀ Explain when you do a follow up analysis and what you are looking for during this analysis. Then attach a follow up analysis if your analysis requires one. **If the test finds a statistically significant result, do a follow-up analysis that compares the observed and expected counts and look for the largest components of the chi-square statistic.**

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### Follow Up Analysis:

- 🌟 In my test, I rejected  $H_0$  and therefore will do a follow-up analysis that compares the observed and expected counts and look for the largest components of the chi-square statistic.
- 🌟 The  $\chi^2$  components for orange, green, and brown contributed 10.7 to the overall  $\chi^2=13.2$ . It appears that there were too many orange and not enough green and brown to support the companies claim for their published color distribution.
- 🌟 Another issue may be since I was analyzing a small bag of M&M's that these size bags were not truly randomly dispersed.

