ANOTHER METHOD TO SOLVE QE'S with the form: $\boxed{x^2 = n}$

10.4 Use Square Roots to Solve Quadratic Equations

Goal • Solve a quadratic equation by finding square roots.

Your Notes

3 TYPES OF SOLUTIONS

SOLVING $x^2 = N$ BY TAKING SQUARE ROOTS – where "N" is a real number.

- If $N > 0$, then $x^2 = {}^{+}N$ has 2 solutions: $\boxed{x = \pm\sqrt{N}}$
- If $N = 0$, then $x^2 = 0$ has 1 solution: $\boxed{x = 0}$.
- If $N < 0$, then $x^2 = {}^{-}N$ has NO solution.

NOTE: THERE ARE 3 TYPES OF SOLUTIONS.

STEPS TO USE THIS METHOD

1. ISOLATE x^2
2. TAKE THE $\sqrt{\ }$ OF BOTH SIDES
3. The $\sqrt{\ }$ can be + and −
4. Check all solutions in THE ORIGINAL EQ.

Example 1 *Solve quadratic equations*

Solve the equation. LEAVE SOLUTIONS IN SIMPLE RADICAL FORM!

a $z^2 - 5 = 4$ — Write original equation. ISOLATE "z"

$\sqrt{z^2} = \sqrt{9}$ — ① Add 5 to each side.

$z = \pm 3$ — ② ③ Take square roots of each side.

$z = 3, -3$ — Simplify. The solutions are 3 and −3.

Write either way

C: $z = 3$
$(3)^2 - 5 = 4$
$4 = 4$ ✓

C: $z = -3$
$(-3)^2 - 5 = 4$
$4 = 4$ ✓

b $r^2 + 7 = 4$ — Write original equation.

$\sqrt{r^2} = \sqrt{-3}$ — Subtract 7 from each side.

* We can not take the SQ. ROOT OF A NEGATIVE #

So, → ANSWER → $\boxed{R = \text{NO SOLUTION}}$

c $25k^2 = 9$ — Write original equation.

$\frac{25k^2}{25} = \frac{9}{25}$

$\sqrt{k^2} = \sqrt{\frac{9}{25}}$ — Divide each side by 25.

$k = \pm\frac{\sqrt{9}}{\sqrt{25}}$ — Take square roots of each side.

$\boxed{k = \pm\frac{3}{5}}$ — Simplify. The solutions are $3/5$ and $-3/5$.

EXAMPLES ① 2 SOLUTIONS
② 1 SOLUTION

Your Notes

Checkpoint Solve the equation. AND CHECK!

STEP I

ISOLATE x^2

STEP II

TAKE SQROOT OF BOTH SIDES

STEP III

USE CALC + check all solutions IN ORIGINAL EQ.

* WRITE THE Check Final step.

1. $3x^2 = 108$	2. $t^2 + 17 = 17$	3. $81p^2 = 4$
$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{108}{3}$	$t^2 + \cancel{17} = 17$ $\cancel{-17} \quad -17$	$\frac{\cancel{81}p^2}{\cancel{81}} = \frac{4}{81}$
$\sqrt{x^2} = \sqrt{36}$	$\sqrt{t^2} = \sqrt{0}$	$\sqrt{p^2} = \sqrt{\frac{4}{81}}$
$\boxed{x = \pm 6}$	$\boxed{t = 0}$	$p = \pm \frac{\sqrt{4}}{\sqrt{81}}$
Remember $(6)^2 = 36$ $(-6)^2 = 36$	C: $T = 0$ $0^2 + 17 = 17$ $17 = 17$ ✓	$p = \pm 2/9$
Check C: $x = 6$ $3(6)^2 = 108$ $108 = 108$ ✓		CALC Check C: $p = 2/9$ $4 = 4$ ✓
C: $x = -6$ $3(-6)^2 = 108$ $108 = 108$ ✓		C: $p = -2/9$ $4 = 4$ ✓

HW ⇓

C: $x = 6$
$108 = 108$ ✓

C: $x = -6$
$108 = 198$ ✓

Example 2 *Approximate solutions of a quadratic equation*

Solve $4x^2 + 3 = 23$. Round the solutions to the nearest hundredth.

When do you round? ALWAYS

ROUND ON THE FINAL STEP TO MINIMIZE ROUNDING ERROR

STEP I

ISOLATE X

STEP II

TAKE √ OF BOTH SIDES

Solution

$4x^2 + 3 = 23$ — Write original equation.

$\quad -3 \quad -3$

$4x^2 = 20$ — Subtract 3 from each side.

$\sqrt{x^2} = \sqrt{5}$ — Divide each side by 4.

EXACT SOLUTION

$\boxed{x = \pm\sqrt{5}}$ — Take square roots of each side.

$x \approx \pm 2.236$ — Use a calculator. Round to the nearest hundredth.

This is in simple radical form

This is an approx. solution

The solutions are about 2.24 and −2.24.

USE CALC TO CHECK!

C: $\boxed{x = 2.24}$

C: $4(2.24)^2 + 3 = 23$

$23.07 \approx 23$ ✓

C: $\boxed{x = -2.24}$

C: $4(-2.24)^2 + 3 = 23$

$23.07 \approx 23$ ✓

* The checks will not be exact due to rounding error. but should be close

Checkpoint Solve the equation. Round the solutions to the nearest hundredth.

4. $2x^2 - 7 = 9$ $+7 \quad +7$ $2x^2 = 16$ $\div 2 \quad \div 2$ $\sqrt{x^2} = \sqrt{8}$ $\boxed{x \approx \pm 2.83}$	5. $6g^2 + 1 = 19$ $-1 \quad -1$ $6G^2 = 18$ $\div 6 \quad \div 6$ $\sqrt{G^2} = \sqrt{3}$ $\boxed{G \approx \pm 1.73}$

Use CALC To check

C: $\boxed{x = +2.83}$

$9.02 \approx 9$ ✓

C: $\boxed{x = -2.83}$

$9.02 \approx 9$ ✓

C: $\boxed{G = +1.73}$

$18.96 \approx 19$ ✓

C: $\boxed{G = -1.73}$

$18.96 \approx 19$ ✓

Your Notes

STEP I
ISOLATE THE BINOMIAL

STEP II
Take the SQ ROOT OF BOTH SIDES

Remember when you take the SQ ROOT OF A PERFECT SQUARE → THE RESULT IS 2 Solutions: + and −

STEP III
Solve for X

STEP IV
Split into 2 problems

STEP V
Check both solutions with Calc IN THE ORIGINAL EQ.

Example 3 Solve a quadratic equation

Solve $5(x+1)^2 = 30$. Round the solutions to the nearest hundredth.

Solution

$$\frac{5(x+1)^2}{5} = \frac{30}{5}$$ Write original equation.

① Divide each side by 5.

$$\sqrt{(x+1)^2} = \sqrt{6}$$

② Take square roots of each side.

$$x + 1 = \pm\sqrt{6}$$
$$\quad -1 \qquad -1$$

③ Subtract 1 from each side.

$$x = -1 \pm \sqrt{6}$$

The solutions are $-1+\sqrt{6}$ ≈ 1.449 and $-1-\sqrt{6}$ ≈ −3.449.

$\boxed{x \approx 1.45, -3.45}$

CHECK To check the solutions:

- USE ORIGINAL EQUATION
- USE CALC.

C: $X = 1.45$
$5(1.45+1)^2 = 30$
$30.01 \approx 30$ ✓

C: $X = -3.45$
$5(-3.45+1)^2 = 30$
$30.01 \approx 30$ ✓

Checkpoint Solve the equation. Round the solutions to the nearest hundredth, if necessary.

6. $\frac{\cancel{3}(m-4)^2}{\cancel{3}} = \frac{12}{3}$ $\sqrt{(m-4)^2} = \sqrt{4}$ $m - 4 = \pm 2$ $+4 \qquad +4$ $m = 4 \pm 2$ $m = 4+2$ $m = 4-2$ $m = 6$ $m = 2$ C: 12=12 ✓ C: 12=12 ✓	7. $\frac{\cancel{4}(a-3)^2}{\cancel{4}} = \frac{32}{4}$ $\sqrt{(a-3)^2} = \sqrt{8}$ $a - 3 = \pm\sqrt{8}$ $+3 \qquad +3$ $a = 3 \pm \sqrt{8}$ $a = 3+\sqrt{8}$ $a = 3-\sqrt{8}$ $a \approx 5.83$ $a \approx .17$ C: 32.04 ≈ 32 ✓ C: 32.04 ≈ 32 ✓