Section 10.3B
Inference for Means: Paired Data

Day 2

- **After this section, you will be able to PERFORM significance tests for paired data.**

- Comparative studies are more convincing than single-sample investigations. For that reason, one-sample inference is less common than comparative inference. Study designs that involve making two observations on the same individual, or one observation on each of two similar individuals, result in **paired data**.

- When paired data result from measuring the same quantitative variable twice, as in the job satisfaction study, we can make comparisons by analyzing the differences in each pair. If the conditions for inference are met, we can use one-sample *t* procedures to perform inference about the mean difference μ_d.

- These methods are sometimes called **paired *t* procedures**.

- **Example:** Caffeine Withdrawal
- ***Carrying Out a Paired T- Test***

EXAMPLE: Researchers designed an experiment to study the effects of caffeine withdrawal. They recruited 11 volunteers who were diagnosed as being caffeine dependent to serve as subjects. Each subject was barred from coffee, colas, and other substances with caffeine for the duration of the experiment. During one two-day period, subjects took capsules containing their normal caffeine intake. During another two-day period, they took placebo capsules. The order in which subjects took caffeine and the placebo was randomized. At the end of each two-day period, a test for depression was given to all 11 subjects. Researchers wanted to know whether being deprived of caffeine would lead to an increase in depression

Results of a caffeine deprivation study

Subject	Depression (caffeine)	Depression (placebo)	Difference (placebo – caffeine)
1	5	16	
2	5	23	
3	4	5	
4	3	7	
5	8	14	
6	5	24	
7	0	6	
8	0	3	
9	2	15	
10	11	12	
11	1	0	

- **Example:** Caffeine Withdrawal
- ***Carrying Out a Paired T- Test***

3

+

1) State Hypotheses and Sketch Graph:

2) Check Conditions:

3) Calculations: Test statistic and P-value:

4

4) Conclusion:

APPENDIX:

Reading Notes to supplement the Presentation–

- **READ and STUDY the Following Points and see me with any questions.**

+

- **Example:** Caffeine Withdrawal
- ***Carrying Out a Paired T- Test***

Researchers designed an experiment to study the effects of caffeine withdrawal. They recruited 11 volunteers who were diagnosed as being caffeine dependent to serve as subjects. Each subject was barred from coffee, colas, and other substances with caffeine for the duration of the experiment. During one two-day period, subjects took capsules containing their normal caffeine intake. During another two-day period, they took placebo capsules. The order in which subjects took caffeine and the placebo was randomized. At the end of each two-day period, a test for depression was given to all 11 subjects. Researchers wanted to know whether being deprived of caffeine would lead to an increase in depression.

Results of a caffeine deprivation study

Subject	Depression (caffeine)	Depression (placebo)	Difference (placebo – caffeine)
1	5	16	11
2	5	23	18
3	4	5	1
4	3	7	4
5	8	14	6
6	5	24	19
7	0	6	6
8	0	3	3
9	2	15	13
10	11	12	1
11	1	0	- 1

1) Set Up Hypotheses: If caffeine deprivation has no effect on depression, then we would expect the actual mean difference in depression scores to be 0. We want to test the hypotheses where

μ_d = the true mean difference (placebo – caffeine) in depression score.

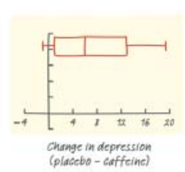
H_0: $\mu_d = 0$
H_a: $\mu_d > 0$

Since no significance level is given, we'll use $\alpha = 0.05$.

2) Check Conditions: If conditions are met, we should do a paired *t* test for μ_d.

✓*Random* researchers randomly assigned the treatment order—placebo then caffeine, caffeine then placebo—to the subjects.

✓*Normal* We don't know whether the actual distribution of difference in depression scores (placebo - caffeine) is Normal. With such a small sample size (n = 11), we need to examine the data to see if it's safe to use *t* procedures.



The histogram has an irregular shape with so few values; the boxplot shows some right-skewness but not outliers; and the Normal probability plot looks fairly linear. With no outliers or strong skewness, the *t* procedures should be pretty accurate.

✓*Independent* We aren't sampling, so it isn't necessary to check the *10% condition*. We will assume that the changes in depression scores for individual subjects are independent. This is reasonable if the experiment is conducted properly.

✓ ***σ is unknown*** We must use a t-statistic

3) Mechanics: The sample mean and standard deviation are $\bar{x}_d = 7.364$ and $s_d = 6.918$

Test statistic $$t = \frac{\bar{x}_d - \mu_0}{s_d / \sqrt{n}} = \frac{7.364 - 0}{6.918/\sqrt{11}} = 3.53$$

***P*-value** According to technology, the area to the right of $t = 3.53$ on the t distribution curve with df = 11 – 1 = 10 is 0.0027.

4) Conclude:
With a *P*-value of 0.0027, which is much less than our chosen $\alpha = 0.05$, we have convincing evidence to reject H_0: $\mu_d = 0$.
We can therefore conclude that depriving these caffeine-dependent subjects of caffeine caused an average increase in depression scores.

Paired T-Tests

Summary

In this section, we learned that…

- ✓ If we somehow know σ, we can use a z test statistic and the standard Normal distribution to perform calculations. In practice, we typically do not know σ. Then, we use the **one-sample t statistic**

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$$

  with *P*-values calculated from the t distribution with n - 1 degrees of freedom.

- ✓ Analyze **paired data** by first taking the difference within each pair to produce a single sample. Then use one-sample t procedures.