CHAPTER 6 RANDOM VARIABLES (p 408-409)

Review ANSWER SHEET

R6.1 KNEES:

VALUE (x)	1	2	3	4	5	
Prob (x)	.1	.2	.3	.3	$\boxed{.1}$	= 1.00

A $P(X=5) = 1-(.1+.2+.3+.3) = 1-.9 = \boxed{.1}$

B $P(X\le 2) = P(x=1)+P(x=2) = .1+.2 = .3$

P(BOTH have pain 1 or 2) $= (.3)(.3) = .09$

C $\mu_X = 1(.1)+2(.2)+3(.3)+4(.3)+5(.1) = 3.1$

$\sigma_X^2 = (1-3.1)^2(.1)+(2-3.1)^2(.2)+(3-3.1)^2(.3)+$
$(4-3.1)^2(.3)+(5-3.1)^2(.1) = \sqrt{1.29} \Rightarrow \boxed{\sigma_X = 1.136}$

$\boxed{\mu_X = 3.1 \quad \sigma_X = 1.136}$

R6.2 $\mu_X = 550°C \qquad \sigma_X = 5.7°C$

A $D = X - 550 \qquad \mu_D = 550-550 \qquad \boxed{\mu_D = 0°C}$ $\boxed{\sigma_D = 5.7°C}$

B $Y = 9/5X + 32$

$\mu_Y = 9/5(550)+32 \qquad \sigma_Y = 9/5(5.7)$

$\boxed{\mu_Y = 1,022°F} \qquad \boxed{\sigma_Y = 10.26°F}$

remember, adding a constant to a single RV does NOT change the variability.

SOURCE TPS

R6.3

X = the payout for a single \$1 bet

$\mu_x = \$.70$ $\sigma_x = \$6.58$

Check $\mu_x + \sigma_x$

L1 Payout \$ (x)	0	0	\$1	\$3	\$120
L2 Prob (x)	.308	.433	.213	.043	.003

$\mu_x = L3 = L1 * L2 \Rightarrow$ 2ND CALC 1VAR $\Sigma = \$.702$ ✓

L4 $\sigma^2_x = (L1 - .702)^2(L2) \Rightarrow$ $\Sigma = 43.307$

$\sigma_x = \sqrt{43.307} = \$6.58$

(A) THE AVERAGE PAYOUT ON A \$1 BET IS \$.70.
THE AMOUNT THAT AN INDIVIDUALS PAYOUT VARY FROM THIS IS \$6.58, ON AVERAGE.

(B) Y = The amount of Jerry's payout (\$5)

$\mu_y = 5\mu_x = 5(.70)$
$\sigma_y = 5\mu_y = 5(6.58)$

$\boxed{\mu_y = \$3.50 \quad \sigma_y = \$32.90}$

(C) W = the amount of Maria's payout (5 - \$1 bets)

$\mu_w = \mu_x + \mu_x + \mu_x + \mu_x + \mu_x = 5\mu_x = 5(.70) = \$3.50$

$\sigma^2_w = 5 \cdot (\sigma_x)^2 = 5(6.58) = 216.482$

$\boxed{\mu_w = \$3.50 \quad \sigma_w = \$14.71}$

(d) The AVERAGE PAYOUT FOR BOTH Jerry and Maria at \$3.50 However the CASINO would probably prefer Maria since there is less variability in her strategy. They are less likely to get great amounts from her but also less likely to have to pay great amounts to her

R 6.4

A T = Capping-machine torque (in-lb) $N(7, .9)$
C = Cap strength - torque to break cap (in/lb) $N(10, 1.2)$

They are independent because the machine that makes the cap and the machine that applies the torque are not the same.

B $D = C - T$

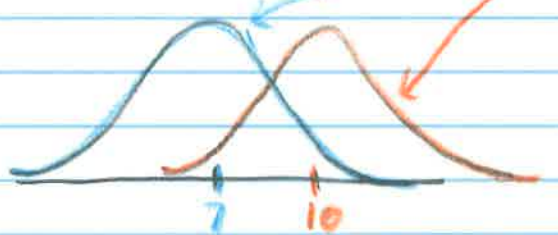
$\mu_D = \mu_C - \mu_T = 10 - 7 = 3$ $\boxed{\mu_D = 3 \text{ in/lb}}$

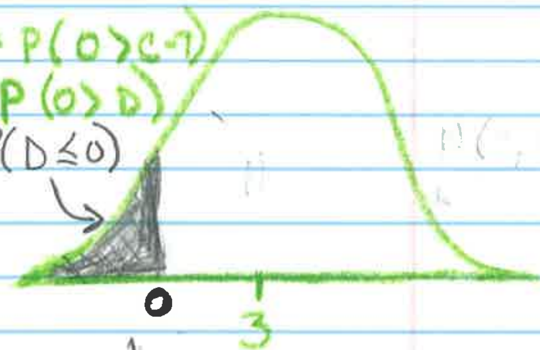
$\sigma^2_D = \sigma^2_C - \sigma_T^2 = .9^2 + 1.2^2 = 2.25$

$\sigma_D = \sqrt{2.25}$ $\boxed{\sigma_D = 1.5 \text{ in/lb}}$

C P(Cap breaks) = $P(T > C) = P(0 > C-T)$

$P(0 > D)$





$N(3, 1.5)$

$P(D \leq 0)$ = normalcdf(-E99, 0, 3, 1.5) = $\underline{\underline{.02275}}$

Using the New Random Variable, D, with a normal distribution with mean = 3 in/lb and standard deviation = 1.5 in/lb, The probability that the cap will break is about 2% (.02275).

R6.5 ORANG M+M's

ⓐ B - EITHER ORANGE OR NOT

I - Assuming the candies are well mixed, the color of 1 candy chosen should NOT TELL US ANYTHING ABOUT THE COLOR OF ANOTHER

N - FIXED NUMBER OF TRIALS $N=8$

S - FIXED Probability $p=.2$

CONCLUSION: X = # of orange candies you get is a RV.

ⓑ $\mu_X = np = 8(.2) = 1.6$ "We expect to find 1.6 orange M+MS in a sample size of 8."

ⓒ $\sigma_X = \sqrt{np(1-p)}$

$= \sqrt{8(.2)(.8)} = 1.13$ "In individual samples of size 8, the number of orange M+M's will vary by 1.13, on average."

R6.6 ⓐ $P(X=0) \Rightarrow P(X=K) = \binom{n}{k} p^K (1-p)^{N-K}$

$P(X=0) = \binom{8}{0} (.2)^0 (1-.2)^8$

$\binom{8}{0} = 8 \; nCr \; 0 = 1$

under Math prb nCr

$\rightarrow 1 \, (1) \, (.8)^8 = .1677$

SINCE THE PROBABILITY IS ABOUT 17%, IT WOULD NOT BE SURPRISING TO GET NO ORANGE M+M'S.

ⓑ $P(X \geq 5) = 1 - P(X \leq 4) = 1 - \text{binomcdf}(8, .2, 4) = 1 - .9896$

$\boxed{P(X \geq 5) = .0104}$ SINCE THE PROBABILITY IS ABOUT 1%, IT WOULD BE SOMEWHAT SURPRISING TO FIND 5 OR MORE M+M'S THAT ARE ORANGE.

$\boxed{R6.7}$ Y = Number of spins to get a "washabi bomb"

12 EQUAL SECTIONS
9 SUSHI $9/12 = 3/4$
3 WASHABI $3/12 = 1/4$

Geometric Distribution
$G(.25)$

$$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \underbrace{.4375}_{\text{geometcdf}(.25,2)} = \boxed{.5625}$$

The probability it takes 3 or more spins to land on "washabi bomb" is 56.25%

$\boxed{R6.8}$ (a) X = # of heads in 10,000 tosses

$\mu_X = 1/2(10{,}000) = 5{,}000$ heads

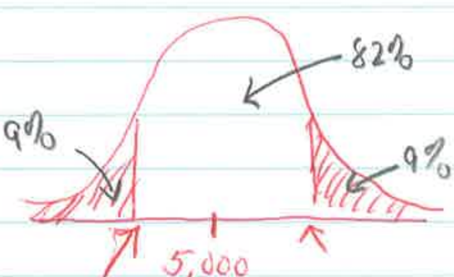
$\sigma_X = \sqrt{npq} = \sqrt{2500} = 50$ heads

(b) The conditions for approximating a normal distribution are met:

$np \geq 10$
$1/2(5000) = 2500 \geq 10$ ✓

$n(1-p) \geq 10$
$1/2(5000) = 2500 \geq 10$ ✓

(c) $N(5{,}000, 50)$ — Well balanced coin



4933
normalcdf
$(-1E99, 4933, 5000, 50) =$
$.09$

5067
normalcdf
$(5067, E99, 5000, 50) =$
$.09$

$\boxed{P(X \leq 4933 \text{ or } X \geq 5067) = .18}$

Conclusion: IF THE COIN WERE WELL BALANCED, GETTING 5,067 heads or more (or 4,933 heads or fewer) would happen 18% of THE time in 10,000 tosses.

This IS NOT particularly surprising, so we do not have evidence that the coin was not balanced.