NAME:

ANSWER KEY

Chapter Review Exercises

These exercises are designed to help you review the important ideas and methods of the chapter. Relevant learning objectives are provided in bulleted form before each exercise.

- Use a probability distribution to answer questions about possible values of a random variable.
- Calculate the mean and standard deviation of a discrete random variable.

R6.1. Knees Patients receiving artificial knees often experience pain after surgery. The pain is measured on a subjective scale with possible values of 1 (low) to 5 (high). Let X be the pain score for a randomly selected patient. The following table gives part of the probability distribution for X.

Value:	1	2	3	4	5
Probability:	0.1	0.2	0.3	0.3	??

(a) Find $P(X = 5)$.

(b) If two patients who received artificial knees are chosen at random, what's the probability that both of them report pain scores of 1 or 2? Show your work.

(c) Compute the mean and standard deviation of X. Show your work.

- Describe the effects of transforming a random variable by adding or subtracting a constant and multiplying or dividing by a constant.
- Find the mean and standard deviation of the sum or difference of independent random variables.

R6.2. A glass act In a process for manufacturing glassware, glass stems are sealed by heating them in a flame. The temperature of the flame can be adjusted to one of five different settings. Here is the probability distribution of the flame ~~temperature X (in~~ degrees Celsius) for a randomly chosen glass:

Temperature:	540°	545°	550°	555°	560°
Probability:	0.1	0.25	0.3	0.25	0.1

$$\mu_X = 550°C \qquad \sigma_X = 5.7°C$$

(a) The target temperature is 550°C. What are the mean and standard deviation of the number of degrees off target, $D = X - 550$?

(b) A manager asks for results in degrees Fahrenheit. The conversion of X into degrees Fahrenheit is given by $y = \frac{9}{5}X + 32$. What are the mean μ_Y and the standard deviation σ_Y of the temperature of the flame in the Fahrenheit scale?

- Interpret the mean and standard deviation of a random variable.
- Find the mean and standard deviation of the sum or difference of independent random variables.

R6.3. Keno In a game of 4-Spot Keno, the player picks 4 numbers from 1 to 80. The casino randomly selects 20 winning numbers from 1 to 80. The table below shows the possible outcomes of the game and their probabilities, along with the amount of money (Payout) that the player wins for a \$1 bet. If X = the payout for a single \$1 bet, you can check that $\mu_X = \$0.70$ and $\sigma_X = \$6.58$.

Matches:	0	1	2	3	4
Probability:	0.308	0.433	0.213	0.043	0.003
Payout:	\$0	\$0	\$1	\$3	\$120

(a) Interpret the values of μ_X and σ_X in context.

(b) Jerry places a single \$5 bet on 4-Spot Keno. Find the expected value and the standard deviation of his winnings.

(c) Marla plays five games of 4-Spot Keno, betting \$1 each time. Find the expected value and the standard deviation of her total winnings.

(d) Based on your answers to (b) and (c), which player would the casino prefer? Justify your answer.

- Determine whether two random variables are independent.
- Find probabilities involving the sum or difference of independent Normal random variables.

R6.4. Applying torque A machine fastens plastic screw-on caps onto containers of motor oil. If the machine applies more torque than the cap can withstand, the cap will break. Both the torque applied and the strength of the caps vary. The capping-machine torque T follows a Normal distribution with mean 7 inch-pounds and standard deviation 0.9 inch-pounds. The cap strength C (the torque that would break the cap) follows a Normal distribution with mean 10 inch-pounds and standard deviation 1.2 inch-pounds.

(a) Explain why it is reasonable to assume that the cap strength and the torque applied by the machine are independent.
(b) Let the random variable $D = C - T$. Find its mean and standard deviation.
(c) What is the probability that a cap will break while being fastened by the machine? Show your work.

Exercises R6.5 and R6.6 refer to the following setting. According to the Mars candy company, 20% of its plain M&M's candies are orange. Assume that the company's claim is true. Suppose that you reach into a large bag of plain M&M's (without looking) and pull out 8 candies. Let X = the number of orange candies you get.

- Determine whether the conditions for a binomial random variable are met.
- Calculate the mean and standard deviation of a binomial random variable. Interpret these values in context.

R6.5. Orange M&M's
(a) Explain why X is a binomial random variable.
(b) Find and interpret the expected value of X.
(c) Find and interpret the standard deviation of X.

- Compute and interpret probabilities involving binomial distributions.

R6.6. Orange M&M's
(a) Would you be surprised if none of the candies were orange? Compute an appropriate probability to support your answer.
(b) How surprising would it be to get 5 or more orange candies? Compute an appropriate probability to support your answer.

- Find probabilities involving geometric random variables.

R6.7. *Sushi Roulette* In the Japanese game show *Sushi Roulette*, the contestant spins a large wheel that's divided into 12 equal sections. Nine of the sections have a sushi roll, and three have a "wasabi bomb." When the wheel stops, the contestant must eat whatever food is on that section. To win the game, the contestant must eat one wasabi bomb. Find the probability that it takes 3 or more spins for the contestant to get a wasabi bomb. Show your method clearly.

- Determine whether the conditions for the Normal approximation to a binomial distribution are met.
- When appropriate, use the Normal approximation to estimate probabilities in a binomial setting.

R6.8.* Is this coin balanced? While he was a prisoner of war during World War II, John Kerrich tossed a coin 10,000 times. He got 5067 heads. If the coin is perfectly balanced, the probability of a head is 0.5.
(a) Find the mean and the standard deviation of the number of heads in 10,000 tosses, assuming the coin is perfectly balanced.
(b) Explain why the Normal approximation is appropriate for calculating probabilities involving the number of heads in 10,000 tosses.
(c) Is there reason to think that Kerrich's coin was not balanced? To answer this question, use a Normal distribution to estimate the probability that tossing a balanced coin 10,000 times would give a count of heads at least this far from 5000 (that is, at least 5067 heads or no more than 4933 heads).

CHAPTER 7 RANDOM VARIABLES

ANSWER SHEET

R6.1 KNEES:

VALUE (X)	1	2	3	4	5	
Prob (X)	.1	.2	.3	.3	$\boxed{.1}$	= 1.00

A $P(X=5) = 1-(.1+.2+.3+.3) = 1-.9 = \boxed{.1}$

B $P(X \le 2) = P(X=1) + P(X=2) = .1 + .2 = .3$

P(BOTH have pain 1 or 2) $= (.3)(.3) = .09$

C $\mu_X = 1(.1) + 2(.2) + 3(.3) + 4(.3) + 5(.1) = 3.1$

$\sigma_X^2 = (1-3.1)^2(.1) + (2-3.1)^2(.2) + (3-3.1)^2(.3) +$
$(4-3.1)^2(.3) + (5-3.1)^2(.1) = \sqrt{1.29} \Rightarrow \boxed{\sigma_X = 1.136}$

$\boxed{\mu_X = 3.1 \quad \sigma_X = 1.136}$

R6.2 $\mu_X = 550°C$ $\sigma_X = 5.7°C$

A $D = X - 550$ $\mu_D = 550 - 550$ $\boxed{\mu_D = 0°C}$ $\boxed{\sigma_D = 5.7°C}$

B $Y = 9/5 X + 32$

$\mu_Y = 9/5(550) + 32$ $\sigma_Y = 9/5(5.7)$

$\boxed{\mu_Y = 1,022°F}$ $\boxed{\sigma_Y = 10.26°F}$

remember, adding a constant to a single RV does NOT change the variability.

SOURCE TPS

R6.3 X = the payout for a single \$1 bet

$\mu_X = \$.70$ $\quad\sigma_X = \$6.58$

Check $\mu_X + \sigma_X$

L1 Payout \$ (x)	0	0	\$1	\$3	\$120
L2 Prob (x)	.308	.433	.213	.043	.003

$\mu_X = L3 = L1 * L2 \Rightarrow$ 2ND CALC 1VAR $\Sigma = \$.702$ ✓d

L4 $\sigma^2_X = (L1 - .702)^2 (L2) \Rightarrow$ $\Sigma = \sqrt{43.307}$

$\sigma_X = \sqrt{43.307} = \$6.58$

(A) THE AVERAGE PAYOUT ON A \$1 BET IS \$.70. THE AMOUNT THAT AN INDIVIDUALS PAYOUT VARY FROM THIS IS \$6.58, ON AVERAGE.

(B) Y = The amount of Jerry's payout (\$5)

$\mu_Y = 5\mu_X = 5(.70)$
$\sigma_Y = 5\mu_Y = 5(6.58)$

$\boxed{\mu_Y = \$3.50 \quad \sigma_Y = \$32.90}$

(C) W = the amount of Maria's payout (5 - \$1 bets)

$\mu_W = \mu_X + \mu_X + \mu_X + \mu_X + \mu_X = 5\mu_X = 5(.70) = \$3.50$
$\sigma^2_W = 5 \cdot (\sigma_X)^2 = 5(6.58) = 216.482$

$\boxed{\mu_W = \$3.50 \quad \sigma_W = \$14.71}$

(d) The AVERAGE PAYOUT FOR BOTH Jerry and Maria at \$3.50 However the CASINO would probably prefer Maria since there is less variability in her strategy. They are less likely to get great amounts from her but also less likely to have to pay great amounts to her

R 6.4

A T = Capping-machine torque (in-lb) $N(7, .9)$
C = Cap strength - torque to break cap (in/lb) $N(10, 1.2)$

They are independent because the machine that makes the cap and the machine that applies the torque are not the same.

B $D = C - T$ $\quad \mu_D = \mu_C - \mu_T = 10 - 7 = 3$ $\quad \boxed{\mu_D = 3 \text{ in/lb}}$

$\sigma_D^2 = \sigma_C^2 - \sigma_T^2 = .9^2 + 1.2^2 = 2.25$

$\sigma_D = \sqrt{2.25}$ $\quad \boxed{\sigma_D = 1.5 \text{ in/lb}}$

C P(Cap breaks) = P(T > C) = P(0 > C-T)

P(0 > D)

$N(3, 1.5)$

$P(D \le 0)$ = normalcdf

$(-E99, 0, 3, 1.5) = \underline{\underline{.02275}}$

USING THE New Random Variable, D,
with a normal distribution with mean = 3 in/lb and standard deviation = 1.5 in/lb, The probability that the cap will break is about 2% (.02275).

R6.5 ORANG M+M'S

(a) B - EITHER ORANGE OR NOT
I - Assuming the candies are well mixed, the color of 1 candy chosen should NOT TELL US ANYTHING ABOUT THE COLOR OF ANOTHER
N - FIXED NUMBER OF TRIALS $N=8$
S - FIXED Probability $p=.2$

CONCLUSION: X = # of orange candies you get is a RV.

(b) $\mu_X = np = 8(.2) = 1.6$ "We expect to find 1.6 orange M+M's in a sample size of 8."

(c) $\sigma_X = \sqrt{np(1-p)}$
$= \sqrt{8(.2)(.8)} = 1.13$ "In individual samples of size 8, the number of orange M+M's will vary by 1.13, on average."

R6.6 (a) $P(X=0) \Rightarrow P(X=K) = \binom{n}{k} p^K (1-p)^{N-K}$

$P(X=0) = \binom{8}{0} (.2)^0 (1-.2)^8$

$\binom{8}{0} = 8 \text{ nCr } 0 = 1$
under (Math) (prb) nCr

$1 (1) (.8)^8 = .1677$

SINCE THE PROBABILITY IS ABOUT 17%, IT WOULD NOT BE SURPRISING TO GET NO ORANGE M+M'S.

(b) $P(X \geq 5) = 1 - P(X \leq 4) = 1 - \text{binomcdf}(8, .2, 4) = 1 - .9896$

$\boxed{P(X \geq 5) = .0104}$ SINCE THE PROBABILITY IS ABOUT 1%, IT WOULD BE SOMEWHAT SURPRISING TO FIND 5 OR MORE M+M'S THAT ARE ORANGE.

R6.7 Y = Number of spins to get a "washabi bomb"

12 EQUAL SECTIONS
9 SUSHI $9/12 = 3/4$
3 WASHABI $3/12 = 1/4$

Geometric Distribution
$G(.25)$

$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - \underbrace{.4375}_{\text{geometcdf}(.25,2)} = \boxed{.5625}$

The probability it takes 3 or more spins to land on "washabi bomb" is 56.25%

R6.8 (a) X = # of heads in 10,000 tosses

$\mu_X = 1/2(10000) = 5{,}000$ heads

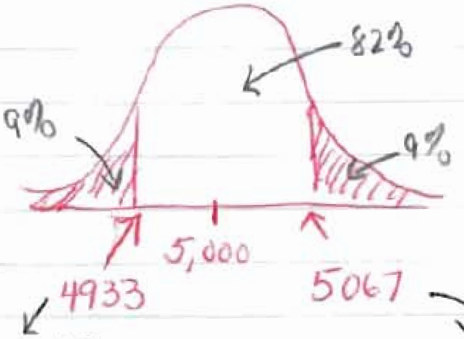
$\sigma_X = \sqrt{npq} = \sqrt{2500} = 50$ heads

(b) The conditions for approximating a normal distribution are met:

$np \geq 10$
$1/2(5000) = 2500 \geq 10$ ✓

$n(1-p) \geq 10$
$1/2(5000) = 2500 \geq 10$ ✓

(c) $N(5{,}000, 50)$ — Well balanced coin



normalcdf $(-1E99, 4933, 5000, 50) =$
$.09$

normalcdf $(5067, E99, 5000, 50) =$
$.09$

$\boxed{P(X \leq 4933 \text{ or } X \geq 5067) = .18}$

Conclusion: IF THE COIN WERE WELL BALANCED, GETTING 5,067 heads or more (or 4,933 heads or fewer) would happen 18% of THE Time in 10,000 tosses.

This IS NOT particularly surprising, so we do NOT have evidence that the coin was NOT balanced.