AP Statistics – 6.3B2
Goal: Understanding Binomial RV's

Remember: Binomial Models are DISCRETE RV's.
Examples:
$P(X<2) = P(X=0) + P(X=1)$
$P(X\leq2) = P(X=0) + P(X=1) + P(X=2)$
$P(X>3) \leftrightarrow P(X\geq4)$
$P(X\geq3) = 1-P(X\leq2)$ $P(X>3) = 1-P(X\leq3)$

I. **Binomial Setting:** CYU (page 385):

1. LET X = # OF ACES → BINOMIAL RV w/ $n=10$ $p=4/52$

B = BINARY - SUCCESS OR FAILURE | YES - SUCCESS IS AN ACE ; FAILURE NOT ACE.
I = INDEPENDENT | YES - SAMPLE WITH REPLACEMENT
N = FIXED NUMBER OF TRIALS | YES - FIXED TRIALS - 10
S = FIXED PROBABILITY OF SUCCESS | YES - FIXED PROBABILITY 4/52

2. Let Y = # over 6 ft tall → NOT BINOMIAL RV (NOT INDEPENDENT)

B = ✓ SUCCESS - OVER 6 FT FAILURE - UNDER 6 FT
I = NOT INDEPENDENT - SELECTING WITHOUT REPLACEMENT FOR A SMALL SAMPLE ✗
N = ✓ FIXED TRIALS - 3
S = ✓ PROBABILITY WILL NOT CHANGE FROM PERSON TO PERSON

3. LET W = # of 5s rolled → NOT BINOMIAL RV (NOT FIXED PROB.)

B = ✓ SUCCESS - ROLL a 5 FAILURE - NOT A 5
I = ✓ INDEPENDENT SINCE ROLLING A DIE - 1 TRIAL DOES NOT AFFECT ANOTHER TRIAL
N = ✓ NUMBER TRIALS FIXED - 5
S = NOT FIXED PROBABILITY 6 SIDED DIE 1/6
8 SIDED DIE 1/8

II. **Binomial Probabilities:** ***Example**: Rolling Doubles*

In many games involving dice, rolling doubles is desirable. Rolling doubles mean the outcomes of two dice are the same, such as 1&1 or 5&5. The probability of rolling doubles when rolling two dice is 6/36 = 1/6. Suppose that a game player rolls the dice 4 times, hoping to roll doubles.

(a) What is the probability that all 4 rolls are **not** doubles?

- Define the RV: LET X = THE NUMBER OF DOUBLES IN 4 ROLLS OF 2 DICE.
- Have the Binomial Conditions been met?
  B - ✓ SUCCESS - ROLL DOUBLES FAILURE - NOT DOUBLES
  I - ✓ DICES ARE INDEPENDENT - 1 ROLL DOES NOT AFFECT ANOTHER ROLL
  N - ✓ FIXED TRIALS - 4
  S - ✓ FIXED PROBABILITY $= 6/36 = 1/6$
- State the parameters of the Binomial Distribution X HAS A BINOMIAL DISTRIBUTION WITH $n=4$ $p=1/6$
- Give the probability statement:
  P(NO DOUBLES) ⇒ $P(X=0)$
- How many ways can you roll doubles 0 times in 4 attempts?
  1 WAY - THE ONLY OUTCOME - F F F F
- Calculate the probability

$P(X=0) = P(F \cdot F \cdot F \cdot F)$ ← since independent, we can MULTIPLY PROBABILITIES

$= P(F) = 1 - 1/6 = 5/6$

$= (5/6)^4$

$\boxed{P(X=0) = .482}$

***Example**: Rolling Doubles (continued)*

(b) Find the probability that the player gets doubles once in four attempts.

- Give the probability statement: $P(X=1)$
- How many ways can you roll doubles 0 times in 4 attempts?

1ST TRY S F F F
2ND TRY F S F F
3RD TRY F F S F
4TH TRY F F F S

4 POSSIBLE OUTCOMES TO ROLL DOUBLES ONCE IN 4 TRIES.

- Calculate the probability THE LONG WAY:

$P(SFFF) = (1/6)(5/6)(5/6)(5/6)$
$P(FSFF) = (1/6)(5/6)^3$
$P(SSFS) = (1/6)(5/6)^3$
$P(FFFS) = (5/6)^3(1/6)$

$P(X=1) = 4(1/6)(5/6)^3 = .386$

HOW TO FIND THE # OF OUTCOMES WITH CALC:

$\binom{n}{k} = \boxed{nCr}$

See green sheet
n = # trials
k = # successes

> math
> prb

FIND: steps
$\binom{4}{1} =$ 4 math PRB 3: nCr 1 = 4

(c) Find the probability that the player gets doubles twice in four attempts.

- Give the probability statement: $P(X=2)$
- How many ways can you roll doubles 2 times in 4 attempts?

$\binom{4}{2} = 4\,nCr\,2 =$ 6 possible outcomes

- Calculate the probability

See Green Sheet

$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$P(X=2) = 6 (1/6)^2 (5/6)^2 = .116$

Tip: Check with: binompdf(4, 1/6, 2)

(d) Should the player be surprised if he gets doubles more than twice in four attempts? Justify your answer.

$P(X>2) = P(X=3) + P(X=4)$

$= \binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) + \binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0$

$= 4\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right) + 1\left(\frac{1}{6}\right)^4 \cdot 1$

Can also be written $P(X\geq3)$

$P(X>2) = .015 + .00077 \approx \boxed{.016}$

CONCLUSION: SINCE THERE IS ONLY A 1.6% CHANCE OF GETTING MORE THAN 2 DOUBLES IN 4 ROLLS, THE PLAYER SHOULD BE SURPRISED IF THIS HAPPENS.

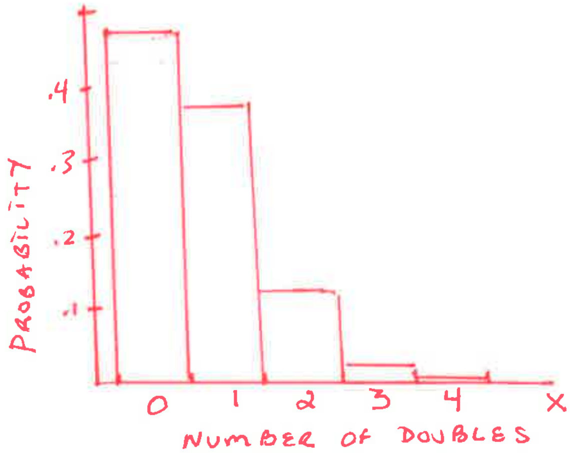
(e) Summarize the probability distribution of the Random Variable X in the following table

Let X = number of doubles in 4 attempts, (X follows a binomial distribution with n = 4 and p = 1/6)

Value (x)	0	1	2	3	4
Probability	.482	.386	.116	.015	.001

← L1
← L2

(f) Create a histogram the probability distribution



Remember:

HISTOGRAM
STAT PLOT
XLIST – L1
FREQ – L2

WINDOW
XMIN = 0
XMAX = 5
X SCL = 1

YMIN = 0
YMAX = .5
YSCL = .1

(g) Find the Mean and Standard Deviation. Clearly show your work.

Remember 1-Var Stat LIST: L1 FREQLIST: L2

$E(X) = \mu_X = \sum x_i p_i = 0(.482) + \dots + 4(.001) = .667$

$VAR(X) = \sigma_X^2 = \sum (x_i - \mu_X)^2 \cdot p_i = (0 - .667)^2(.482) + \dots + (4 - .667)^2(.001) = .556$

$SD(X) = \sigma_X = \sqrt{.556} = .746$

Green Sheet
Binomial Dist
$\mu_X = np = 4(1/6) = .667$
$\sigma_X = \sqrt{np(1-p)} = \sqrt{4(1/6)(5/6)} = .746$

III. **Binomial Problem start to finish:** **Example**: ***Tastes as Good as the Real Thing?***

The makers of a diet cola claim that its taste is indistinguishable from the full calorie version of the same cola. To investigate, an AP Statistics student named Emily prepared small samples of each type of soda in identical cups. Then, she had volunteers taste each cola in a random order and try to identify which was the diet cola and which was the regular cola. Overall, 23 of the 30 subjects made the correct identification.

If we assume that the volunteers really couldn't tell the difference, then each one was guessing with a 1/2 chance of being correct. Let X = the number of volunteers who correctly identify the colas.

(a) Explain why X is a binomial random variable.

BINARY? YES. GUESS CORRECTLY OR NOT

INDEPENDENT? YES. RANDOMLY ASSIGNED. ASSUMES VOLUNTEERS CANNOT TELL THE DIFFERENCE.

NUMBER? YES. THERE ARE 30 TRIALS

SUCCESS? YES. THE PROBABILITY OF GUESSING CORRECTLY IS ALWAYS 50%

This is a Binomial Distribution WITH n=30 and p=.5. → B(30.5)

(b) Find the mean and the standard deviation of X. Interpret each value in context.

$1-p=q$

$\mu_X = np = 30(.5) = 15$

$\sigma_X = \sqrt{np(1-p)} = \sqrt{30(1/2)(1/2)} = 2.74$

IF THIS EXPERIMENT WAS REPEATED MANY TIMES AND THE VOLUNTEERS WERE RANDOMLY GUESSING, THE AVERAGE NUMBER OF CORRECT GUESSES WOULD BE ABOUT 15 AND THE NUMBER OF CORRECT GUESSES WOULD VARY FROM 15 BY ABOUT 2.74, ON AVERAGE.

(c) Of the 30 volunteers, 23 made correct identifications. Does this give convincing evidence that the volunteers can taste the difference between the diet and regular colas?

$P(X \geq 23) = 1 - P(X \leq 22)$ ← the binomial cdf ALWAYS Comes from the left

$= 1 - \text{binomcdf}(30, .5, 22)$

$= .0026$

THERE IS A VERY SMALL CHANCE THAT THERE WOULD BE 23 OR MORE Correct GUESSES IF THE VOLUNTEERS COULDN'T TELL THE DIFFERENCE IN COLA'S. THEREFORE, WE HAVE CONVINCING EVIDENCE THAT THE VOLUNTEERS CAN TASTE THE DIFFERENCE.

IV. What you need to know about binomial and geometric RV's and their distributions

Remember: Binomial and Geometric Models are DISCRETE RV's. See page 1 for examples

Binomial Setting (BINS)	Geometric Setting (BITS)
• **Binary? Each observation falls into one of two categories: success or failure.**	• **Binary? Each observation falls into one of two categories: success or failure.**
• **Independent? The *n* observations are all independent.**	• **Independent? The *n* observations are all independent.**
• **Number? There is a fixed number *n* of observations.**	• **Trials?** The variable of interest is **"the number of trials required to obtain the 1st success."**
• **Success? The probability of success, *p*, is the same for each observation.**	• **Success? The probability of success, *p*, is the same for each observation.**

Independence – knowing the result of one trial does not have any effect on the result of any other trial.

Binomial Distribution	Geometric Distribution
• B(n,p) **where** ○ **n is the number of trials and** ○ **p is the fixed probability**	• G(p) **where** ○ **p is the fixed probability**

Variables used in formulas below *X=random variable* *n = number of trials*	*p = probability of success* *q = (1-p) = means "probability of failure"* *k = # of successes*
Binomial Probability To find the number of possible outcome: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ **Learn how to use calculator. No need to memorize formula:** $\binom{4}{2} = 4\ nCr\ 2 = 6$ **To find the probability for "k" successes:** $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$	**Geometric Probability** $P(X=k) = (1-p)^{k-1}p$ * ***where k = number of trials until the first success.*** **Probability it takes more than n trials to see the 1st success :** $P(x>k) = (1-p)^k$
Binompdf(n,p,k) – "Point" density function *Remember to state distribution B(n,p)*	**Geometpdf(p,k)** – "Point" density function *Remember to state distribution G(p)*
Binomcdf(n,p,k) – Cumulative density function *Remember to state distribution B(n,p)*	**Geometcdf(p,k)** – Cumulative density function *Remember to state distribution G(p)*
$\mu = np$ $\sigma = \sqrt{np(1-p)}$ **Conditions: $np \geq 10$ and $n(1-p) \geq 10$** * Better approximation as *n* gets larger.	Don't memorize the formulas for the geometric mean and standard deviation. $\mu_Y = E(Y) = \frac{1}{p}$