Section 6.4 Conditional Probability

Learning Objectives

After this section, you should be able to…

- ✓ DEFINE conditional probability
- ✓ COMPUTE conditional probabilities

Conditional Probability Defined

- When we want the probability of an event from a *conditional* distribution, we write ***P*(B|A)** and pronounce it "**the probability of B *given* A**."
- A probability that takes into account a given condition is called a **conditional probability**.

Definition:

The probability that one event happens given that another event is already known to have happened is called a **conditional probability.** Suppose we know that event *A* has happened. Then the probability that event *B* happens *given* that event *A* has happened is denoted by $P(B \mid A)$.

Read | as "given that" or "under the condition that"

How to find conditional probability of an event B *given the event A.*

1. We restrict our attention to the outcomes in A.
2. We then find the fraction of *those* outcomes that are A and B.

$$P(\mathbf{B}|\mathbf{A}) = \frac{P(\mathbf{A} \cap \mathbf{B})}{P(\mathbf{A})}$$

- **Note: *P*(A) cannot equal 0, since we know that A has occurred.**

Example 1 – DECK OF CARDS

It is important to understand the direction of conditioning

EXAMPLE: I draw a card and look at it. I tell you it is red.

- What is probability it is a heart, given red?

P(heart | red) = P(heart and red) / P(red) =

- And what is the probability it is red, given a heart?

P(red| heart) = P(red and heart) / P(heart) =

Example 2: Using two-way table

Consider the Grade Distributions below. Define events

E: the grade comes from an EPS course, and

B: the grade is lower than a B.

	Grade Level			
School	A	B	Below B	Total
Liberal Arts	2,142	1,890	2,268	6300
Engineering and Physical Sciences	368	432	800	1600
Health and Human Services	882	630	588	2100
Total	3392	2952	3656	*10000*

Find *P*(*B*)

$P(B) = 3656 / 10000 = 0.3656$

Find *P*(*E* | *B*)

$P(E \mid B) = 800 / 3656 = 0.2188$

Find *P*(*B* | *E*)

$P(B \mid E) = 800 / 1600 = 0.5000$

Example 3: Using two-way table

Use the table to answer the questions:

	Jeans	Other	Total
MALE	12	5	17
FEMALE	8	11	19
TOTAL	20	16	36

1) **What is the probability a male wears Jeans?**

2) **What is the probability that someone wearing jeans is male?**

3) **Are being male and wearing jeans disjoint?**

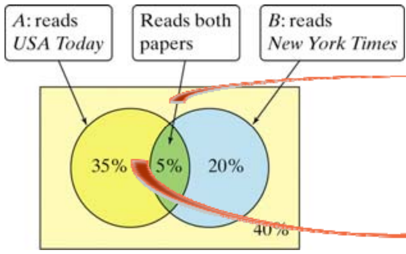
Example 4: Using Venn Diagrams

Residents of a large apartment complex can be classified based on the events

A: reads *USA Today* and

B: reads the *New York Times*.

The Venn Diagram below describes the residents. What is the probability that a randomly selected resident who reads *USA Today* also reads the *New York Times*?



$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = 0.05$$

$$P(A) = 0.40$$

$$P(B \mid A) = \frac{0.05}{0.40} = 0.125$$

There is a 12.5% chance that a randomly selected resident who reads *USA Today* also reads the *New York Times*.