6.4 HW

NAME: ______________________

EXERCISES

SHOW WORK CLEARLY

1. Homes. Real estate ads suggest that 64% of homes for sale have garages, 21% have swimming pools, and 17% have both features. What is the probability that a home for sale has DRAW A VENN DIAGRAM

a) a pool or a garage?
b) neither a pool nor a garage?
c) a pool but no garage?

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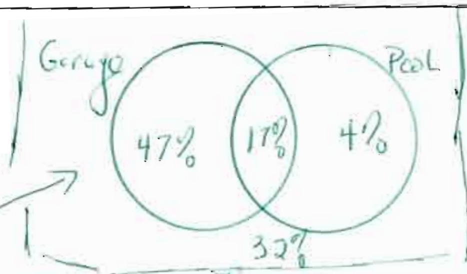

(a) P(Pool or Garage) = .47 + .17 + .04 = .68

or = .64 + .21 − .17

(b) P(neither Pool or Garage) = 1 − .68 = .32

or

(c) P(Pool but No Garage) = 4% from Venn diagram

6. Birth order. A survey of students in a large Introductory Statistics class asked about their birth order (1 = oldest or only child) and which college of the university they were enrolled in. Here are the results:

College	Birth Order: 1 or only	Birth Order: 2 or more	Total
Arts & Sciences	34	23	57
Agriculture	52	41	93
Human Ecology	15	28	43
Other	12	18	30
Total	113	110	223

Suppose we select a student at random from this class. What is the probability that the person is

a) a Human Ecology student?
b) a firstborn student? → $113/223 \approx .507$
c) firstborn *and* a Human Ecology student?
d) firstborn *or* a Human Ecology student?

$43/223 \approx .193$

$\frac{15}{223} \approx .067$

P(1st Born or Ecology) =

$\frac{113}{223} + \frac{43}{223} - \frac{15}{223} = \frac{141}{223} \approx .632$

8. Pets. In its monthly report, the local animal shelter states that it currently has 24 dogs and 18 cats available for adoption. Eight of the dogs and 6 of the cats are male. Find each of the following conditional probabilities if an animal is selected at random:

a) The pet is male, given that it is a cat.
b) The pet is a cat, given that it is female.
c) The pet is female, given that it is a dog.

	CATS	DOGS	
MALE	6	8	14
FEMALE	12	16	28
TOTAL	18	24	42

(a) P(Male | cat) = 6/18 ≃ .3333
(b) P(Cat | female) = 12/28 ≃ .429
(c) P(female | dog) = 16/24 ≃ .667

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9. Health. The probabilities that an adult American man has high blood pressure and/or high cholesterol are shown in the table.

Cholesterol	Blood Pressure: High	Blood Pressure: OK	
High	0.11	0.21	.32
OK	0.16	0.52	.68
	.27	.73	1.00

What's the probability that
a) a man has both conditions?
b) a man has high blood pressure?
c) a man with high blood pressure has high cholesterol?
d) a man has high blood pressure if it's known that he has high cholesterol?

(a) P(both Conditions) = .11
(b) P(high blood pressure) = .11 + .16 = $\boxed{.27}$
(c) P(HIGH CHOLESTEROL | HIGH BP) = $\frac{.11}{.27}$ ≃ .407
(d) P(HIGH BP | HIGH CHOLESTEROL) = $\frac{.11}{.32}$ ≃ .344

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10. Death penalty. The table shows the political affiliations of American voters and their positions on the death penalty.

Party	Death Penalty: Favor	Death Penalty: Oppose	
Republican	0.26	0.04	.30
Democrat	0.12	0.24	.36
Other	0.24	0.10	.34
	.62	.38	1.00

a) What's the probability that
  i) a randomly chosen voter favors the death penalty?
  ii) a Republican favors the death penalty?
  iii) a voter who favors the death penalty is a Democrat?
b) A candidate thinks she has a good chance of gaining the votes of anyone who is a Republican or in favor of the death penalty. What portion of the voters is that?

(a)
(i) P(favor death penalty) = .62
(ii) P(favor death penalty | Rep) = $\frac{.26}{.30}$ ≃ .867
(iii) P(Dem | favors death penalty) $\frac{.12}{.62}$ ≃ .194

(b) P(Republic or Favor Death Penalty) = .30 + .62 − .26 = .66

The candidate could expect about 66% of the votes

12. **Birth order, take 2.** Look again at the data about birth order of Intro Stats students and their choices of colleges shown in Exercise 6.

a) If we select a student at random, what's the probability the person is an Arts and Sciences student who is a second child (or more)?
b) Among the Arts and Sciences students, what's the probability a student was a second child (or more)?
c) Among second children (or more), what's the probability the student is enrolled in Arts and Sciences?
d) What's the probability that a first or only child is enrolled in the Agriculture College?
e) What is the probability that an Agriculture student is a first or only child?

(a) P(ARTS + Science + 2ND CHILD) = $\frac{23}{223} \approx .103$

(b) P(Second child | Arts + Science) = $\frac{23}{57} \approx .404$

(c) P(Arts + Science | 2ND Child) = $\frac{23}{110} \approx .209$

(d) P(AGRICULTURE | 1ST BORN) = $\frac{52}{113} \approx .460$

(e) P(1ST BORN | AGRICULTURE) = $\frac{52}{93} \approx .559$

REVIEW PROBLEMS FROM SECTIONS 6.1 to 6.3

8. **Crash.** Commercial airplanes have an excellent safety record. Nevertheless, there are crashes occasionally, with the loss of many lives. In the weeks following a crash, airlines often report a drop in the number of passengers, probably because people are afraid to risk flying.

a) A travel agent suggests that since the law of averages makes it highly unlikely to have two plane crashes within a few weeks of each other, flying soon after a crash is the safest time. What do you think?

THERE IS NO SUCH THING AS THE "LAW OF AVERAGES"! THE OVERALL PROBABILITY OF AN AIRPLANE CRASH DOES NOT CHANGE DUE TO RECENT CRASHES.

9. **Fire insurance.** Insurance companies collect annual payments from homeowners in exchange for paying to rebuild houses that burn down.

a) Why should you be reluctant to accept a $300 payment from your neighbor to replace his house should it burn down during the coming year?
b) Why can the insurance company make that offer?

A IT WOULD BE FOOLISH FOR YOU TO INSURE YOUR NEIGHBORS HOUSE FOR $300, IT IS UNLIKELY YOUR NEIGHBORS HOUSE WOULD BURN DOWN, THE RISK IS PROBABLY NOT WORTH $300.

B INSURANCE COMPANIES INSURE MANY PEOPLE AND MOST OF THEM NEVER MAKE A CLAIM. SO THE RELATIVE RISK TO THE INSURANCE COMPANY IS LOW.

17. **College admissions.** For high school students graduating in 2007, college admissions to the nation's most selective schools were the most competitive in memory. (*The New York Times*, "A Great Year for Ivy League Schools, but Not So Good for Applicants to Them," April 4, 2007). Harvard accepted about 9% of its applicants, Stanford 10%, and Penn 16%. Jorge has applied to all three. Assuming that he's a typical applicant, he figures that his chances of getting into both Harvard and Stanford must be about 0.9%.

a) How has he arrived at this conclusion?
b) What additional assumption is he making?
c) Do you agree with his conclusion?

(A) HE MULTIPLIED HARVARD'S 9% AND STANFORD'S 10% TO GET THE .9%

(B) HE ASSUMES ACCEPTANCE TO HARVARD AND STANFORD ARE INDEPENDENT EVENTS

(C) HIS ASSUMPTION IS WRONG. STUDENTS THAT MEET THE CRITEREA ARE MORE LIKELY TO BE ACCEPTED TO ALL THESE COLLEGES. SINCE THE DECISIONS ARE NOT INDEPENDENT, You Can't Multiply.

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32. **Blood.** The American Red Cross says that about 45% of the U.S. population has Type O blood, 40% Type A, 11% Type B, and the rest Type AB.

a) Someone volunteers to give blood. What is the probability that this donor
   1) has Type AB blood?
   2) has Type A or Type B?
   3) is not Type O?
b) Among four potential donors, what is the probability that
   1) all are Type O?
   2) no one is Type AB?
   3) they are not all Type A?
   4) at least one person is Type B?

$P(O) = 45\%$
$P(A) = 40\%$
$P(B) = 11\%$
$P(AB) = 4\%$

$\boxed{A}$ (1) $P(AB) = 1 - .45 - .40 - .11 = .04$

(2) $P(A \cup B) = .4 + .11 = .51$

(3) $P(\text{NOT } O) = 1 - .45 = .55$

$\boxed{B}$

(1) $P(4 \text{ TYPE } O) = (.45)^4 \approx \boxed{.041}$

(2) $P(\text{NO ONE } AB) = (1 - .04)^4 = (.96)^4 \approx .849$

(3) $P(\text{They are not all Type } A) = 1 - (.4)^4 = .9744$

(4) $P(\text{at least 1 Type } B) = 1 - (1 - .11)^4 = 1 - (.89)^4 \approx .373$

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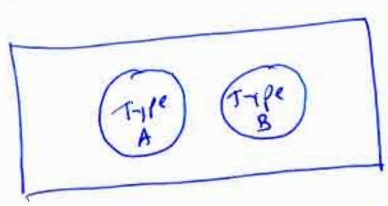
34. **Disjoint or independent?** In Exercise 32 you calculated probabilities involving various blood types. Some of your answers depended on the assumption that the outcomes described were *disjoint;* that is, they could not both happen at the same time. Other answers depended on the assumption that the events were *independent;* that is, the occurrence of one of them doesn't affect the probability of the other. Do you understand the difference between disjoint and independent?

a) If you examine one person, are the events that the person is Type A and the person is Type B disjoint, independent, or neither?
b) If you examine two people, are the events that the first is Type A and the second Type B disjoint, independent, or neither?
c) Can disjoint events ever be independent? Explain.

$\boxed{A}$ DISJOINT. 1 PERSON CAN NOT BE BOTH TYPE A AND TYPE B.

$\boxed{B}$ INDEPENDENT. KNOW THE FIRST PERSON IS TYPE A DOES NOT INFLUENCE THE PROBABILITY THE SECOND PERSON HAVING TYPE B.

$\boxed{C}$



- DISJOINT EVENTS CAN NEVER BE INDEPENDENT
- ONCE YOU KNOW THE FIRST EVENT, THEN OTHER ONE CAN NOT OCCUR. RESULTS OF THE FIRST EVENT INFLUENCES THE OTHER.