Section 6.2 and 6.3
Probability Rules

Learning Objectives

After this section, you should be able to…

- ✓ DEFINE and APPLY basic rules of probability
- ✓ CONSTRUCT Venn diagrams and DETERMINE probabilities
- ✓ DETERMINE probabilities from two-way tables

Randomness, Probability, and Simulation

What Probability IS…

- **Probability is defined as the long run relative frequency of occurrence.**
- **Probability involves random events**, repeated again and again.
- **The "Law of Averages" is FALSE**! Future outcomes are not affected by past behavior. For example if you toss a coin 10 times and get 10 heads, this has no impact and it is not more likely the next toss will be tails.
- The "Law of Large Numbers" is TRUE. That is the "Law of Large Numbers," means short run anomalies get drowned out in the long run based on a large number of trials.

Probability Models

Descriptions of chance behavior contain two parts:

The **sample space *S*** of a chance process is the **set of all possible outcomes**.

A **probability model** is a description of some chance process that consists of two parts: a sample space *S* and a probability for each outcome.

EXAMPLE: Roll a dice

- **Sample Space (list of al outcomes): s={1, 2, 3, 4, 5, 6}**

Definition: An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. **Events are usually designated by capital letters**, like *A, B, C,* and so on.

If *A* is any event, we write its probability as P(*A*). **EXAMPLES**:

- **Suppose we define event B as "rolling a 5."** **P(B)=1/6**
- **Suppose we define event C as "rolling a 5 or 6."** **P(C)=2/6 or 1/3**
- **Suppose we define event D as "Not rolling a 5."** **P(B)=1-1/6 = 5/6**

Basic Rules of Probability

All probability models must obey the following rules:

- **The probability of any event is a number between 0 and 1.**
- **All possible outcomes together must have probabilities whose sum is 1.**
- **If all outcomes in the sample space are equally likely, the probability that event *A* occurs can be found using the formula**

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **The probability that an event does NOT occur is 1 minus the probability that the event does occur.**
- **If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.**

Definition: Two events are **mutually exclusive (disjoint)** if they have **no outcomes in common and so can never occur together**. *The coin is either heads or tails. It can NOT be both.*

NOW… Rules USING Probability Notation:

- For any event A, $0 \le P(A) \le 1$.
- If S is the sample space in a probability model,

  $P(S) = 1$.
- In the case of equally likely outcomes,

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- **Complement rule:** $P(A^C) = 1 - P(A)$
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive,

  $P(A \text{ or } B) = P(A) + P(B)$.

Example: Distance Learning

Probability Rules

Distance-learning courses are rapidly gaining popularity among college students. Randomly select an undergraduate student who is taking distance-learning courses for credit and record the student's age. Here is the probability model:

Age group (yr):	18 to 23	24 to 29	30 to 39	40 or over
Probability:	0.57	0.17	0.14	0.12

(a) Show that this is a legitimate probability model.

Each probability is between 0 and 1 and sum to 1

0.57 + 0.17 + 0.14 + 0.12 = 1

(b) Find the probability that the chosen student is not in the traditional college age group (18 to 23 years).

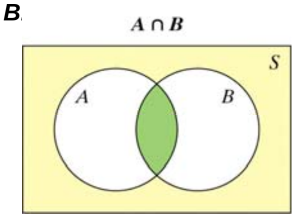
***P*(not 18 to 23 years) = 1 – *P*(18 to 23 years)**

= 1 – 0.57 = 0.43

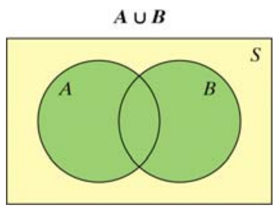
Venn diagrams are very helpful in describing probability rules.

Probability Rules

The intersection of events *A* and *B* ($A \cap B$) is the set of all outcomes in both events *A and B*.



The union of events *A* and *B* ($A \cup B$) is the set of all outcomes in either event *A or B*.

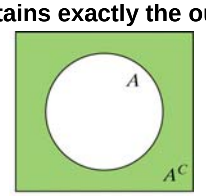


Hint: To keep the symbols straight, remember ∪ for union and ∩ for intersection.

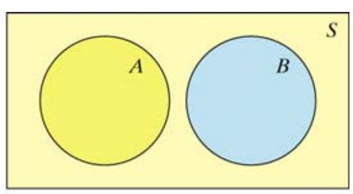
Venn Diagrams (continued)

Probability Rules

The complement A^C contains exactly the outcomes that are not in *A*.



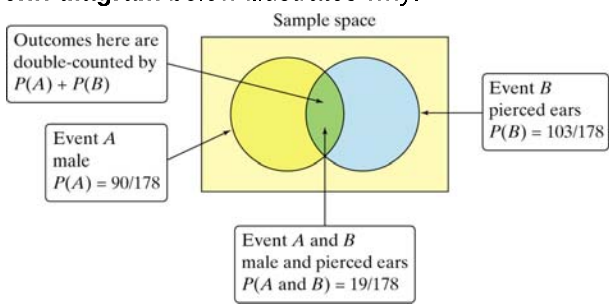
The events *A* and *B* are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.



The General Addition Rule

In the previous illustrates the fact that we can't use the addition rule for mutually exclusive events unless the events have no outcomes in common.

The **Venn diagram** below illustrates why.



General Addition Rule for Two Events

If A and B are any two events resulting from some chance process, then

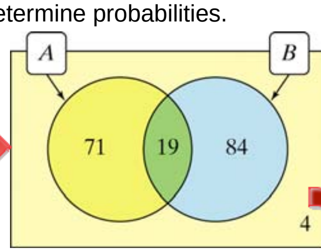
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

+ Probability Rules

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**



Define events *A*: is male and *B*: has pierced ears.

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle *A*, outside circle *B*	Male and no pierced ears	$A \cap B^C$	71
Inside circle *B*, outside circle *A*	Female and pierced ears	$A^C \cap B$	84
Outside both circles	Female and no pierced ears	$A^C \cap B^C$	4

Two-Way Tables and Probability

When finding probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

Suppose we choose a student at random. Find the probability that the student

Gender	Pierced Ears? Yes	No	Total
Male	19	71	90
Female	84	4	88
Total	103	75	178

(a) has pierced ears.

(b) is a male with pierced ears.

(c) is a male or has pierced ears.

Define events

***A*: is male and**

***B*: has pierced ears.**

Two-Way Tables and Probability

Event *A*: is male

Event *B*: has pierced ears

(a) Each student is equally likely to be chosen. 103 students have pierced ears. So,

***P*(pierced ears) = *P*(*B*) = 103/178.**

(b) We want to find *P*(male *and* pierced ears), that is, *P*(*A* and *B*).

Look at the intersection of the "Male" row and "Yes" column. There are 19 males with pierced ears. So,

***P*(*A* and *B*) = 19/178.**

(c) We want to find *P*(male *or* pierced ears), that is, *P*(*A* or *B*).

*There are 90 males in the class and 103 individuals with pierced ears. However, 19 males have pierced ears – **don't count them twice**!*

P(*A* or *B*) = (19 + 71 + 84)/178. So,

***P*(*A* or B) = 174/178**

Multiplication Rule:

- For two independent events **A** and **B**, the probability that *both* **A** and **B** occur is the product of the probabilities of the two events.
- $P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$, provided that **A** and **B** are independent.
- **We will cover the formal definition of independence in the next class.**

Probability Rules

Summary

- ✓ A **probability model** describes chance behavior by listing the possible outcomes in the **sample space *S*** and giving the probability that each outcome occurs.
- ✓ An **event** is a subset of the possible outcomes in a chance process.
- ✓ For any event A, $0 \le P(A) \le 1$
- ✓ $P(S) = 1$, where S = the sample space
- ✓ If all outcomes in S are equally likely, $P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$
- ✓ $P(A^C) = 1 - P(A)$, where A^C is the **complement** of event A; that is, the event that A does not happen.

Probability Rules

Summary (continued)

- ✓ The **intersection ($A \cap B$)** of events *A* and *B* consists of outcomes in both *A* and *B*.
- ✓ The **union ($A \cup B$)** of events *A* and *B* consists of all outcomes in event *A*, event *B*, or both.
- ✓ A **two-way table** or a **Venn diagram** can be used to display the sample space for a chance process.
- ✓ Events *A* and *B* are **mutually exclusive (disjoint)** if they have no outcomes in common. If *A* and *B* are disjoint, $P(A \text{ or } B) = P(A) + P(B) = P(A \cup B)$.
- ✓ The **general addition rule** can be used to find $P(A \text{ or } B)$:

  $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- ✓ In the special case of *independent* events, $P(A \cap B)=P(A) \bullet P(B)$