

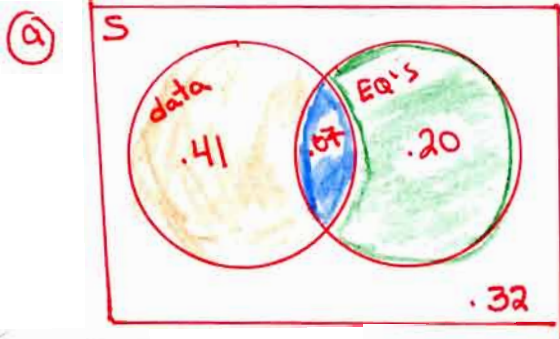


**JUST CHECKING**

1. Back in Chapter 1 we suggested that you sample some pages of this book at random to see whether they held a graph or other data display. We actually did just that. We drew a representative sample and found the following:

- 48% of pages had some kind of data display,
- 27% of pages had an equation, and
- 7% of pages had both a data display and an equation.

- a) Display these results in a Venn diagram.
- b) What is the probability that a randomly selected sample page had neither a data display nor an equation?
- c) What is the probability that a randomly selected sample page had a data display but no equation?



(b)  $P(\text{Neither}) = .32$

(c)  $P(\text{Data but no Equations}) = .41$



**JUST CHECKING**

2. The American Association for Public Opinion Research (AAPOR) is an association of about 1600 individuals who share an interest in public opinion and survey research. They report that typically as few as 10% of random phone calls result in a completed interview. Reasons are varied, but some of the most common include no answer, refusal to cooperate, and failure to complete the call.

Which of the following events are independent, which are disjoint, and which are neither independent nor disjoint?

- a) A = Your telephone number is randomly selected. B = You're not at home at dinnertime when they call.
- b) A = As a selected subject, you complete the interview. B = As a selected subject, you refuse to cooperate.
- c) A = You are not at home when they call at 11 a.m. B = You are employed full-time.

(A) INDEPENDENT - The outcomes of phone # selected and NOT HOME DO NOT Relate to each other

(B) DISJOINT (MUTUALLY EXCLUSIVE) - you either complete the survey or you do NOT

(C) NEITHER - ① you could be home at 11am and be self employed  
 ② EVENTS COULD BE DEPENDENT - You are employed so you are not at home at 11am

## JUST CHECKING

3. Remember our sample of pages in this book from the earlier Just Checking . . . ?

48% of pages had a data display.

27% of pages had an equation, and

7% of pages had both a data display and an equation.

- Make a contingency table for the variables *display* and *equation*.
- What is the probability that a randomly selected sample page with an equation also had a data display?
- Are having an equation and having a data display disjoint events?
- Are having an equation and having a data display independent events?

		EQUATIONS		TOTAL
		YES	NO	
data DISPLAY	YES	7%	41%	48%
	NO	20%	32%	52%
	TOTAL	27%	73%	100%

- [B] P(AN EQUATION PAGE has data)  
 $P = \frac{7}{27}$  - data on EQ pages  
 $P \approx .259$  (This is conditional prob)
- [C] Not disjoint 7% have both

Read - Fill in from given info  
 Remaining % can be calculated

- [D] NOT INDEPENDENT  
 • AT THIS POINT, THEY WOULD BE INDEPENDENT IF WE

SAW THE DISTRIBUTION FOR 1 VAR. IS THE SAME FOR ALL CATEGORIES FOR THE OTHER.

We will learn a more formal definition soon.

## JUST CHECKING

4. Opinion polling organizations contact their respondents by telephone. Random telephone numbers are generated, and interviewers try to contact those households. In the 1990s this method could reach about 69% of U.S. households. According to the Pew Research Center for the People and the Press, by 2003 the contact rate had risen to 76%. We can reasonably assume each household's response to be independent of the others. What's the probability that . . .

- Contact Rate = .76      Fail to contact = .24 (1-.76)
- the interviewer successfully contacts the next household on her list?
  - the interviewer successfully contacts both of the next two households on her list?
  - the interviewer's first successful contact is the third household on the list?
  - the interviewer makes at least one successful contact among the next five households on the list?

LLN ONLY WORKS IN THE LONG RUN, NOT IN THE SHORT RUN. THERE IS NO CHANGE IN THE PROBABILITY

(a)  $P(\text{NEXT CONTACT}) = .76$

(b)  $P(\text{TWO CONTACTS}) = (.76)(.76) = (.76)^2 = .5776$

(c)  $P(\text{1ST CONTACT ON THE 3RD TRY}) = (.24)(.24)(.76) = .043776$

(d)  $P(\text{at least 1 contact in 5}) = 1 - (.24)^5 \approx .9992$

$$P(US) = .40$$

$$P(JAPAN) = .30$$

$$P(GERMAN) = .10$$

$$P(OTHER) = .20$$

Events are Mutually Exclusive  
 Since Probability = 1

**48** Class Examples

Suppose that 40% of cars in your area are manufactured in the United States, 30% in Japan, 10% in Germany, and 20% in other countries. If cars are selected at random, find the probability that:

- A car is not U.S.-made.

$$P(\text{NOT US}) = 1 - P(US) = .60$$

The probability that a car selected at random is NOT U.S. is 60%

- It is made in Japan or Germany. *ADD*

$$P(Japan) + P(German) = .3 + .1 = .40$$

The probability the car was made in Japan or Germany is 40%

- You see two in a row from Japan.

$$P(Japan) \cdot P(Japan) = (.3)(.3) = .09$$

The probability that 2 randomly selected cars were both made in Japan is 9%

- None of three cars came from Germany.

→ Take Complement  $P(\text{German}^c) = .90$

$$P(\text{No German Cars in 3 cars}) = (.9)^3 = .729$$

The probability that none of the 3 randomly selected cars in Germany is 72.9%

- At least one of three cars is U.S.-made.

→ means to work backwards

$$P(\text{at least 1 US in 3}) = 1 - P(\text{NO US in 3}) = 1 - (.6)^3 = .784$$

The probability that at least 1 car is made in US is 78.4%

- The first Japanese car is the fourth one you choose.

$$P(\text{NOT Japan}) = 1 - .3 = .7$$

$$P(\text{1st J is the 4th car}) = P(\text{NOT J}) \cdot P(\text{NOT J}) \cdot P(\text{NOT J}) \cdot P(J) = (.7)^3(.3) = .1029$$

The probability the 1st Japanese car is the 4th car chosen is 10.29%

Class Examples

5) If students are familiar with card games, a deck of cards makes a good frame of reference for many of the issues in this chapter.

- One card is drawn. What is the probability it is an ace or red? (General Addition Rule)

$$P(\text{ace}) + P(\text{Red}) - P(\text{Red Ace}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$$

- PROB OF A DIAMOND OR RED? NOT MUTUALLY EXCLUSIVE  
 $P(\text{diamond}) = \frac{13}{52}$      $P(\text{Red}) = \frac{26}{52}$     ALL diamonds are red     $\frac{1}{2}$

- PROB OF A HEART OR CLUB? MUTUALLY EXCLUSIVE

$$\frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$$

4

4

$$P(\text{Correct Answer}) = .25$$

$$P(\text{Wrong Answer}) = .75$$

6) Five multiple choice questions, each with four possible answers, appear on your history exam. What is the probability that if you just guess, you

a. get none of the questions correct?  $(.75)^5 = .2373$

b. get all of the questions correct?  $(.25)^5 = .00098$

c. get at least one of the questions wrong?  $1 - (.25)^5 = .999$

d. get your first incorrect answer on the fourth question?  $(.25)^3 (.75) = .0117$

7) The Masterfoods company manufactures bags of Peanut Butter M&M's. They report that they make 10% each brown and red candies, and 20% each yellow, blue, and orange candies. The rest of the candies are green.

$$P(\text{Brown}) = .10$$

$$P(\text{Red}) = .10$$

$$P(\text{Yellow}) = .20$$

$$P(\text{Orange}) = .20$$

$$P(\text{Blue}) = .20$$

a. If you pick a Peanut Butter M&M at random, what is the probability that

i. it is green?  $P(\text{GREEN}) = 1 - .1 - .1 - .2 - .2 - .2 = .20$

ii. it is a primary color (red, yellow, or blue)?  $P(R, Y, B) = .1 + .2 + .2 = .50$

iii. it is not orange?  $P(\text{NOT Orange}) = 1 - .2 = .80$

b. If you pick four M&M's in a row, what is the probability that

i. they are all blue?  $P(\text{all blue}) = (.2)^4 = .0016$

ii. none are green?  $P(\text{NONE GREEN}) = (.8)^4 = .4096$

iii. at least one is red?  $P(\text{at least 1 red}) = 1 - P(\text{NOT Red}) = 1 - (.9)^4 = .3439$

iv. the fourth one is the first one that is brown?  $(.9)^3 (.1) = .0729$

c. After picking 10 M&M's in a row, you still have not picked a red one. A friend says that you should have a better chance of getting a red candy on your next pick since you have yet to see one. Comment on your friend's statement.

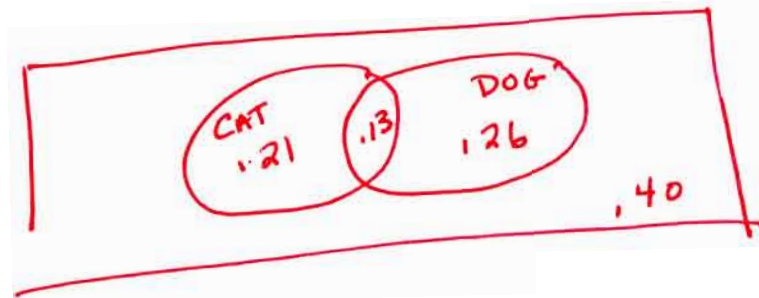
THERE IS NO SUCH RULE AS LAW OF AVERAGES.  
SHORT RUN EVENTS DO NOT PREDICT THE FUTURE.

- 8) According to the American Pet Products Manufacturers Association (APPMA) 2003-2004 National Pet Owners Survey, 39% of U.S. households own at least one dog and 34% of U.S. households own at least one cat. Assume that 60% of U.S. households own a cat or a dog.
- a. What is the probability that a randomly selected U.S. household owns neither a cat nor a dog?

$$P(\text{NO CAT OR DOG}) = .40 \quad \text{see venn diagram}$$

- b. What is the probability that a randomly selected U.S. household owns both a cat and a dog?

$$P(\text{BOTH CAT + DOG}) = .13$$



- 9) A survey of an introductory statistics class in Autumn 2003 asked students whether or not they ate breakfast the morning of the survey. Results are as follows:

		Breakfast		
		Yes	No	Total
Sex	Male	66	66	132
	Female	125	74	199
	Total	191	140	331

- a. What is the probability that a randomly selected student is female?

$$P(\text{female}) = \frac{199}{331} \approx .601$$

- b. What is the probability that a randomly selected student ate breakfast?

$$P(\text{Breakfast}) = \frac{191}{331} \approx .577$$

- c. What is the probability that a randomly selected student is a female who ate breakfast?

$$P(\text{female} \cap \text{breakfast}) = \frac{125}{331} = .378$$

- 10) A survey of local car dealers revealed that 64% of all cars sold last month had CD players, 28% had alarm systems, and 22% had both CD players and alarm systems.
- a. What is the probability one of these cars selected at random had neither a CD player nor an alarm system?

$$P(\text{Neither}) = .30$$

See Venn diagram

- b. What is the probability that a car had a CD player unprotected by an alarm system?

$$P(\text{CD and Not Alarm}) = .42$$

- c. What is the probability a car with an alarm system had a CD player?

THIS IS AN EXAMPLE OF CONDITIONAL PROBABILITY BUT YOU DO NOT NEED TO KNOW THE FORMULA. YOU CAN USE LOGIC AND THE VENN DIAGRAM:

$$P(\text{CD given they have an alarm system also}) = P(\text{both})/P(\text{alarm}) = .22/.28 = .786$$

So, the probability a car with an alarm system has a CD is about 78.6%.

