Section 6.0 and 6.1
Introduction to Randomness, Probability, and Simulation

Learning Objectives

After this section, you should be able to…

- ✓ DESCRIBE the idea of probability
- ✓ DESCRIBE myths about randomness
- ✓ DESCRIBE simulations

Source: TPS 4th edition (CH5)

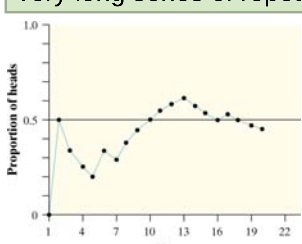
The Idea of Probability

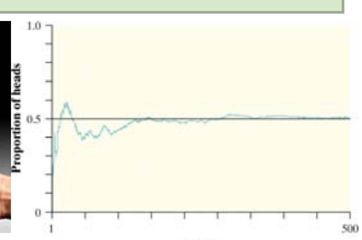
Chance behavior is unpredictable in the short run, but has a regular and predictable pattern in the long run.

The **law of large numbers** says that if we observe more and more repetitions of any chance process, the proportion of times that a specific outcome occurs approaches a single value.

Definition:

The **probability** of any outcome of a chance process is a number between 0 (never occurs) and 1(always occurs) that describes the proportion of times the outcome would occur in a very long series of repetitions.





Myths about Randomness

The idea of probability seems straightforward. However, there are several myths of chance behavior we must address.

The myth of short-run regularity:

The idea of probability is that randomness is predictable in the long run. Our intuition tries to tell us random phenomena should also be predictable in the short run. However, probability does not allow us to make short-run predictions.

The myth of the "law of averages":

Probability tells us random behavior evens out in the long run. Future outcomes are not affected by past behavior. That is, past outcomes do not influence the likelihood of individual outcomes occurring in the future.

Simulation (will be covered in Section 6.7)

The imitation of chance behavior, based on a model that accurately reflects the situation, is called a **simulation**.

Performing a Simulation

State: What is the question of interest about some chance process?

Plan: Describe how to use a chance device to imitate one repetition of the process. Explain clearly how to identify the outcomes of the chance process and what variable to measure.

Do: Perform many repetitions of the simulation.

Conclude: Use the results of your simulation to answer the question of interest.

We can use physical devices, random numbers, and technology to perform simulations.

Summary

- ✓ A chance process has outcomes that we cannot predict but have a regular distribution in many distributions.
- ✓ The **law of large numbers** says the proportion of times that a particular outcome occurs in many repetitions will approach a single number.
- ✓ The long-term relative frequency of a chance outcome is its **probability** between 0 (never occurs) and 1 (always occurs).
- ✓ Short-run regularity and the law of averages are myths of probability.
- ✓ A **simulation** is an imitation of chance behavior.

Section 6.1
Probability: What Are the Chances?

Learning Objectives

After this section, you should be able to…

- ✓ DESCRIBE chance behavior with a probability model and Basic probability vocabulary (sample space, events, outcomes)
- ✓ DESCRIBE basic rules of probability (AND, OR, NOT)
- ✓ CONSTRUCT Venn diagrams

Probability Models

We used simulation to imitate chance behavior. Fortunately, we don't have to always rely on simulations to determine the probability of a particular outcome.

Descriptions of chance behavior contain two parts:

Definition:

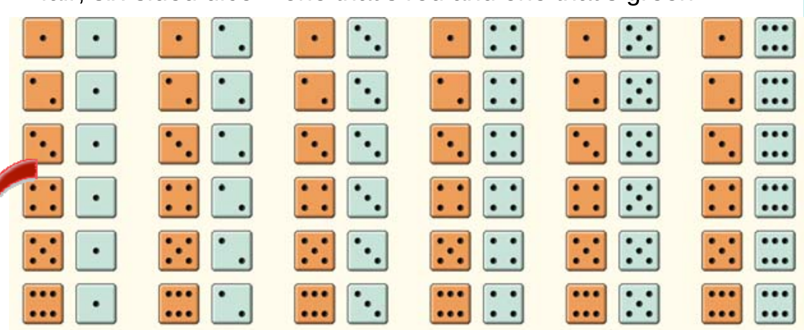
The **sample space *S*** of a chance process is the set of all possible outcomes.

A **probability model** is a description of some chance process that consists of two parts: a sample space *S* and a probability for each outcome.

Probability Rules

Example: Roll the Dice

Give a probability model for the chance process of rolling two fair, six-sided dice – one that's red and one that's green.



Sample Space 36 Outcomes

Since the dice are fair, each outcome is equally likely. Each outcome has probability 1/36.

Probability Rules

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Probability Models

Probability models allow us to find the probability of any collection of outcomes.

Definition:

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A, B, C,* and so on.

If *A* is any event, we write its probability as P(*A*).

In the dice-rolling example, suppose we define event *A* as "sum is 5."

There are 4 outcomes that result in a sum of 5.
Since each outcome has probability 1/36, P(*A*) = 4/36.

Probability Rules

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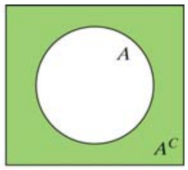
TREE Diagrams

- Your Notes:

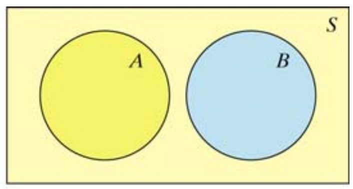
Venn Diagrams and Probability

Because Venn diagrams have uses in other branches of mathematics, some standard vocabulary and notation have been developed.

NOT: **The complement A^C contains exactly the outcomes that are not in *A*.**



The events *A* and *B* are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.

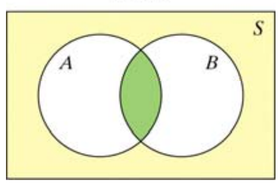


Probability Rules

Venn Diagrams and Probability

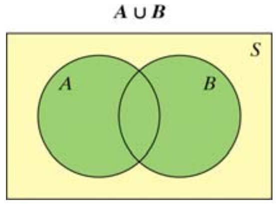
AND: **The intersection of events *A* and *B* ($A \cap B$) is the set of all outcomes in both events *A and B*.**

$A \cap B$



OR: **The union of events *A* and *B* ($A \cup B$) is the set of all outcomes in either event *A or B*.**

$A \cup B$



Hint: To keep the symbols straight, remember $\cup$ for union and $\cap$ for intersection.

Probability Rules