AP Statistics - 5.3 (part 1)	Name: KEY
Conditional Probability and Independence Activity	Date:

I. Review 5.2: *"Who Owns a Home?"*

A:

	High School Graduate	Not a High School Graduate	Total
B: Homeowner	221	119	340
Not a Homeowner	89	71	160
Total	310	190	$\boxed{500}$

The events of interest in this scenario were A: is a high school graduate and B: owns a home

Find the Probabilities for example

1. P(A)= 310/500 = $\boxed{.62}$
2. P(B)= 340/500 = $\boxed{.68}$
3. P(A ∩ B)= P(A and B) = 221/500 = .442 INTERSECT ∩
4. P(A U B)= P(A or B) = P(A) + P(B) − P(A∩B) = .62 + .68 − .442 = $\boxed{.858}$ UNION U

II. What is Conditional Probability? *"Who Owns a Home?"*

1) If we know that a person owns a home, what is the probability that the person is a high school graduate? 340 own homes among this group 221 HS Grads

P(is a hs grad given own home) = $\frac{221}{340}$ = .65 or 65%

NOTATION: P(A | B) "probability of A given B"

2) If we know that a person is a high school graduate, what is the probability that the person owns a home? 310 HS grads among this group 221 own a home

P(owns a home given a H.S grad) = $\frac{221}{310}$ = .71 or about 71%

NOTATION: P(B | A)

*** READ THE QUESTIONS VERY CAREFULLY!!!

TEXT EXAMPLE (page 313): *"Who has pierced ears?"* A=male B=~~HS grad~~ PIERCED EARS

1) P(is male GIVEN has pierced ears)= P(A | B) = 19/103 about 18.4%
KNOWN STUDENT HAS PIERCED EARS (THIS IS THE GIVEN); want probability male.

2) P(has pierced ears GIVEN male)= P(B|A) = 19/90 about 21.1%
KNOW MALE (THE GIVEN); want probability has pierced ears

3) P(male)= P(A) = 90/178 = .506 P(pierced ears)= P(B) = 103/178 = .578

DEFINITION: Conditional Probability (p313)

THE PROBABILITY THAT ONE EVENT HAPPENS GIVEN THAT ANOTHER EVENT IS ALREADY KNOWN TO HAVE HAPPENED.

* SUPPOSE WE KNOW THAT EVENT "A" HAS HAPPENED (THE GIVEN)
* THEN CALCULATE THE CONDITIONAL PROBABILITY FOR B:

P(B | A) $\boxed{\text{Probability OF B Given A}}$

III. Using Hypothetical Tables for Probability

Example: Suppose that at a large high school, we know that 75% of the vehicles in the parking lot is an American-made vehicle and that 70% of the drivers are students. Also, the probability that a randomly selected driver is a student or drives an American-made vehicle is 0.95.

Create: a "Hypothetical 100 Table" or "Hypothetical 1000 Table" to eliminate need for decimals.

P(A or S) = .95

1) Fill in Given
2) Fill in empty cells

Car / Driver

	Student	Teachers	Total
American	50	25	75 ← P(A)
Non-American	20	100 − 95 = 5	25
Total	70 ← P(S)	30	100

GIVEN [FILL IN TABLE]
A = American
O = OTHER
S = STUDENT
T = TEACHER
P(A) = .75 (75%)
P(S) = .70 (70%)
P(A or S) = (Formal)
P(A ∪ S) = .95 (95%) (INFORMAL)

Questions: You must write the correct probability notations for full credit and show work.

Suppose a driver is selected at random:

1) What is the probability that the driver is a student?

$P(S) = .70$ (Given)

2) What is the probability that the driver drives a Non- American car?

$P(O) =$ ⓐ Table $= .25$
ⓑ Formula: $100 - P(A) = 100 - 75 = .25$

3) What is the probability that the driver drives an American or Non- American car?

$P(A \text{ OR } O) = P(A \cup O) = 1.00$

4) What is the probability that the driver is teacher or drives a Non- American car?

$P(T \text{ OR } O) = P(T \cup O) =$ ⓐ add cells $= 25 + 5 + 20 = .50$
ⓑ Formula $P(T) + P(O) - P(T \cap O) = 30 + 25 - 5 = .50$

5) What is the probability that the driver is teacher and drives a Non- American car?

$P(T \text{ and } O) = P(T \cap O) = .5 = .05 \leftarrow 5/100$

6) If the driver is a student, what is the probability that they drive an American car?

Conditional Probability $= \dfrac{P(\text{and})}{P(\text{given})} = \dfrac{P(S \text{ and } A)}{P(S)} = \dfrac{50}{70} = 5/7 = .714$

7) If the driver drives a Non- American car, what is the probability that the driver is a student?

AND → $P(O \text{ and } S)$ / Given → $P(O)$

$\dfrac{P(O \text{ and } S)}{P(O)} = \dfrac{20}{25} = 4/5 = .80$

8) Are the events "driver is a student" and "car driven is American" independent?

$\boxed{\text{INDEPENDANCE FORMULA: } P(A \text{ and } B) = P(A)\,P(B)}$

$P(S) = .70$
$P(A) = .75$
$P(A \text{ and } S) = .50$

$.50 = (.7)(.75)$
$.50 \neq .525$ ← There fore NOT independent

~~III~~ IV. What is Conditional Probability and Independence?

TEXT EXAMPLE (page 315): *Toss a fair coin* Suppose you toss a fair coin twice

DEFINE EVENTS: A: 1^{ST} TOSS HEADS
B: 2^{ND} TOSS HEADS $\Longrightarrow$ $P(A) = 1/2$ AND $P(B) = 1/2$

What's $P(B|A)$?

* THE COIN HAS NO MEMORY SO $P(B|A) = 1/2$
* THEREFORE $P(B|A) = P(B)$. THE 2 EVENTS ARE INDEPENDENT

*** KNOWING THAT THE 1ST TOSS WAS A HEAD DOES NOT AFFECT THE PROBABILITY THAT THE SECOND TOSS IS HEADS. HENCE THEY ARE INDEPENDENT

[WRITE DEFINITION BELOW – 1ST BULLET]

TEXT EXAMPLE (page 315): *Who has pierced ears?*

See the Prior Page: A = Males
B = pierced ears
$P(A) = .506$
$P(A|B) = .184$
$P(B) = .578$
$P(B|A) = .211$

ARE THE FOLLOWING TRUE?
$P(A|B) = P(A)$ and
$P(B|A) = P(B)$

NOTICE: $P(A|B) \neq P(A)$ AND $P(B|A) \neq P(B)$

* THE CONDITIONAL PROBABILITIES ARE VERY DIFFERENT FROM THE UNCONDITIONAL PROBABILITIES. (KNOW ONE EVENT GIVES US INFO ABOUT THE OTHER).
* THEREFORE THESE EVENTS ARE NOT INDEPENDENT.

DEFINITION: Independent Events (p315)

* 2 EVENTS A and B are independent if the occurrence of 1 event has NO effect on the chance that the other event will happen (the coin example)

* USING PROBABILITIES TO DETERMINE INDEPENDENCE: $P(A|B) = P(A)$ and $P(B|A) = P(B)$ (pierced ear example)

* SAID ANOTHER WAY: 2 EVENTS ARE INDEPENDENT IF KNOWING THE OUTCOME OF 1 EVENT DOES NOT GIVE YOU ANY ADDITIONAL INFO ABOUT THE PROBABILITY THAT THE OTHER EVENT WILL OCCUR.

~~IV.~~ V. **More Independence Problems**

~~CYA~~ CYU (page 317): *Determine if the events are independent and justify your answer.*

1) *Deck of Cards* – Event A: 1st Heart; Event B: 2nd Heart; **Sampling** WITH REPLACEMENT

INDEPENDENT SINCE WE ARE PUTTING THE 1ST CARD BACK + RESHUFFLING

Knowing what the 1ST card was will not tell us anything about what the 2ND card will be.

2) *Deck of Cards* – Event A: 1st Heart; Event B: 2nd Heart; **Sampling** WITHOUT REPLACEMENT

NOT INDEPENDENT ONCE WE KNOW THE SUIT OF THE 1ST CARD, THEN WE WILL HAVE MORE INFORMATION ABOUT THE SUIT OF THE 2ND CARD. THE PROBABILITY OF GETTING A HEART ON THE 2ND CARD WILL CHANGE DEPENDING ON WHAT THE 1ST CARD WAS.

3) Gender and Handed

INDEPENDENT

$P(\text{RIGHT}) = \frac{24}{28} = .\frac{6}{7} \Longleftrightarrow P(\text{RIGHT} \mid \text{FEMALE}) = \frac{18}{21} = \frac{6}{.7}$

$P(\text{FEMALE}) = \frac{21}{28} = .75 \Longleftrightarrow P(\text{FEMALE} \mid \text{RIGHT}) = \frac{18}{24} = .75$

SINCE THE Conditions are met, we conclude Female and Right handed are independent. Once we know the chosen person is female, this does not tell us anything about right handed or not.

Example: *Allergies*

Is there a relationship between gender and having allergies? To find out, we used the random sampler at the United States Census at School website to randomly select 40 US high school students who completed a survey. The two-way table shows the gender of each student and whether the student has allergies. **Problem:** ***Are the events "female" and "allergies" independent? Justify your answer.***

	Female	Male	Total
Allergies	10	8	18
No Allergies	13	9	22
Total	23	17	40

DEFINE EVENTS: A = ALLERGY

F = FEMALE

$P(F) = \frac{23}{40} = .576 \neq P(F \mid A) = \frac{10}{18} = .556$

$P(A) = \frac{18}{40} = .45 \neq P(A \mid F) = \frac{10}{23} = .43$

ANSWER TO PROBLEM

* These probabilities are close but NOT EQUAL. So THE EVENTS Female and Allergies are NOT INDEPENDENT.

* KNOW THAT A STUDENT WAS FEMALE SLIGHTLY LOWERED THE PROBABILITY THAT SHE HAS ALLERGIES.

UNDERSTANDING THE PROBLEM:

* DOES KNOWING A STUDENT'S GENDER AFFECT THE PROBABILITY THAT THE STUDENT HAS ALLERGIES?

① IF A STUDENT IS FEMALE, THEN THE PROBABILITY SHE HAS ALLERGIES: $P(A \mid F) = .435$

② COMPARED TO THE UNCONDITIONAL PROBABILITY $P(A) = .45$

* SINCE THESE ARE DIFFERENT, THE EVENTS ARE NOT INDEPENDENT.

INDEPENDENT EVENTS

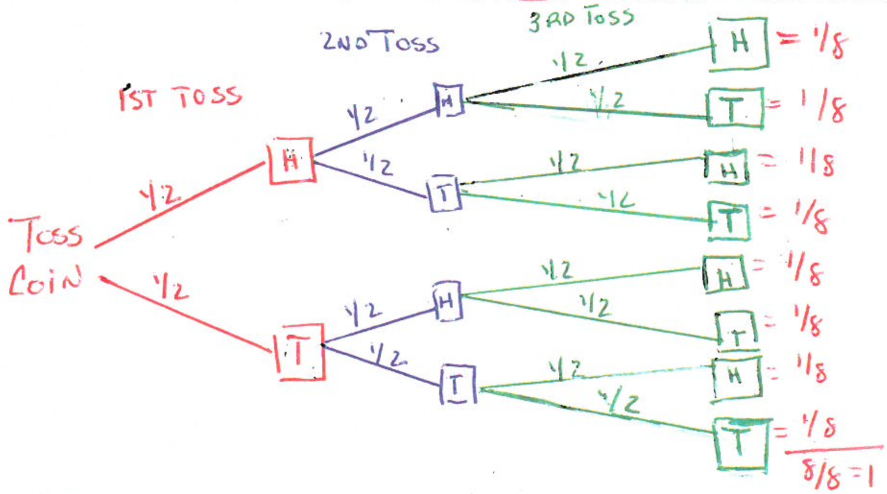
~~IV.~~ VI **Tree Diagrams and the ~~General~~ Multiplication Rule** $\cap$

1) Draw a tree diagram to display the sample space of tossing a coin 3 times and find the probabilities.

Define Events:

Event 1: 1ST TOSS Event 2: 2ND TOSS Event 3: 3RD TOSS

Are these Events Independent? YES COINS HAVE NO MEMORY



P(3 heads)= $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 1/8$

The sample space has 8 possible outcomes

DEFINITION: Multiplication Rule for Independent Events (p321)

IF A and B are independent events, then the probability that A and B occur is $P(A \cap B) = P(A \text{ and } B) = P(A) \cdot P(B)$

VI (CONT.) TREE DIAGRAMS AND GENERAL MULTIPLICATION RULE:

2) **Example:** *Picking Two Sneezers*

In the ***Allergies*** example, we used a two-way table that classified 40 students according to their gender and whether they had allergies. **Problem:** Suppose we chose 2 students at random.

(a) Draw a tree diagram that shows the sample space for this chance process.

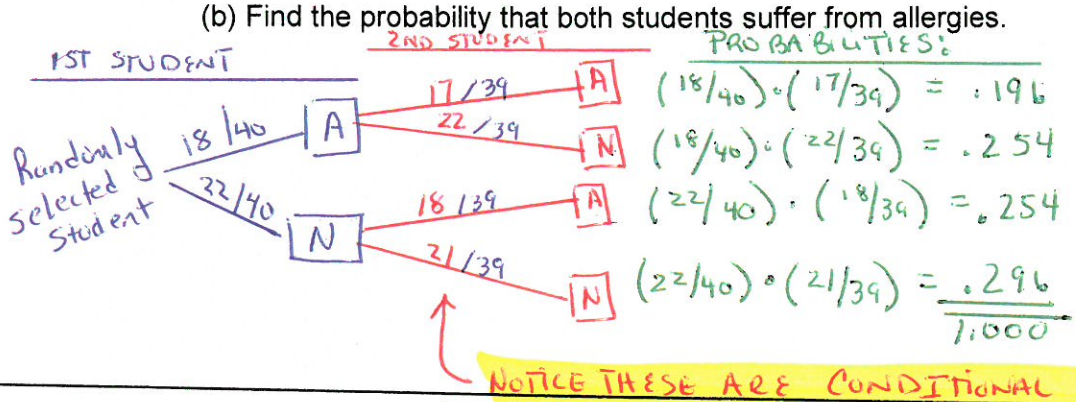
(b) Find the probability that both students suffer from allergies.

$\boxed{A}$ = ALLERGY

$\boxed{N}$ = NOT ALLERGY

$P(A) = 18/40$

$P(N) = 22/40$



* P(2 ALLERGY STUDENTS) = P(1ST STUDENT W/ ALLERGY AND 2ND STUDENT W/ ALLERGY) =
P(1ST STUDENT W/ ALLERGY) · P(2ND STUDENT W/ ALLERGY | 1ST STUDENT W/ ALLERGY) =
$= (18/40)(17/39) = .196$

THERE IS ABOUT A 20% CHANCE OF SELECTING 2 STUDENTS WITH ALLERGIES.

DEFINITION: General Multiplication Rule (p319)

The probability that events A and B both occur can be found using this rule: $P(A \cap B) = P(A \text{ and } B) = P(A) \cdot P(B|A)$