5.3 Influences

- **Correlation r is not resistant.**
  - One unusual point in the scatterplot greatly affects the value of r.
- **Extrapolation is not very reliable.**
- **LSRL also not resistant.**
  - A point extreme in the x direction with no other points near it pulls the line toward itself.
    - This point is influential.

Outliers and Influential Observations in Regression

An **outlier** is an observation that lies outside the overall pattern of the other observations. Points that are outliers in the *y* direction of a scatterplot have large regression residuals, but other outliers need not have large residuals.

An observation is **influential** for a statistical calculation if removing it would markedly change the result of the calculation. Points that are outliers in the *x* direction of a scatterplot are often influential for the least-squares regression line.

Beware correlations based on averages

Correlations based on averages are usually too high when applied to individuals.

- **Example**:
  - If we **plot** the **average height of young children** against their age in months,
    - we will see a very strong positive association with correlation near 1.
  - **But individual children of the same age vary a great deal in height.**
    - A **plot** of height against age for **individual children will show much more scatter and lower correlation** than the plot of average height against age.

Review This Problem:
Work Through
Example – Corrosion and Strength

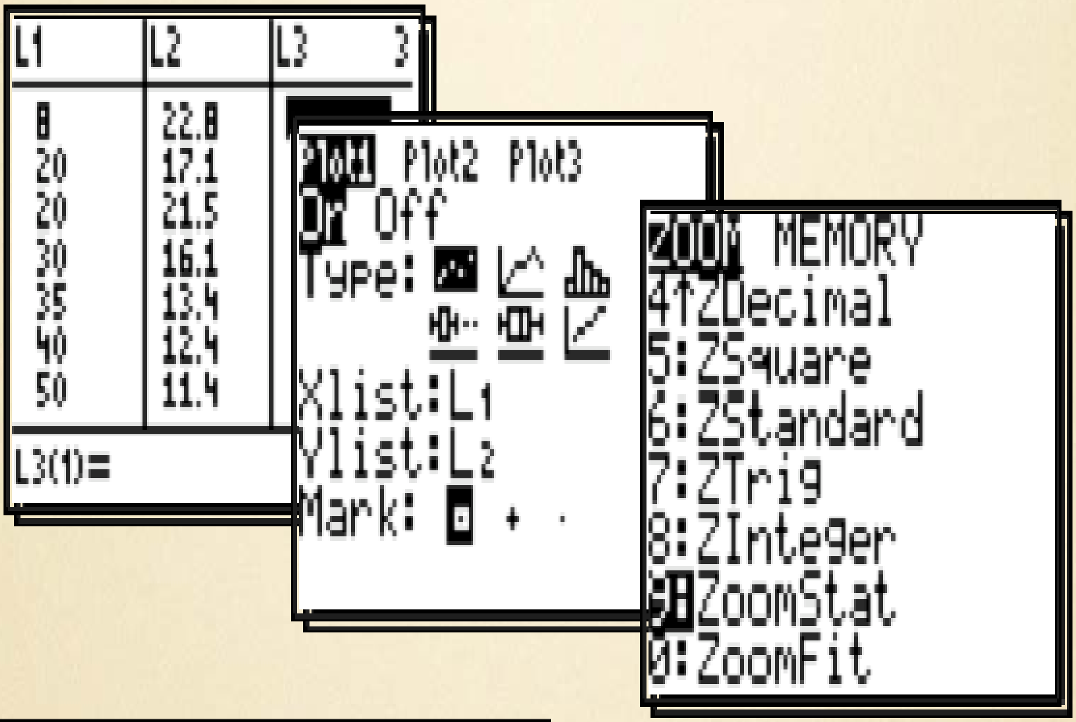
- Consider the following data from the article, "The Carbonation of Concrete Structures in the Tropical Environment of Singapore" (Magazine of Concrete Research (1996):293-300 which discusses how the corrosion of steel(caused by carbonation) is the biggest problem affecting concrete strength:

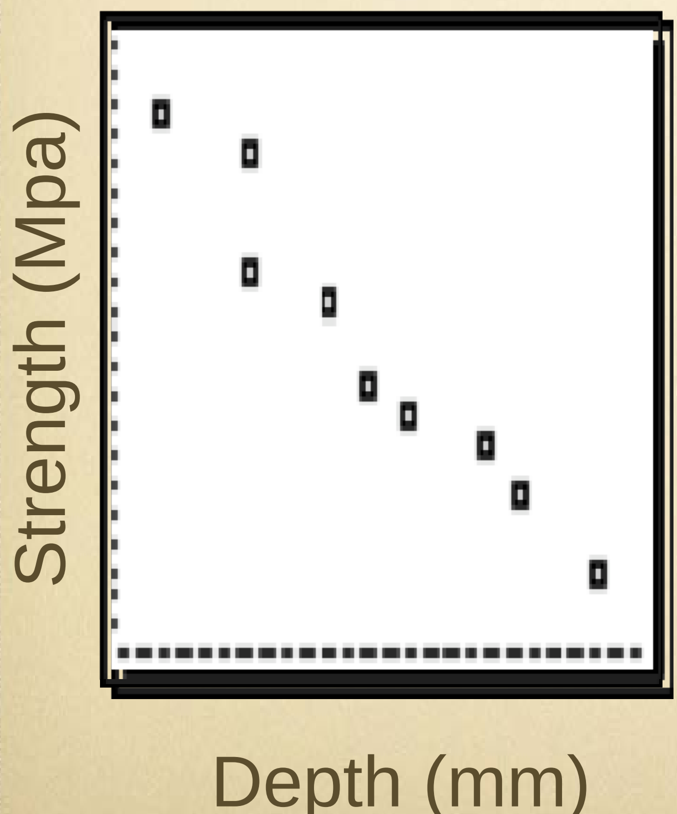
  - x= carbonation depth in concrete (mm)
  - y= strength of concrete (Mpa)

x	8	20	20	30	35	40	50	55	65
y	22.8	17.1	21.5	16.1	13.4	12.4	11.4	9.7	6.8

- **Define the Explanatory and Response Variables.**
- **Plot the data and describe the association.**
- **Answers must be in context for given problem.**

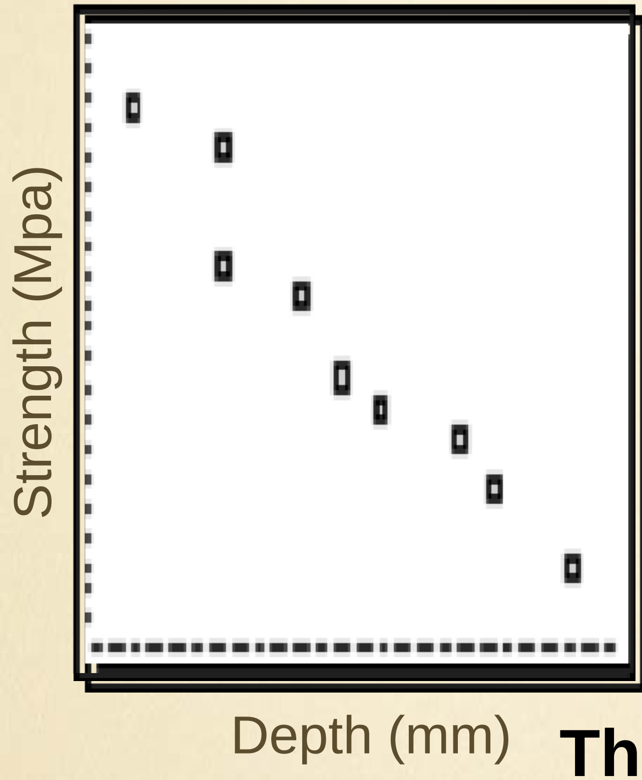
1) Create a Graph to visualize and describe the Association

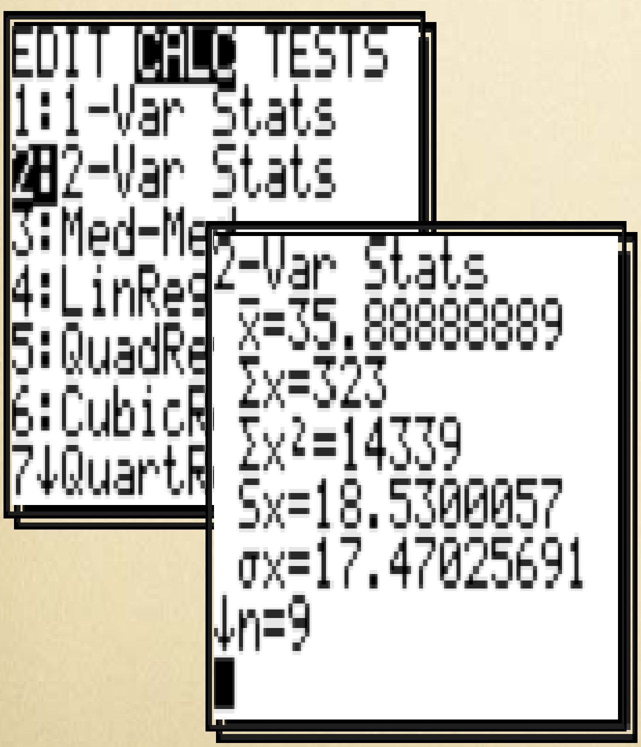




There is a strong, negative, linear relationship between depth of corrosion and concrete strength. As the depth increases, the strength decreases at a constant rate.

2) Find the Means and Standard Deviations for Depth and Strength and describe in context.





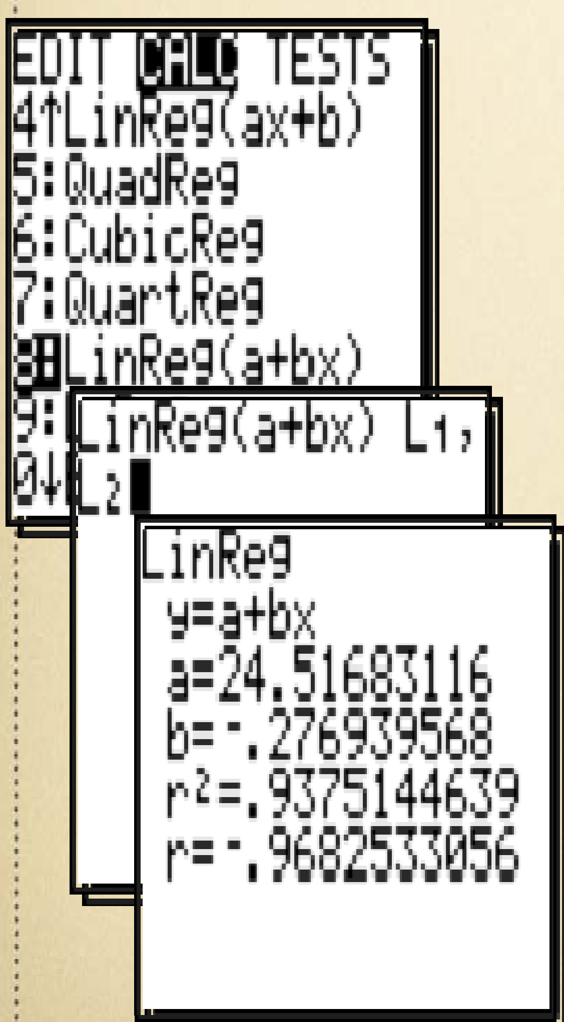
The mean depth of concrete is 35.89mm with a standard deviation of 18.53mm.

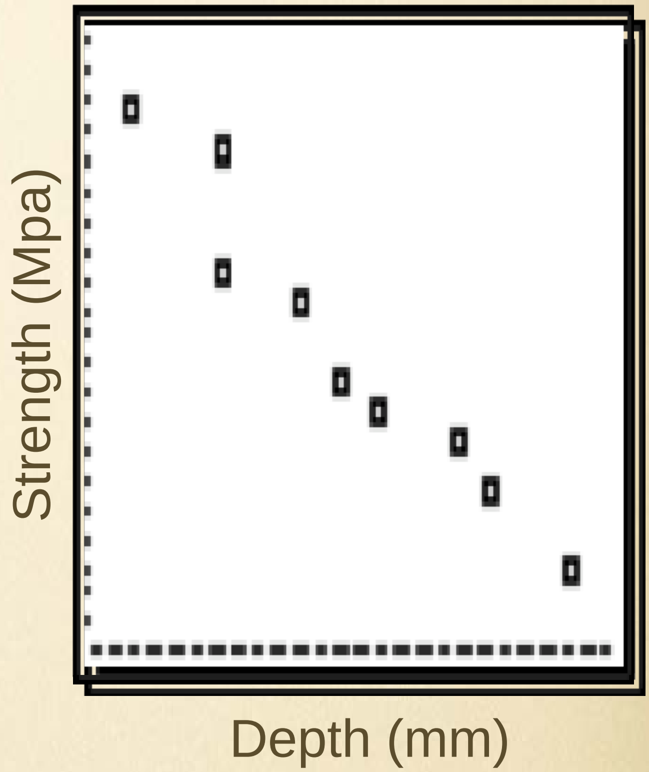
The mean strength of concrete is 14.58 Mpa with a standard deviation of 5.29 Mpa.

3) Find the Correlation Coefficient and describe in context.

4) Find the equation of the Least Squares Regression Line

The correlation coefficient (r=-.968) quantifies there is a Strong, Negative, LINEAR association between depth of concrete corrosion and strength of concrete.



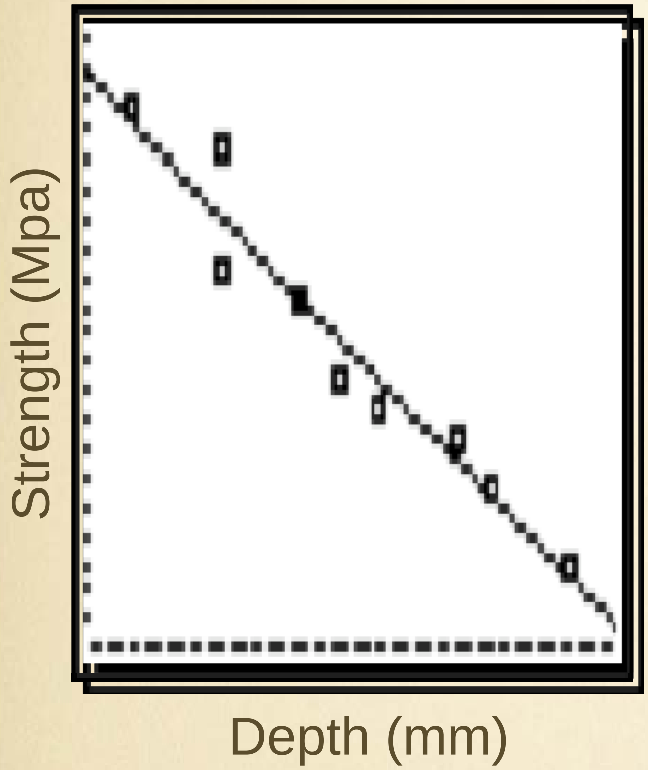


LSRL Equation:

$$\widehat{strength}=24.52-0.28(depth)$$

y **x**

5) Describe LSRL in Context

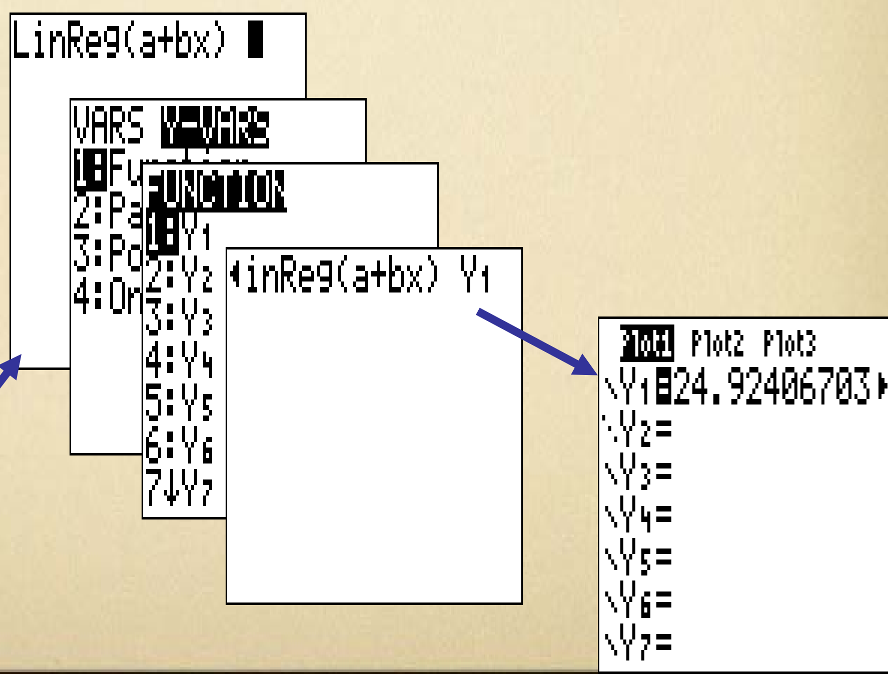


LSRL Equation:

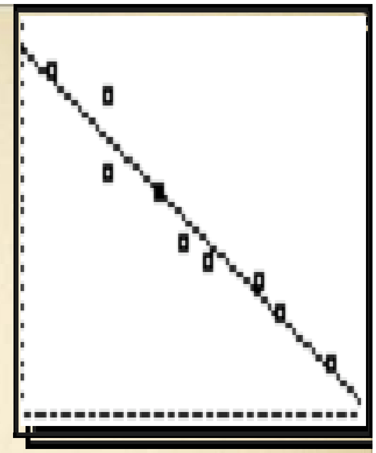
$\widehat{\text{strength}} = 24.52 - 0.28(\text{depth})$

The slope is b=-0.28.
For every increase of 1mm in depth of concrete corrosion,
we predict a 0.28 Mpa decrease in strength of the concrete.

- **Use these steps to store the LSRL in Y_1 and overlay it on the scatterplot.**



6) Use the prediction model (LSRL) to determine the following:



- What is the predicted strength of concrete with a corrosion depth of 25mm?

  - strength=24.52+(-0.28)depth
  - strength=24.52+(-0.28)(25)
  - **The predicted strength is17.59 Mpa.**

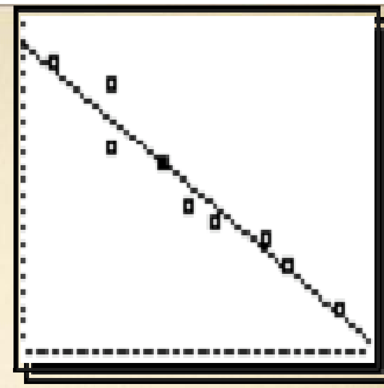
- What is the predicted strength of concrete with a corrosion depth of 40mm?

  - strength=24.52+(-0.28)(40)
  - **The predicted strength is13.44 Mpa.**

  - **How does this prediction compare with the observed strength at a corrosion depth of 40mm?**

RESIDUALS

7) Interpret Residuals *(from previous slide)*



- We calculated the **predicted strength** when the corrosion depth was 40mm to be **13.44 Mpa**

- From the given data table, we can find the **observed strength** when corrosion=40mm is to be **12.4mm**

- **The prediction did not match the observation.**
  - That is, there is **“error” or “residual” between our prediction and the actual observation.**
  - RESIDUAL = Observed y - Predicted y
  - The residual when corrosion=40mm is:
    - residual = 12.4 - 13.44
    - **residual = -1.04**

Assessing the Model

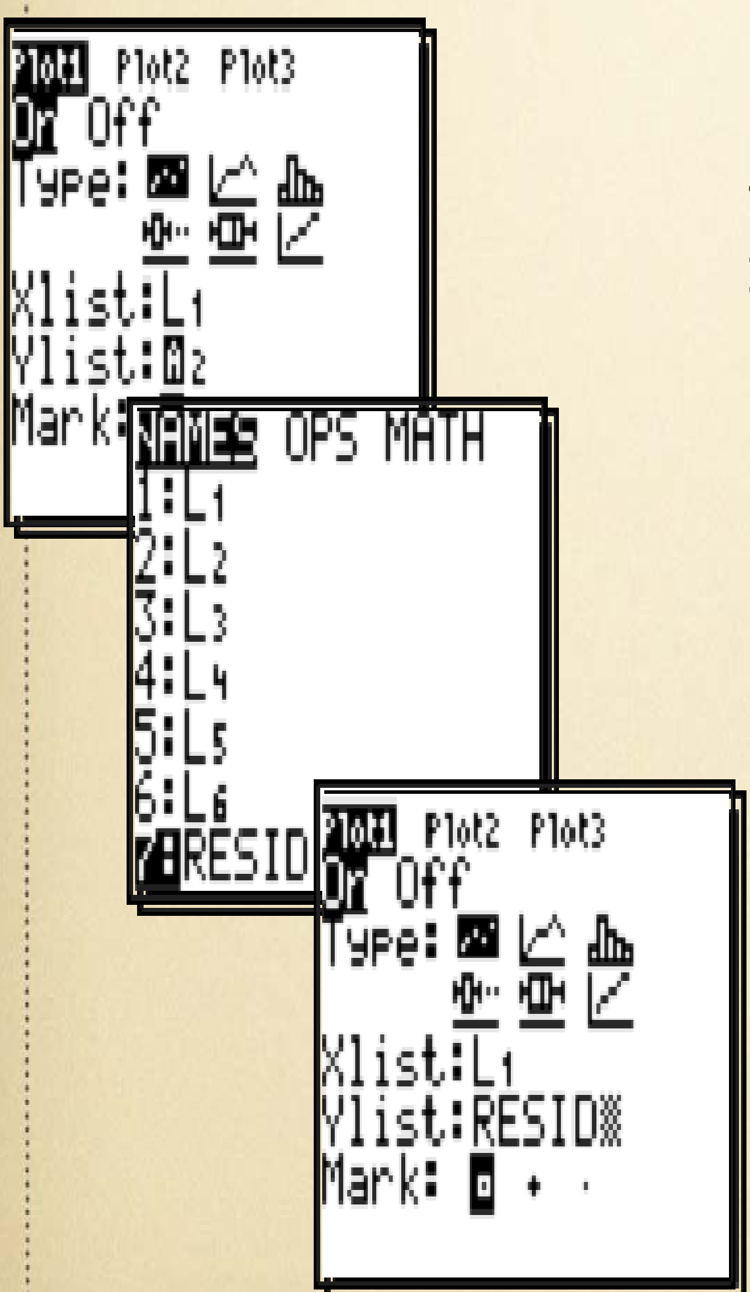
8) Is the model appropriate?
9) What is the strength of the model?

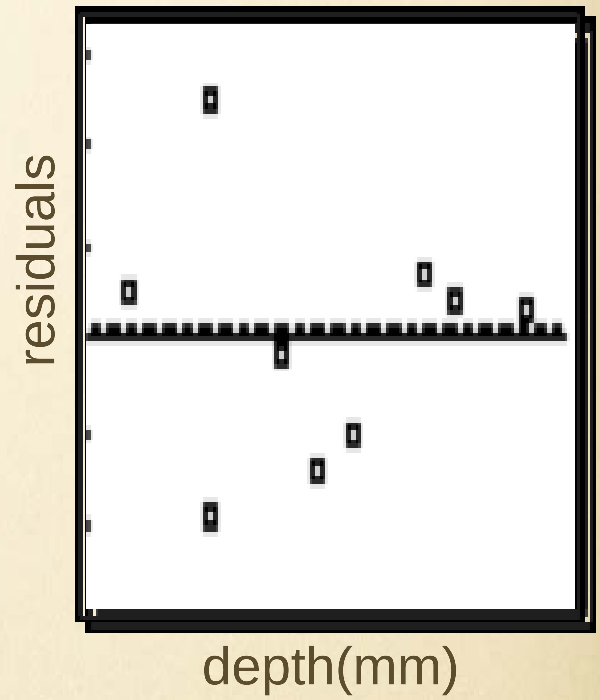
- Is the LSRL the most appropriate prediction model for strength?
  - ✓ r suggests it will provide strong predictions...
  - ✓ can we do better?

- To determine this, we need to study the residuals generated by the LSRL.
  - Make a residual plot.
  - Look for a pattern.
  - If no pattern exists, the LSRL may be our best bet for predictions.
  - If a pattern exists, a better prediction model may exist...

8) Review the Residual Plot to see if our model is appropriate

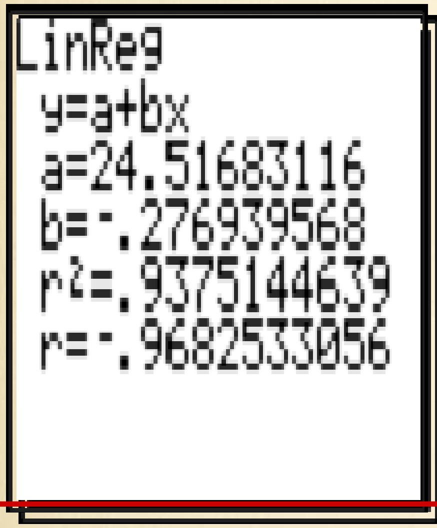
- ☑ Construct a Residual Plot for the (depth,strength) LSRL.





- ☑ There appears to be **no pattern** to the residual plot...
- ☑ therefore, the LSRL may be our best prediction model.

9) Review the Coefficient of Determination (r^2) to assess he strength of our model

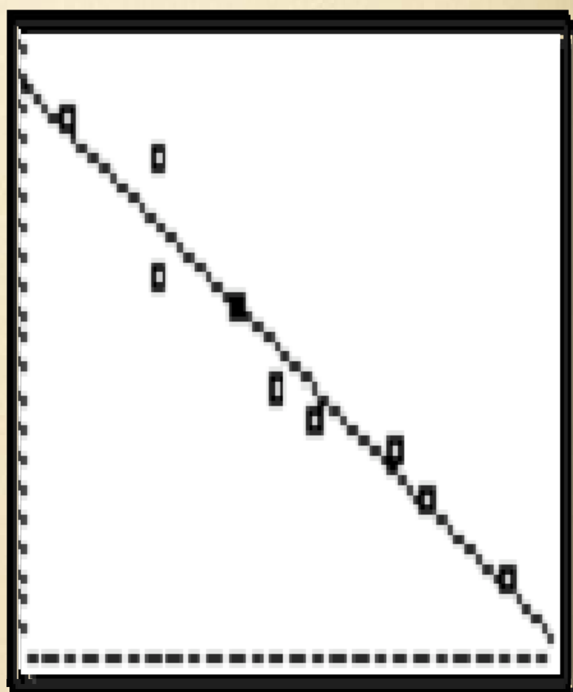


- We know what "r" tells us about the linear association between depth and strength.
- **What about r^2?**

r^2 =.9375

(in context) **93.75% of the variability in predicted strength can be explained by the LSRL on depth.**

(6.25% of the variability can NOT be explained by our model. This is a very strong model.)



Strength (Mpa)

Depth (mm)

Summary

- When exploring a bivariate relationship:
  - Make and interpret a scatterplot:
    - Strength, Direction, Form
  - Describe x and y:
    - Mean and Standard Deviation in Context
  - Find the Least Squares Regression Line.
    - Write in context.
  - Construct and Interpret a Residual Plot.
  - Interpret r and r^2 in context.
  - Use the LSRL to make predictions...