Solve Two-Step Equations

Goal • Solve two-step equations.

IDENTIFYING OPERATIONS FOR 2 STEP EQUATIONS
Identify the operations involved in the equation

3x + 7 = 19. First always simplify both sides

1. Step 1: Undo Add/Subtraction
2. Step 2: Undo Mult/Division
3. You must always check by substituting in the original equation.

Example 1 Solve a two-step equation

Solve 3x + 7 = 19.

Solution

Write original equation.
Subtract 7 from each side.
Simplify.
Divide each side by 3
Simplify.
The solution is Circle it.

CHECK

Write original equation.
Substitute 4 for x.
Multiply 3 by 4.
Simplify. Solution checks.

When solving a two-step equation, apply the inverse operations in the reverse order of the order of operations.
**Checkpoint** Solve the two-step equation. Check your solution.

1. \[ \frac{r}{4} - \frac{12}{y^2} = -5 \]
   \[ \frac{r}{4} = \frac{-5+y^2}{12} \]
   \[ r = \frac{28}{4} \]
   \[ r = 28 \]

2. \[ 7k - 14 = 42 \]
   \[ \frac{7k}{7} = \frac{42+14}{7} \]
   \[ k = 8 \]

3. \[ \frac{28}{4} - 12 = -5 \]
   \[ -7 - 12 = -5 \]
   \[ -5 = -5 \]

**Example 2** Solve a two-step equation by combining like terms.

Solve \( 4a + 3a = 63 \).

**Solution**

\[ 4a + 3a = 63 \] \[ \text{Write original equation.} \]

Combine like terms. \[ 4a + 3a = 7a \]

Divide each side by \( 7 \).

\[ a = 9 \] \[ \text{Simplify.} \]

The solution is \( a = 9 \).

**CHECK**

Write original equation.

\[ 4a + 3a = 63 \]

\[ 4(9) + 3(9) = 63 \]

\[ 36 + 27 = 63 \]

\[ 63 = 63 \]

**Checkpoint** Solve the equation. Check your solution.

3. \[ 5z + 4z = 36 \]
   \[ \frac{9z}{9} = \frac{36}{9} \]
   \[ z = 4 \]
   \[ 5(4) + 4(4) = 36 \]
   \[ 36 = 36 \]

4. \[ 5b - 2b = 9 \]
   \[ \frac{3b}{3} = \frac{9}{3} \]
   \[ b = 3 \]

\[ 5(3) - 2(3) = 9 \]

\[ 15 - 6 = 9 \]

\[ 9 = 9 \]
3.3 Solve Multi-Step Equations

**Goal**  
Solve multi-step equations.

---

**Example 1**  
Solve an equation by combining like terms

Solve $3t + 5t - 5 = 11$.

**Solution**

\[
8t - 5 = 11
\]

\[
+5 +5
\]

\[
8t = 16
\]

\[
\frac{8t}{8} = \frac{16}{8}
\]

\[
t = 2
\]

**Simplify**

Write original equation.
Combine like terms.
Add 5 to each side.
Simplify.
Divide each side by 8.
Simplify.

The solution is 

\[
\text{Check: } 3(2) + 5(2) - 5 = 11
\]

\[
\frac{11}{11} = 11
\]

**STEP 1**

ALWAYS
Simplify
BOTH SIDES

---

**Example 2**  
Solve an equation using the distributive property

Solve $5a + 3(a + 2) = 22$.

**Show All Steps**

\[
5a + 3a + 6 = 22
\]

\[
8a + 6 = 22
\]

\[
\frac{8a}{2} = \frac{16}{2}
\]

\[
a = 2
\]

**C:**

\[
5(2) + 3(2 + 2) = 22
\]

\[
10 + 3(4) = 22
\]

\[
10 + 12 = 22
\]

\[
22 = 22
\]
Example 3  Multiply by a reciprocal to solve an equation

Solve $\frac{3}{4}(a - 5) = 9$.

Solution

Write original equation.

Multiply each side by $\frac{4}{3}$.

Two ways to solve this problem

1. Distribute $\frac{3}{4}$

2. Multiply both sides by the reciprocal

Checkpoint  Solve the equation. Check your solution.

C: $\frac{1}{2}(4x - 2) = 7$

$2x - 1 = 7$

$2x = 8$

$x = 4$

C: $\frac{1}{2}(4 \cdot 4 - 2) = 7$

$\frac{1}{2}(16 - 2) = 7$

$\frac{1}{2}(14) = 7$

$7 = 7$
3.2 HW

6) \( G = 8 \)
14) \( n = 21 \)

8) \( Q = 1 \)
16) \( P = 6 \)

10) \( W = 20 \)
18) \( X = 6 \)

12) \( Z = -12 \)

3.3 HW

4) \( Z = -5 \)

10) \( m = -7 \)

12) \( Z = 2 \)

14) \( m = 3 \)

16) \( C = 5 \)

19) \( D = 12 \)

38) **KJ:** $32.50/ticket
   plus $3.30/ticket
   plus $5.90 per order
   Total spending = $220.70

**DEFINE VARIABLE:**
\( X = \) tickets bought

**DEFINE EQ:**
\[
(32.50 + 3.30)X + 5.90 = 220.70
\]
\[
X = 6
\]

**BOUGHT 6 TICKETS**
3.4 Solve Equations with Variables on Both Sides

**Goal**  • Solve equations with variables on both sides.

**VOCABULARY**

Identity means that an equation is true for any value of \( x \).

Solution \( x = \) all real numbers or \( x = \mathbb{R} \).

---

**Example 1** Solve an equation with variables on both sides

Solve \( 15 + 4a = 9a - 5 \).

**Solution**

1. Collect variables on one side of the equation and constant terms on the other to solve equations with variables on both sides.

2. Always simplify both sides.

3. Get the variable on one side.

Write original equation.

Subtract \( 4a \) from each side.

Add 5 to each side.

Divide each side by 5.

The solution is \( a = 4 \).

CHECK

Substitute \( a = 4 \) for \( a \).

Multiply.

Solution checks.
Example 2  Solve an equation with grouping symbols

Solve $4t - 12 = 6(t + 3)$.

Solution

$4t - 12 = 6(t + 3)$

Write original equation.

*Distributive property*

$4t - 12 = 6t + 18$

*Subtract $4t$ from each side.

$-12 = 2t + 18$

*Subtract $18$ from each side.

$-30 = 2t$

*Divide each side by $2$.

$\frac{-30}{2} = \frac{2t}{2}$

$-15 = t$

$\Box$ Checkpoint  Solve the equation. Check your solution.

$C: 3(1)+7=8(1)+2$

$\frac{-7}{2} = \frac{5b}{2}$

$10 = 10 \checkmark$

Example 3  Identify the number of solutions of an equation

Solve the equation, if possible.

a. $4x + 5 = 4(x + 5)$

Original equation

Distributive property

$4x + 5 = 4x + 20$

$5 \neq 20$ (F)

$x = \text{no solution}$

or $x = 4$

b. $6x - 3 = 3(2x - 1)$

Original equation

Distributive property

$6x - 3 = 6x - 3$

$-3 = -3$ (T)

$x = \text{all real numbers}$

Check

$X = 0$  $ightarrow$  $-3 = -3$ \checkmark

$X = 1$  $ightarrow$  $3 \neq 3$ \xmark

$X = 2$  $ightarrow$  $9 = 9$ \checkmark

$X = 3$  $ightarrow$  $15 = 15$ \checkmark

$X = 4$  $ightarrow$  $21 \neq 21$ \xmark

SPECIAL CASES

Check mentally but do not need to write.

When the variable drops out and the numbers do not equal then there is no solution.

When the variable drops out and the numbers are equal then there is an infinite number of solutions.
Your Notes

**Checkpoint** Solve the equation, if possible.

3. \[ \frac{1}{2} (4t - 6) = 2t \]
   \[
   \frac{4t}{2} - \frac{6}{2} = 2t
   \]
   \[
   2t - 3 = 2t
   \]
   \[
   -3 \neq 0
   \]

**T = No Solution**

4. \[ 10m - 4 = -2(2 - 5m) \]
   \[
   10m - 4 = -4 + 10m
   \]
   \[
   -4 = -4(1)
   \]

**X = All Real Numbers**

**Book Answer: Identity**

---

**Steps for Solving Linear Equations**

1. **Step 1** Use the distributive property to remove any grouping symbols.
2. **Step 2** Simplify the expression on each side of the equation.
3. **Step 3** Use the properties of equality to collect the variable terms on one side of the equation and the constant terms on the other side of the equation.
4. **Step 4** Use the properties of equality to solve for the variable.
5. **Step 5** Check your solution in the original equation.
4) \( K = 1 \)  
   \( C: Z = 2 \)  

6) \( m = 3 \)  
   \( C: 20 = 20 \)  

8) \( P = -6 \)  

10) \( H = 3 \)  
   \( C: 8 = 8 \)  

12) \( R = 5 \)  
   \( C: 84 = 84 \)  

14) \( n = 7 \)  
   \( C: 45 = 45 \)  

16) \( w = \text{no solution} \)  

20) \( z = \text{no solution} \)  

22) \( x = -33 \)  
   \( C: -656 = -656 \)  

24) \( y = \text{all real numbers} \) 

26) \( G = \text{all real numbers} \) 

46) \( k = \)  

\[
\begin{align*}
3x + 7 & = 3(9) + 7 = 34 \\
4x - 2 & = 4(9) - 2 = 34 \\
4x - 2 & = 3x + 7 \\
-3x & = -7 \\
x - 4 & = 7 \\
42 + 3 & \\
x & = 9 \\
P & = 4(34) = 136 \\
\text{Perimeter} & = 136 \text{ units}
\end{align*}
\]

41) \( 14 - \frac{1}{5}(J - 10) = \frac{2}{5}(25 + J) \)
   \[
   14 - \frac{1}{5}J + 2 = \frac{2}{5}J + 10
   \]
   \[
   -\frac{1}{5}J + 16 = \frac{2}{5}J + 10
   \]
   \[
   +\frac{1}{5}J \\
   +\frac{1}{5}J
   \]
   \[
   16 = \frac{3}{5}J + 10
   \]
   \[
   -10
   \]
   \[
   \frac{6}{5} = \frac{3}{5}J
   \]
   \[
   \times 5 \times 5
   \]
   \[
   30 = 3J
   \]
   \[
   -30
   \]
   \[
   J = 10
   \]

\[
C: 14 - \frac{1}{3}(10 - 10) = \frac{2}{3}(25 + 10)
\]
   \[
   \frac{2}{3} \times (35) \n   \]
   \[
   14 = 14 \]
3.5 Write Ratios and Proportions

**Goals**
- Find ratios and write and solve proportions.

**VOCABULARY**

<table>
<thead>
<tr>
<th>Ratio</th>
<th>The use of division to compare 2 quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion</td>
<td>2 RATIOs THAT ARE EQUIVALENT. EXAMPLE</td>
</tr>
</tbody>
</table>

**RATIOS**
1. A ratio uses **Division** to compare two quantities.
2. The ratio of two quantities, \( a \) and \( b \), where \( b \) is not equal to 0, can be written in three ways:
   - \( \frac{a}{b} \)
   - \( a:b \)
   - \( a \) to \( b \)
3. Each ratio is read "the ratio of \( a \) to \( b \)."
4. Ratios should be written in **Simplest** form.

**Example 1**

**Write a ratio**

**Cell Phone Use** A person makes 6 long distance calls and 15 local calls in 1 month.

a. Find the ratio of long distance calls to local calls.

b. Find the ratio of long distance calls to all calls.

**Solution**

a. \[
\frac{\text{long distance calls}}{\text{local calls}} = \frac{6}{15} = \frac{2}{5}
\]

b. \[
\frac{\text{long distance calls}}{\text{all calls}} = \frac{6}{21} = \frac{2}{7}
\]

\[6 + 15\]
**Checkpoint** Shawn and Myra are selling tickets to their school's talent show. Shawn sold 36 tickets, and Myra sold 44 tickets. Find the specified ratio.

1. The number of tickets Shawn sold to the number of tickets Myra sold
   \[
   \frac{\text{Shawn}}{\text{Myra}} = \frac{36}{44} = \frac{9}{11}
   \]

2. The number of tickets Myra sold to the number of tickets Shawn and Myra sold
   \[
   \frac{\text{Myra}}{\text{Total}} = \frac{44}{80} = \frac{11}{20}
   \]

**Example 2** Solve a proportion

Solve the proportion \( \frac{x}{15} = \frac{3}{5} \).

**Solution**

\[
\frac{x}{15} = \frac{3}{5}
\]

- Write original proportion.
- Cross multiply and divide.
- \( \frac{5x}{5} = \frac{15 \cdot 3}{5} \)
- \( x = 9 \)

**Checkpoint** Solve the proportion. Check your solution.

3. \( \frac{9}{4} = \frac{c}{28} \)

   \[
   \frac{9 \cdot 28}{4} = c
   \]
   \[c = 63\]

4. \( \frac{a}{32} = \frac{7}{8} \)

   \[
   \frac{8a}{7} = \frac{32}{8}
   \]
   \[a = 28\]

Copyright © McDougal Littell/Houghton Mifflin Company.
Example 3  Solve a multi-step problem

Swimming Pool  A empty swimming pool is being filled with water. After 5 minutes the pool has 400 gallons of water. If the pool has a volume of 11,200 gallons, how long does it take to fill the empty pool?

Solution

Step 1  Write a proportion involving two ratios that compare the amount of water in the pool to the amount of time.

\[
\frac{400}{5} = \frac{11,200}{x}
\]

Ratio is gallons to minutes

Step 2  Solve the proportion.

\[
\frac{400}{400} \times \frac{x}{400} = \frac{11,200}{400}
\]

\[
x = 140
\]

The pool is full after 140 minutes. OR 2 HOURS AND 20 MIN

Checkpoint  Complete the following exercise.

5. An Olympic sized pool has a volume of 810,000 gallons. If it is filled at the same rate as the pool in Example 3, how long will it take to fill the pool?

Rate: \[\frac{400\text{ gallons}}{5\text{ minutes}} = \frac{810,000\text{ gal}}{M}\]

\[\frac{400}{400} = \frac{5(810,000)}{400.}
\]

\[M = 10125\text{ minutes} \text{ or } 168.75\text{ hrs}\]
3.6 Solve Proportions Using Cross Products

Goal • Solve proportions using cross products.

VOCABULARY
Cross product

Cross Product:
\[
\frac{a}{b} = \frac{c}{d}
\]

NOTE IF THE CROSS PRODUCTS ARE EQUAL THEN IT IS A TRUE PROPORTION.

CROSS PRODUCTS PROPERTY
Words The cross products of a proportion are \textit{EQUAL}.
Example \begin{align*}
\frac{5}{6} \cdot 10 &= \frac{6}{12} \cdot 10 \\
50 &= 60 \quad (\text{They are } =) 
\end{align*}

Algebra If \( \frac{a}{b} = \frac{c}{d} \) where \( b \neq 0 \) and \( d \neq 0 \), then \( ad = bc \).
**Example 1**  Solve a proportion using cross products

Solve the proportion \( \frac{5}{y} = \frac{15}{75} \).

**Solution**

\[
\frac{5}{y} = \frac{15}{75} \\
5y = 15 \\
\frac{5y}{15} = \frac{15}{15} \\
y = 3 \\
\]

The solution is \( y = 3 \).

Write original proportion.
Cross products property
Simplify.

\[ \frac{5}{25} = \frac{15}{75} \]

\[ \frac{1}{5} = \frac{1}{3} \]

**Example 2**  Write and solve a proportion

Plant Food  To feed your plants, you need to mix 3 tablespoons of plant food with 16 ounces of water. If it takes 80 ounces of water to feed all of your plants, how many tablespoons of plant food are needed?

**Solution**

**Step 1**  Define variable

**Plant Food**

KI:

3T food with 16 oz water

80 oz to feed plants

**Step 2**  Variable

\( x \) = \# oz of plant food

\( \uparrow \) must have units

**Step 3**  Write a proportion involving two ratios that compare the amount of plant food with the amount of water.

\[ \frac{3}{16} = \frac{x}{80} \]

\( \leftarrow \text{amount of plant food} \)

\( \leftarrow \text{amount of water} \)

**Step 4**  Solve the proportion.

\[ \frac{16x}{16} = \frac{3 \times 80}{16} \]

\[ x = 15 \]

Write proportion.
Cross product property
Simplify.

**Step 5**  Write answer in a sentence

You need 15 tablespoons of plant food for 80 ounces of water.
Your Notes

\[ \frac{5}{4} = \frac{25}{45} \]

\[ \frac{5}{q} = \frac{25}{45} \]

\[ q = \frac{4}{9} \]

\[ \frac{5}{q} = \frac{25}{45} \]

\[ \frac{5}{4} = \frac{3}{9} \]

\[ \frac{5}{2} = \frac{3}{6} \]

\[ b = \frac{9}{5} \]

3. In Example 2, suppose it takes 120 ounces to feed all of the plants. How many tablespoons of plant food are needed?

\[ \frac{3}{10} = \frac{x}{120} \]

\[ \frac{120x}{3} = \frac{120}{10} \]

\[ x = \frac{90}{4} = 22.5 \]

Need 22.5 tablespoons of plant food.

Example 3

Use a scale model

Scale Model: An architect creates a scale model of a school. The school is 50 feet high. The ratio of the model to the actual school is 1 foot to 75 feet. Estimate the height of the model.

Solution

Write and solve a proportion to find the height \( h \) of the scale model.

\[ \frac{h}{50} = \frac{1}{75} \]

\[ \text{height of model (feet)} \]

\[ \text{actual height (feet)} \]

Cross products property

Simplify.

The height of the scale model is \( \frac{2}{3} \) foot, or 8 inches.

Checkpoint

Complete the following exercise.

4. In Example 3, suppose the ratio of the model to the actual school is 1 foot to 100 feet. Estimate the height of the model.

\[ \frac{\text{MODEL}}{\text{ACTUAL}} = \frac{H}{50} = \frac{1}{100} \]

\[ \frac{100H}{50} = \frac{50}{100} \]

\[ H = \frac{1}{2} \text{ ft or 6 in} \]
\( \frac{11}{w} = \frac{33}{w+24} \)

\( 11(w+24) = 33w \)

\( 264 = 22w \)

\( w = 12 \)

\( \frac{11}{12} = \frac{33}{12+24} \)

\( \frac{11}{12} = \frac{33}{36} \)

\( \frac{11}{12} = \frac{11}{12} \)

\( \frac{7}{3} = \frac{2x+5}{x} \)

\( 7x = 3(2x+5) \)

\( X = 15 \)

\( \frac{24}{52+4} = \frac{4}{2-1} \)

\( 24(z-1) = 4(5z+4) \)

\( 4z = 40 \)

\( z = 10 \)

\( \frac{x-8}{2} = \frac{11-4c}{11} \)

\( 11(x-8) = -2(11-4c) \)

\( 11c - 88 = -22 + 8c \)

\( 3c = 66 \)

\( C = 22 \)

\( \frac{2}{3} = \frac{4v+4}{2v+14} \)

\( 2(2v+14) = -3(4v+4) \)

\( 4v + 28 = -12v - 12 \)

\( 16v = -40 \)

\( v = -2.5 \)

\( \frac{k-8}{7+k} = \frac{-1}{5} \)

\( 5(k-8) = -1(k+7) \)

\( 5k - 40 = -k - 7 \)

\( 6k = 33 \)

\( k = 5.5 \)

\( \frac{m+1}{4} = \frac{3m+6}{7} \)

\( 7(m+1) = 4(3m+6) \)

\( 7m + 7 = 12m + 24 \)

\( -17 = 5m \)

\( m = -3.4 \)

\( \frac{7.2}{8} = \frac{x}{x} \)

\( X = 18 \)

\( \frac{34}{2} = \frac{30}{x} \)

\( x = 5 \)

\( \frac{12 \text{ Biscuits}}{2 \text{ C flour}} = \frac{30 \text{ Biscuits}}{x \text{ C flour}} \)

\( X = 5 \)

**Need 5 Cups flour**

\( \text{wp proportion} \rightarrow \frac{12}{2} = \frac{30}{x} \)

\( \frac{7.2}{8} = \frac{x}{20} \)

\( x = 18 \)

**Will take 18 mins. To upload 20 photo's**
Problem 35: \[
\frac{1\text{ cm}}{15\text{ km}} = \frac{6\text{ cm}}{D}
\]
Distance = 90 km

Problem 36: \[
\frac{1\text{ cm}}{15\text{ km}} = \frac{3.2\text{ cm}}{D}
\]
Distance is 48 km

Problem 37: \[
\frac{1\text{ cm}}{15\text{ km}} = \frac{0.5\text{ cm}}{D}
\]
Distance is 7.5 km

Problem 38: \[
\frac{1\text{ cm}}{15\text{ km}} = \frac{4.7\text{ cm}}{D}
\]
Distance is 70.5 km

Problem 39: KI: Model 1 m : 25 m

Empire State is 443.2 m
What is the height of the model?

\[
H = \text{model height (m)}
\]
\[
\frac{1}{25} = \frac{H}{443.2}
\]
\[
H = \frac{443.2 \times 1}{25}
\]
\[
H = 17.73\text{ m}
\]
Model is 17.73 m tall
3.7 Solve Percent Problems

Goal  • Solve percent problems.

Your Notes

SOLVING PERCENT PROBLEMS USING PROPORTIONS

You can represent "a is p percent of b" by using the proportion

$$\frac{a}{b} = \frac{p}{100} \quad \text{or} \quad \frac{IS}{OF} = \frac{\%}{100}$$

where a is a part of the base b and \(\frac{p}{100}\) or p\%, is the percent

Example 1  Find a percent using a proportion

What percent of 50 is 33?

Solution

Write a proportion when 50 is the base and 33 is part of the base.

$$\frac{33}{50} = \frac{p}{100}$$  \(\leftarrow\) Write proportion.

$$\frac{33 \cdot 100}{50} = 660$$

\(\leftarrow\) Cross products property

$$p = 66.0$$

33 is 66\% of 50.

Don't forget \% sign
METHOD I: \[ \frac{IS}{OF} = \frac{?}{100} \]

**Checkpoint** Use a proportion to answer the question.

1. What percent of 80 is 28?
   \[ \frac{P}{100} = \frac{28}{80} \Rightarrow \frac{80P}{80} = \frac{28}{80} \times 4 \]
   \[ P = 35\% \]

2. What percent of 90 is 36?
   \[ \frac{P}{100} = \frac{36}{90} \Rightarrow \frac{90P}{90} = \frac{36}{90} \times 1 \]
   \[ P = 40\% \]

**THE PERCENT EQUATION**

You can represent "a is p percent of b" by using the equation:

\[ \boxed{EQ: \ a = \frac{p\%}{b} \cdot b} \]

where \( a \) is a part of the base \( b \) and \( p\% \) is the percent.

---

**Example 2** Find a percent using the percent equation

What percent of 250 is 100?

\[ \frac{P \cdot 250}{250} = \frac{100}{250} \]

Write percent equation.

\[ P = \frac{100}{250} = \frac{2}{5} \rightarrow \text{decimals} \]

Solve it

\[ P = 0.4 \]

Write decimal as a percent.

\[ 100 \text{ is } 40\% \text{ of } 250 \]

CHECK with proportion method

\[ \frac{P}{40} = \frac{100}{250} \]

\[ 40 \cdot 250 = 100 \cdot 100 \]

Cross products

\[ 19000 = 19000 \]
Your Notes

Example 3  Find a part of a base using the percent equation

What number is 75% of 300?

Solution

\[ N = 0.75 \times 300 \]

\[ N = 225 \]

225 is 75% of 300

Checkpoint Use the percent equation to answer the question.

3. What percent of 75 is 60?

\[ \frac{p \times 75}{75} = \frac{60}{75} \]

\[ p = \frac{60}{75} \times \frac{75}{75} \]

\[ p = 80\% \]

4. What number is 40% of 80?

\[ N = 0.4 \times 80 \]

\[ N = 32 \]

Example 4  Find a base using the percent equation

25 is 125% of what number?

Solution

\[ 25 = 1.25 \times N \]

\[ N = \frac{25}{1.25} \]

\[ N = 200 \]

25 is 125% of 200.
Checkpoint Use the percent equation to answer the question.

5. 60 is 25% of what number?

\[ \frac{60}{N} = \frac{25}{100} \]
\[ \frac{60}{N} = \frac{25}{100} \]
\[ \frac{25N}{25} = \frac{60 \cdot 100}{25} \]
\[ N = 240 \]

6. 75 is 150% of what number?

\[ \frac{75}{N} = \frac{150}{100} \]
\[ \frac{75}{N} = \frac{150}{100} \]
\[ 150N = 75 \cdot 100 \]
\[ N = \frac{7500}{150} \]
\[ N = \frac{150}{3} \]
\[ N = 50 \]

<table>
<thead>
<tr>
<th>TYPES OF PERCENT EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent Problem</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>1 Find a percent.</td>
</tr>
<tr>
<td>2 Find part of a base.</td>
</tr>
<tr>
<td>3 Find a base.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
1. IN THE STATEMENT: 54 is 15% of 360

   **Answer:** Percent is 15.
   Part (Is) is 54.
   Base (of) is 360.

2. “28 is 35% of 80” → Proportion: \( \frac{28}{80} = \frac{35}{100} \)

   Goes in Numerator
   Percent 15, 35
   Goes in denominator

3. \( \frac{27}{75} = \frac{P}{100} \)

   \( P \cdot 75 = 27 \)

   \( P = 36 \)

   \( 36\% \)

7. \( \frac{81}{N} = \frac{54}{100} \)

   \( 81 = 0.54 \cdot N \)

   \( N = 150 \)

9. \( P \cdot 80 = 56 \)

   \( P = 0.7 \)

11. \( P \cdot 153 = 9.18 \)

   \( P = 0.06 \)

13. \( N = 1.15 \cdot 60 \)

   \( N = 69 \)

15. \( 7 = 0.28 \cdot N \)

   \( N = 25 \)

17. \( 41.8 = 4.4 \cdot N \)

   \( N = 9.5 \)
4. \[ \text{EQ} \quad P \cdot 120 = 66 \]
   \[ P = 0.55 \]

6. \[ N = 60 \cdot 85 \]
   \[ N = 51 \]

8. \[ 42 = 2.00 \cdot N \]
   \[ N = 21 \]

10. \[ P \cdot 225 = 99 \]
    \[ P = 0.44 \]

12. \[ N = 0.18 \cdot 150 \]
    \[ N = 27 \]

14. \[ N = 0.82 \cdot 215 \]
    \[ N = 176.3 \]

16. \[ 189 = 0.9 \cdot N \]
    \[ N = 210 \]
3.8 Rewrite Equations and Formulas

Goal: Write equations in function form and rewrite formulas.

VOCABULARY

- Function form: IS THE SAME AS \( y = mx + b \)
  - Given an equation with \( x \) and \( y \), you isolate \( y \) (\( y = \_ \_ \) )

- Literal equation: are equations with 2 or more variables (aka letters)

Example 1: Rewrite an equation in function form

Write \( 2x + 2y = 10 \) in function form.

Solution
Solve the equation for \( y \).

\[
\begin{align*}
2x + 2y &= 10 \\
2y &= -2x + 10 \\
y &= -x + 5
\end{align*}
\]

Write original equation. ISOLATE \( y \)

Subtract \( 2x \) from each side.
Divide each side by \( 2 \). (ALL TERMS DIVIDE BY \( 2 \))

The equation \( y = -x + 5 \) is written in function form.

Example 2: Solve a literal equation

Solve \( a + by = c \) for \( a \).

Solution
Write original equation. \( a + by = c \)

Subtract \( by \) from each side.

The solution is \( a = c - by \).
Example 3 Solve and use a formula

The interest $I$ on an investment of $P$ dollars at an interest rate $r$ for $t$ years is given by the formula $I = Prt$.

a. Solve the formula for the time $t$.

b. Use the rewritten formula to find the time it takes to earn $100$ interest on $1000$ at a rate of $5.0\%$.

Solution

a. Write original formula.

\[ \frac{I}{P} = rt \]

Divide each side by $Pr$.

\[ t = \frac{I}{Pr} \]

b. Substitute $100$ for $I$, $1000$ for $P$, and $5\%$ for $r$ in the rewritten formula.

\[ t = \frac{100}{1000 \cdot 0.05} \]

Substitute.

\[ t = \frac{100}{50} \]

Simplify.

\[ t = 2 \]

It will take $2$ years to earn $100$ in interest.

Checkpoint Write the equation in function form.

\[ 1. \quad 2x + y = 5 \]

\[ y = -2x + 5 \]

\[ 2. \quad 3 + 3y = 9 - 6x \]

\[ \frac{3}{3}y = \frac{6 - 6x}{3} \]

\[ y = -2x + 2 \]

Checkpoint Complete the following exercises.

3. Solve $a + by = c$ for $b$.

\[ b = \frac{c - ay}{y} \]

4. In Example 3, solve the equation for $P$. Find the investment $P$ if $I = 400$, $r = 4\%$, and $t = 4$ years.

\[ P = \frac{I}{rt} \]

\[ P = \frac{400}{0.04 \cdot 4} = \frac{400}{0.16} = 2500 \]
LITERAL EQUATION (write sentence)

1. \( I = \frac{PRT}{t} \)
2. \( \frac{P}{R} \)

To Solve \( I = \frac{PRT}{t} \) for \( t \):

1. Divide each side by \( PR \)
2. New formula \( t = \frac{I}{P} \)

3. \[ ax = bx - c \]

4. \[ a(x + b) = c \]
   \[ ax + ab = c \]
   \[ ax + ab = c - ab \]
   \[ \frac{ax}{a} = \frac{c - ab}{a} \]
   \[ x = \frac{c - ab}{a} \]

5. Cross multiply

6. \[ a = \frac{x}{c} \]
   \[ ab = \frac{bx}{c} \]
   \[ \frac{ab}{c} \]
   \[ x = \frac{ab}{c} \]
\[ \frac{x + b = c}{a} \quad \frac{ax + b = c \cdot x - d}{-c \cdot x^2 + 3 - c \cdot x^2} \]
\[ ax - c \cdot x + b = -d \]
\[ \frac{a \cdot x - c \cdot x = -b - d}{-b - b} \]
\[ x(a - c) = -b - d \]
\[ \frac{X = \frac{-b + d}{a - c}}{\text{or}} \quad \frac{X = \frac{b - d}{c - a}}{a - c} \]

11. \( y = -2x + 7 \)

12. \[ 5x + 4y = 10 \]
\[ -5x \]
\[ -5x - 5x \]
\[ 4y = -10x + 10 \]
\[ \frac{4y}{4} = \frac{-10x + 10}{4} \]
\[ y = -5x + \frac{5}{2} \]
\[ y = -2.5x + 2.5 \]

13. \[ y = -3x + 4 \]

20. \( W = \frac{V}{h} \)

21. \[ S = 2 \cdot B + Ph \]
\[ -2B = -2B \]
\[ S - 2B = \frac{P \cdot h}{P} \]
\[ h = \frac{S - 2B}{P} \]

22. \[ L = 24F \]
\[ \frac{24}{2F} \]
\[ F = \frac{L}{24} \]