

Part 1

- b** This proportion is the result of a sample, so it's a statistic.
- c** This is the definition of a sampling distribution.
- b** When the center of a statistic's sampling distribution is at the parameter value, the statistic is unbiased.
- b** The mean of both the population and the sampling distribution are fixed—only the means of individual samples vary.
- a** Since the 10% condition is satisfied, this is the appropriate formula for the standard deviation. The sample is too small for the sampling distribution of \hat{p} to be Normal.
- e** See definition of central limit theorem on page 450 in text.
- a** The standard deviation of the sampling distribution of means depends on sample size, not population size.
- b** $\mu_{\bar{x}} = \mu = 84$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 6$. Since n is small and we don't know the shape of the population distribution, the shape of the sampling distribution is unknown.

$$9. \quad \mathbf{c} \quad P(\bar{x} > 60) = P\left(z > \frac{60 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = P\left(z > \frac{60 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(z > \frac{60 - 62.5}{\frac{6}{\sqrt{5}}}\right)$$

$$10. \quad \mathbf{d} \quad P(\hat{p} < 0.5) = P\left(z < \frac{0.5 - p}{\sqrt{\frac{(p)(1-p)}{n}}}\right) = P\left(z < \frac{0.5 - 0.53}{\sqrt{\frac{(0.53)(0.47)}{500}}}\right)$$

$$11. \quad \mathbf{(a)} \quad P(62.5 < x < 68.75) = P\left(\frac{62.5 - 65}{5} < z < \frac{68.75 - 65}{5}\right) = P(-0.5 < z < 0.75) = 0.4649$$

$$\mathbf{(b)} \quad P(62.5 < \bar{x} < 68.75) = P\left(\frac{62.5 - 65}{\frac{5}{\sqrt{12}}} < z < \frac{68.75 - 65}{\frac{5}{\sqrt{12}}}\right) = P(-1.73 < z < 2.60) = 0.9535$$

- (c)** Yes. The calculations in both (a) and (b) assumed the Normality of the underlying distribution. In (a) the population was given as Normal. In (b), we would not be able to assume the Normality of the sampling distribution because the sample size is less than 30.

12. (a) $\mu_{\hat{p}} = p = 0.2$; $\sigma_{\hat{p}} = \sqrt{\frac{(0.2)(0.8)}{200}} = 0.028$. (b) We can use this formula if we assume

that there are more than $10(200) = 2000$ listeners who call in to the program.

(c) The probability of getting 50 or more males in 200 callers if the true proportion of males is still 0.20 is $P(\hat{p} > 0.25) = P\left(z > \frac{0.25 - 0.2}{0.028}\right) = P(z > 1.79) = 0.0367$. Roughly 1 out of 25

times, we will get this many or more male callers. This is probably unusual enough to suggest that the true proportion of male listeners is higher than 0.20. 13. (a) The dot at 240 represents the interquartile range of one of the 50 samples taken from this population. (b) We know from the five-number summary that the true population value for IQR is $380 - 215 = 165$. It appears that the distribution of sample IQR's is slightly skewed right, but its median is 165, so it's safe to say that the center of the distribution is near 165. Therefore the sample first quartile is an unbiased estimator.

AP STAT CH 8 Test -- Part 1: MC -- 1/2B

Part 1

1. **b** 19% is the proportion of white cars in the population (the parameter) and 22% is the proportion of white cars in the sample (the statistic).
2. **d** Low bias means the statistic's sampling distribution is centered around the parameter. Low variability means the typical value of the statistic is close to the parameter. (Variability in samples is inevitable, hence **c** is incorrect).
3. **e** A sampling distribution is values of a statistic from all possible samples of a given size from the population. If we think of the statistic as a random variable, this is its probability distribution.
4. **c** Since $n > 30$, the sampling distribution is approximately Normal, with

$$\mu_{\bar{x}} = 112 \text{ and } \sigma_{\bar{x}} = \frac{20}{\sqrt{100}} \approx 1.414.$$
5. **a** Since $n > 30$, the sampling distribution is approximately Normal. None of the other statements is correct.
6. **d** Statement I is true for all infinite populations and if 10% condition is met for finite populations. Statement II is based on the idea of combining multiple Normal distributions, Statement III is the central limit theorem.
7. **a** A restatement of the central limit theorem.
8. **d** The standard deviation of the sampling distribution depends on the sample size but not on the population size (as long as the 10% condition is met).
9. **c** The only condition that is required for using the formula is that samples from a finite population are less than 10% of the population size.

10. **b**
$$P(\bar{x} > 25) = P\left(z > \frac{25 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = P\left(z > \frac{25 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(z > \frac{25 - 20.5}{\frac{15.4}{\sqrt{40}}}\right)$$

11. (a) No. We don't know the shape of the distribution, so we can't calculate this probability.

(b) $\mu_{\bar{x}} = 12$ and $\sigma_{\bar{x}} = \frac{0.4}{\sqrt{50}} \approx 0.0566$. (c) Since $n = 50$, which is greater than 30, we can use the

Normal probability distribution. $P(\bar{x} < 11.9) = P\left(z < \frac{11.9 - 12}{0.0566}\right) = P(z < -1.77) = 0.0384$

(d) If the true mean amount of soda in the cans is 12 ounces, there is about a 4% chance of getting a sample mean as low or lower than 11.9 ounces. This result is unlikely enough to make us suspicious and lead us to conclude that the company is under-filling its cans of soda!

12. (a) $\mu_{\hat{p}} = p = 0.45$. (b) $\sigma_{\hat{p}} = \sqrt{\frac{(0.45)(0.55)}{500}} = 0.0222$. (c) $np = 500(0.45) = 225 > 10$ and

$n(1-p) = 500(0.55) = 275 > 10$. So the sampling distribution is approximately Normal.

(d) $P(\hat{p} > 0.5) = P\left(z > \frac{0.5 - 0.45}{0.0222}\right) = P(z > 2.25) = 0.0122$.

13. SAMPLE RANGE IS NOT AN UNBIASED ESTIMATOR OF POPULATION RANGE.

The population range is $80 - 20 = 60$. The range of a sample will only be this large if the population's minimum and maximum values in the distribution are both in the sample. Otherwise, the sample range will be smaller. Thus the center of the sampling distribution of sample ranges will be somewhere below 60. In this particular case, the median of the distribution is 57.

MULTIPLE-CHOICE QUESTIONS

- A C. The mean of the sampling distribution is equal to the mean of the population and the standard deviation of the sampling distribution is equal to the population standard deviation divided by the square root of the sample size (*Introduction to Statistics & Data Analysis* 3rd ed. pages 450–459/4th ed. pages 504–513).

- B** D. The average of \$372,000 is computed from a sample of 50 homes; therefore it is the value of a statistic (*Introduction to Statistics & Data Analysis* 3rd ed. pages 446–449/4th ed. pages 500–503).
- C** E. Any particular sample mean is not necessarily equal to the population mean. However, the mean of the sampling distribution for random samples of size n will equal the population mean (*Introduction to Statistics & Data Analysis* 3rd ed. pages 450–459/4th ed. pages 504–513).
- D** C. The mean of the sampling distribution equals the mean of the population but the standard deviation of the sampling distribution will be smaller—it is the population standard deviation divided by the square root of the sample size (*Introduction to Statistics & Data Analysis* 3rd ed. pages 450–459/4th ed. pages 504–513).
- E** A. The Central Limit Theorem states that even in skewed populations, if the sample size n is sufficiently large, the sampling distribution will be well approximated by a normal curve (*Introduction to Statistics & Data Analysis* 3rd ed. pages 450–459/4th ed. pages 504–513).



“FRAPPY”
{Free Response AP Problem...Yay!}

The following problem is taken from an actual Advanced Placement Statistics Examination. Your task is to generate a complete, concise statistical response in 15 minutes. You will be graded based on the AP rubric and will earn a score of 0-4. After grading, keep this problem in your binder for your AP Exam preparation.

Big Town Fisheries recently stocked a new lake in a city park with 2,000 fish of various sizes. The distribution of the lengths of these fish is approximately normal.

Scoring:

(a) Big Town Fisheries claims that the mean length of the fish is 8 inches. If the claim is true, which of the following would be more likely?

- A random sample of 15 fish having a mean length that is greater than 10 inches

or

- A random sample of 50 fish having a mean length that is greater than 10 inches

Justify your answer.

E P I

(b) Suppose the standard deviation of the sampling distribution of the sample mean for random samples of size 50 is 0.3 inch. If the mean length of the fish is 8 inches, use the normal distribution to compute the probability that a random sample of 50 fish will have a mean length less than 7.5 inches.

E P I

(c) Suppose the distribution of fish lengths in this lake was nonnormal but had the same mean and standard deviation. Would it still be appropriate to use the normal distribution to compute the probability in part (b)? Justify your answer.

E P I

Total: __/4



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A local radio station plays 40 rock-and-roll songs during each 4-hour show. The program director at the station needs to know the total amount of airtime for the 40 songs so that time can also be programmed during the show for news and advertisements. The distribution of the lengths of rock-and-roll songs, in minutes, is roughly symmetric with a mean length of 3.9 minutes and a standard deviation of 1.1 minutes.

Scoring:

(a) Describe the sampling distribution of the sample mean song lengths for random samples of 40 rock-and-roll songs.

E P I

(b) If the program manager schedules 80 minutes of news and advertisements for the 4-hour (240-minute) show, only 160 minutes are available for music. Approximately what is the probability that the total amount of time needed to play 40 randomly selected rock-and-roll songs exceeds the available airtime?

E P I

Total: __/4