Statistics Practice
AP Statistics Exam Review #4

I. **Confidence Intervals**

1. True or False: For a fixed confidence level, when the sample size increases, the length of the confidence interval for a population mean decreases.

2. True or False: The larger the level of confidence, the shorter the confidence interval.

3. You want to design a study to estimate the proportion of students on your campus who agree with the statement, "The student government is an effective organization for expressing the needs of students to the administration." You will use a 95% confidence interval and you would like the margin of error of the interval to be .05 or less. The minimum sample size required is approximately

$$p = ? \quad \hat{p} = ? \quad use \quad p = .5$$

$$.05 \geq z^*_{.95} \sqrt{\frac{p(1-p)}{n}}$$

$$.05 \geq 1.96 \sqrt{\frac{.5(1-.5)}{n}}$$

$$(\sqrt{n})^2 \geq \left(\frac{1.96 \sqrt{.5(1-.5)}}{.05} \right)^2$$

$$n \geq 384.16$$

$$n = 385$$

4. You have measured the systolic blood pressure of a random sample of 25 employees of a company located near you. A 95% confidence interval for the mean systolic blood pressure for the employees of this company is (122, 138). Which of the following statements gives a valid interpretation of the level of confidence for this interval?
   a. 95% of the sample of employees have a systolic blood pressure between 122 and 138.
   b. 95% of the population of employees have a systolic blood pressure between 122 and 138.
   c. If the procedure were repeated many times, 95% of the resulting confidence intervals would contain the population mean systolic blood pressure.
   d. The probability that the population mean blood pressure is between 122 and 138 is .95

$\bar{X} = 76.5$

5. If the 98% confidence limits for the population mean are 73 and 80, which of the following **could be** the 95% confidence limits?
   73 and 81   72 and 79   72 and 81   d. 74 and 79   ✓

   Smaller than 98% CI

6. A test engineer wants to estimate the mean gas mileage ~ (in miles per gallon) for a particular model of automobile. Eleven of these cars are subjected to a road test, and the gas mileage is computed for each car. A dot-plot of the 11 gas-mileage values is roughly symmetrical and has no outliers. The mean and standard deviation of these values are 25.5 and 3.01, respectively. Assuming that these 11 automobiles can be considered a simple random sample of cars of this model, find the 95% confidence interval for the population mean gas mileage.

approx normal

$\bar{X} = 25.5 \qquad t^* = 2.228 \qquad \bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$

$S = 3.01$

$n = 11 \ (df = 10)$

$25.5 \pm 2.228 \left(\frac{3.01}{\sqrt{11}}\right)$

$25.5 \pm 2.022 = [23.478, \ 27.522]$

7. A 95% confidence interval for the mean reading achievement score for a population of third-grade students is (44.2, 54.2).
   a. What is the margin of error of this interval?
   b. What is the value of the sample mean?

$\boxed{\bar{X} = 49.2}$

Width = 10

$49.2 \pm ME$

$\boxed{ME = 5}$

II. Inference Testing (part 1)

8. A t test is used instead of a z test because **population standard dev. unknown**

9. The claim about the population that we are trying to find evidence for is the **null hypothesis**

10. The probability that we would get our observed result if the null hypothesis were true is the **p-value**

11. A test is **one-tailed** if we are interested only in deviations from the null in one direction.

12. A test is **two-tailed** if we are interested in any deviation from the null.

13. Suppose a test of the hypotheses in a question produces a P-value of 0.11. The correct action would be to **fail to reject H_0**

14. If the null hypothesis is rejected when it is true, a **type I** error has occurred.

15. Failing to reject the null hypothesis when it is false is a **type II** error.

16. A one-sided significance test gives a p-value of .04. From this we can
    a. Reject the null hypothesis with 99% confidence.
    b. Reject the null hypothesis with 95% confidence.
    c. Say that the probability that the null hypothesis is false is .04.
    d. Say that the probability that the null hypothesis is true is .04.
    e. None of these

17. A significance test was performed to test $H_0 : \mu = 2$ versus $H_a : \mu \neq 2$. The test statistic is z = 1.40. What is the p-value for this test?
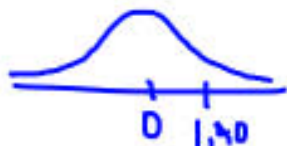
two tailed

$$2p(z > 1.40) = .16$$

18. A manufacturer of balloons claims that p, the proportion of its balloons that burst when inflated to a diameter of up to 12 inches, is no more than 0.05. Some customers have complained that the balloons are bursting more frequently. If the customers want to conduct an experiment to test the manufacturer's claim, which of the following hypotheses would be appropriate?
    a. Ho: $p \neq 0.05$, Ha: $p = 0.05$
    b. Ho: $p = 0.05$, Ha: $p > 0.05$
    c. Ho: $p = 0.05$, Ha: $p \neq 0.05$
    d. Ho: $p = 0.05$, Ha: $p < 0.05$
    e. Ho: $p < 0.05$, Ha: $p = 0.05$
    f. None of these

19. The one-sample t statistic for a test of $H_0 : \mu = 10$ $H_a : \mu < 10$ based on n = 10 observations has a test statistic with the value $t = -2.25$. What is the p-value of this test?

    $df = 9$

    $pvalue = P(t < -2.25) = .026$

20. Which of the following is not an assumption needed to perform a hypothesis test on a single mean using a z – test statistic?
    a. An SRS of size n from the population.
    b. Known population standard deviation, σ.
    c. Either a normal population or a large sample ($n \geq 30$).
    d. The population must be at least 10 times the size of the sample. ← proportion
    e. All are needed.

21. When the test statistic falls outside of the rejection region on a hypothesis test, we
    a. Reject the null hypothesis
    b. Fail to reject the null hypothesis
    c. Accept the null hypothesis
    d. Look for more evidence to reject the null hypothesis
    e. None of these

22. What are the conditions that need to be satisfied for a one-sample significance test involving a population proportion?
    a. The sample must be an SRS from the population of interest. ✓
    b. The population must be at least 10 times the size of the sample. ✓
    c. The number of successes and the number of failures must each be at least 10 (both $n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$). ✓
    d. All of the above
    e. None of the above

1. The p-value of a test of significance is the probability that:
    a) the decision resulting from the test is correct.
    b) 95% of the confidence intervals will contain the parameter of interest.
    c) the null hypothesis is true.
    d) the alternative hypothesis is true.
    (e) None of these describes the p-value.

   ★ pvalue is prob. that your sample results true given H₀ is true

   the p-value is the probability of obtaining a test statistic result at least as extreme as the one that was actually observed,
   assuming that the null hypothesis is true.

2. If we want to claim that a population parameter is different from a specified value, this
   situation can be considered as a one-tailed test.   ≠
    a) True
    (b) False   two

3. In the P-value approach to hypothesis testing, if the P-value is less than a specified significance   α
   level, we fail to reject the null hypothesis.
    a) True
    (b) False   pvalue < α ⇒ reject H₀

4. The manager of a factory wants to compare the mean number of units assembled per employee in a week for two new assembly techniques. Two hundred employees from the factory are randomly selected and each is randomly assigned to one of the two techniques. After teaching 100 employees one technique and 100 employees the other technique, the manager records the number of units each of the employees assembles in one week. Which of the following would be the most appropriate inferential statistical test in this situation?
   a) One-sample z-test   b) Two-sample t-test   c) Matched Pairs t-test
   d) Chi-square goodness-of-fit test   e) One-sample t-test

5. Do you have an insatiable craving for chocolate or some other food? Since many North Americans apparently do, psychologists are designing scientific studies to examine the phenomenon. According to the New York Times (Feb. 22, 1995), one of the largest studies of food cravings involved a survey of 1000 McMaster University (Canada) students. The survey revealed that 97% of the women in the study acknowledged specific food cravings while only 67% of the men did. Assume that 600 of the respondents were women and 400 were men. Is there sufficient evidence to claim that the true proportion of women who acknowledge having food cravings exceed the corresponding proportion for men?   2 prop. z test

$\hat{p}_1$
$\hat{p}_2$

b) Yes. Since the p-value is so small, there is statistical evidence to reject the null hypothesis of no difference between gender population proportions of food cravings.

$H_0 : P_1 = P_2$

$H_a : P_1 > P_2$

1.645

$z = \dfrac{(.97 - .67) - 0}{\sqrt{\dfrac{.97(.03)}{600} + \dfrac{.67(.33)}{400}}} = 12.23$

p value : $p(z > 12.23) = 1.04 \times 10^{-34} \approx 0$

6. We wish to test if a new feed increases the mean weight gain compared to an old feed. At the conclusion of the experiment it was found that the new feed gave a 10 kg bigger gain than the old feed. A two-sample t-test with the proper one-sided alternative was done and the resulting p-value was .082. This means:

    b) there was only an 8.2% chance of observing an increase greater than 10 kg (assuming the null hypothesis was true).

7. A random sample of 288 voters registered in the state of California showed that 141 voted in the last general election. A random sample of 216 registered voters in the state of Colorado showed that 125 voted in the most recent general election. The absolute value of the appropriate test statistic for testing the claim that more registered voters in Colorado voted in the last general election than those in California is

    a) t = 1.98
    b) z = 4.87
    c) z = 1.98
    d) p = .0237

$$\hat{p}_1 = \frac{141}{288} \qquad \hat{p}_2 = \frac{125}{216}$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\dfrac{\frac{141}{288}\left(1-\frac{141}{288}\right)}{288} + \dfrac{\frac{125}{216}\left(1-\frac{125}{216}\right)}{216}}}$$

8. Herbicide A has been used for years in order to kill a particular type of weed, but an experiment is to
be conducted in order to see whether a new herbicide, Herbicide B, is more effective than Herbicide A. Herbicide A will continue to be used unless there is sufficient evidence that Herbicide B is more effective. The alternative hypothesis in this problem is that

a) Herbicide A is more effective than Herbicide B.
b) Herbicide B is more effective than Herbicide A.
c) Herbicide A is not more effective than Herbicide B.
d) Herbicide B is not more effective than Herbicide A.
e) Herbicides A and B differ in effectiveness.

$$H_0 : A = B$$

$$H_a : A < B$$

$$B > A$$

For questions 9-12: At Dream HS, teachers are informed that their grade distribution should model 20% A's, 30% B's, 20% C's, 15% D's, and 15% F's. Mrs. Concerned wonders if her 100 student's grades fit this pattern. Her students earned 26 A's, 34 B's, 30 C's, 6 D's, and 4 F's.

9. Which of the following significance test should be used?
   a) Chi-Square Goodness-of-Fit Test
   b) One Proportion Z Test
   c) Difference of Two Proportions Z Test

	A	B	C	D	F
Obs.	26	34	30	6	4
Exp.	20	30	20	15	15

D 10. The degrees of freedom for this test is

4

$$\frac{(O-E)^2}{E} \qquad \frac{6^2}{20} + \frac{4^2}{30} + \frac{10^2}{20} + \frac{9^2}{15} + \frac{11^2}{15}$$

D 11. The test statistic is $\longrightarrow$ $\boxed{\chi^2 = 20.8}$

D 12. For a significance test performed on this situation, the p-value is

p value : $p(\chi^2 > 20.8) = \chi^2 cdf\ (20.8, 9999999, 4)$

$= .00035$

For each of the following situations in problems 13-19, determine which of the following tests should be used. Assume necessary conditions have been met. (Answers may be used more than once or not at all.)

A. 1 sample z test   B. 1 sample t test   C. 2 sample t test   D. Chi-squared
E. 1 proportion z test   F. 2 proportion z test   G. matched – pairs t test

B

13.   *Consumer Reports* (January 1993) stated that the mean retail cost of an AT&T model 3730 cellular phone was $600. A random sample of 10 stores in Los Angeles had a mean cost of $586.50 with standard deviation of $26.77. Does this indicate that the mean cost in Los Angeles is less than $600?

E

14.   Athabasca Fishing Lodge is located on Lake Athabasca in northern Canada. In one of their recent brochures, the lodge advertises that 75% of their guests catch northern pike over 20 pounds. Suppose last summer 64 out of a random sample of 83 guests did in fact catch northern pikes weighing over 20 pounds. Does this indicate that the population proportion of guests who catch pikes over 20 pounds is different from 75%?

G

15.   A new law has been passed giving city police greater powers in apprehending suspected criminals. For six neighborhoods, the numbers of reported crimes one year before and one year after the new law are shown. Does this indicate that the number of reported crimes have dropped?

Neighborhood	1	2	3	4	5	6
Before	18	35	44	28	22	37
After	21	23	30	19	24	29

D 16. A machine has a record of producing 80% excellent, 16% good, and 4% unacceptable parts. After extensive repairs, a sample of 200 produced 157 excellent, 42 good, and 1 unacceptable part. Have the repairs changed the nature of the output of the machine? χ^2

F 17. A random sample of 378 hotel guests was taken one year ago, and it was found that 178 requested nonsmoking rooms. Recently, a random sample of 516 hotel guests showed that 320 requested nonsmoking rooms. Do these data indicate that the <u>proportion of</u> hotel guests requesting nonsmoking rooms has increased?

C 18. Based on information from *Consumer Reports* (October 1993), a random sample of 86 thirty-gram servings of Duncan Hines Chocolate Chip Cookies had a mean of 132 calories with standard deviation 27 calories. A random sample of 75 Pepperidge Farm Chocolate Chip Cookies (30-gram servings) had a mean of 124 calories and standard deviation of 33 calories. Does this information indicate that there is a difference in the average number of calories for these two cookies brands?

B 19. *Math Horizons* is a publication of the Mathematical Association of America. The 1993 issue reports that in the U.S., graduating math majors who also have studied actuarial science (statistics) have a mean first-year salary of $27,600. Suppose that a random sample of 36 such graduates in the Denver region showed that they were earning an average of $27,810 with standard deviation $915. Does this indicate that the population mean salary in the Denver region is higher than the national average?

20. The following are percentages of fat found in 5 samples of each of two brands of ice cream:

A   5.7. 4.5 6.2 6.3 7.3
B   6.3 - 5.7 5.9 6.4 5.1

Which of the following procedures is appropriate to test the hypothesis of equal average fat content in the two types of ice cream?

a) matched pairs t-test with 5 degrees of freedom
b) two sample t-test with 9 degrees of freedom
c) matched pairs t-test with 4 degrees of freedom
d) two sample t-test with 4 degrees of freedom

21. In the previous problem, a Type II error would be:
a) to conclude that the fat content is different when it actually is
b) to conclude that the fat content is different when it actually is not
c) to conclude that the fat content is the same when it actually is
d) to conclude that the fat content is the same when it actually is not

Fail to reject a false H₀

For questions 22 and 23, use the two-way table specifying favorite ice cream flavors by gender.

OBS

	Male	Female	
Chocolate	32 .	16	48
Vanilla	14	4 .	18
Strawberry	3	10	13
	49	30	79

$EXP = n_r \cdot n_c / n$

29.77	18.23
11.16	6.84
8.06	4.94

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$\chi^2 = 10.7$$

22.   What is the expected number of females who prefer vanilla?

a) 18.2   (b) 6.8   c) 4.9   d) 11.2   e) none of these

23.   We wish to test if there is an association between gender and flavor choice. Which of the following is a valid conclusion from this information?

(a) We have sufficient evidence of an association between gender and ice cream flavor preference at the 5% level.

b) There is insufficient evidence of a relationship between gender and ice cream preference.

c) We have sufficient evidence that there is no association between gender and ice cream flavor preference at the 5% level.

d) There is insufficient evidence to support the claim that gender and ice cream preference are independent.

pvalue: $P(\chi^2 > 10.7) = .00087 < \alpha$

H_0: no association (variables independent)

H_a: is an association

24.   There are 4 TV sets in the student center of a large university. At a particular time each day, four different soap operas (1, 2, 3 and 4) are viewed on these TV sets. It is believed that the percentages of the audience captured by these shows are 25%, 30%, 25%, and 20%, respectively. 300 students are surveyed and the summary of their response follows:

	1	2	3	4
Observed	80	88	79	53

Exp   75 90 75 60

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 1.41$$

Do these observed data fit the belief of percentages who watch each show ($\alpha = 0.05$)?

a) Yes
b) No

$$p(\chi^2 > 1.41) = .704$$

Fail to reject H_0

one sample t test

25. An association of college bookstores reported that the average amount of money spent by students on textbooks for the Fall 2010 semester was $325.16. A random sample of 75 students at the local campus of the state university indicated an average bill for textbooks for the semester in question to be $312.34 with a standard deviation of $76.42. Do these data provide significant evidence that the actual average bill is different from the $325.16 reported?

Based on the results of the significance test, you should
a) Reject H_0 at the 1% significance level
b) Fail to reject H_0 at the 1% significance level but reject H_0 at the 5% significance level
c) Fail to reject H_0 at the 5% significance level

$\bar{x} = 312.34 \quad s = 76.42 \quad n = 75$

$H_0 : \mu = 325.16$

$H_a : \mu \neq 325.16$

$t = \dfrac{312.34 - 325.16}{76.42/\sqrt{75}} = -1.45$

$2 \cdot p(t < -1.45) = .15$