9.8 Factor Polynomials Completely

Goal • Factor polynomials completely.

Your Notes

VOCABULARY

Factor by grouping WHEN YOU HAVE 4 TERMS. GROUP THE 1ST 2 TERMS AND THE LAST 2 TERMS FACTOR OUT A COMMON MONOMIAL AND LOOK FOR A COMMON BINOMIAL

Example 1 *Factor out a common binomial* — 2 TERMS

Factor the expression.

a. $3x(x + 2) - 2(x + 2)$

$(x+2)(3x-2)$

b. $y^2(y - 4) + 3(4 - y)$ $(-y+4)$ $-1(y-4)$

$y^2(y-4) + 3(-1)(y-4)$

$(y-4)(y^2-3)$

Example 2 *Factor by grouping*

Factor the expression.

a. $y^3 + 7y^2 + 2y + 14$

b. $y^2 + 2y + yx + 2x$

Solution

a. $y^3 + 7y^2 + 2y + 14 = (y^3 + 7y^2) + (2y + 14)$ [1] GROUP TERMS

$= y^2(y+7) + 2(y+7)$ [2] FACTOR COMMON BINOMIAL

$= (y+7)(y^2+2)$ [3] FACTOR COMPLETELY

b. $y^2 + 2y + yx + 2x = (y^2+2y) + (yx+2x)$

$= y(y+2) + x(y+2)$

$= (y+2)(y+x)$

Remember that you can check a factorization by multiplying the factors.

Your Notes

Example 3 *Factor by grouping*

Factor $x^3 - 12 + 3x - 4x^2$.

STEP 1: Reorder terms from high to low exponents

STEP 2 GROUP

$x^3 - 4x^2 + 3x - 12$

$x^2(x-4) + 3(x-4)$

$(x-4)(x^2+3)$

Checkpoint Factor the expression.

1. $5z(z-6) + 4(z-6)$

$(z-6)(5z+4)$

2. $2y^2(y-1) + 7(1-y)$

$2y^2(y-1) + 7(-1)(y-1)$

$(y-1)(2y^2-7)$

$(1-y)$, -1 -1

$-1(-1+y)$

$-1(y-1)$

3. $x^3 - 4x^2 + 5x - 20$

$x^2(x-4) + 5(x-4)$

$(x-4)(x^2+5)$

4. $n^3 + 48 + 6n + 8n^2$ ← Reorder Terms

$n^3 + 8n^2 + 6n + 48$

$n^2(n+8) + 6(n+8) =$

$(n+8)(n^2+6)$

VOCABULARY

Factor completely FACTOR A POLYNOMIAL UNTIL IT IS WRITTEN AS A PRODUCT OF UNFACTORABLE POLYNOMIALS WITH INTEGER COEFFICIENTS

NO FRACTIONS OR DECIMALS

GUIDELINES FOR FACTORING POLYNOMIALS COMPLETELY

To factor a polynomial completely, you should try each of these steps.

1. Factor out the GREATEST common monomial factor.
2. Look for a difference of two squares (x^2-9) or a perfect SQUARE TRINOMIAL (EX $x^2+6x+9=(x+3)^2$)
3. Factor a trinomial of the form $ax^2 + bx + c$ into a product of binomial factors.
4. Factor a polynomial with four terms by GROUPING

Example 4 *Factor completely*

Factor the polynomial completely.

a. $x^2 + 3x - 1$

b. $3r^3 - 21r^2 + 30r$

c. $9d^4 - 4d^2$

FACTOR

Solution

a. The terms of the polynomial have no common monomial factor. Also, there are no factors of -1 that have a sum of 3. This polynomial CAN NOT be factored. The answer you WRITE IS "PRIME."

(A) x^2+3x-1

$(x+\quad)(x-\quad)$ X

b. $3r^3 - 21r^2 + 30r = 3R(R^2-7R+10)$ ① FACTOR GCF

$= 3R(R-2)(R-5)$ ② KEEP FACTORING, IF POSSIBLE.

c. $9d^4 - 4d^2 = d^2(9d^2-4)$ ① FACTOR GCF

$= d^2(3d-2)(3d+2)$ ② KEEP FACTORING

Checkpoint **Factor the polynomial.**

IF THE LEADING COEF IS NEGATIVE ALWAYS FACTOR OUT -1

5. $-2x^3 + 6x^2 + 108x$

$\frac{-2x^3}{-2x} + \frac{6x^2}{-2x} + \frac{108x}{-2x}$

$-2x(x^2-3x-54)$

1	54
2	27
3	18
6	9

$-2x(x+6)(x-9)$

6. $\frac{12y^4}{3y^2} - \frac{75y^2}{3y^2} = 3y^2(4y^2-25) =$

$3y^2(2y-5)(2y+5)$

Example 5 *Solve a polynomial equation*

Solve $5x^3 - 25x^2 = -30x$.

Solution

$5x^3 - 25x^2 = -30x$ — Write original equation.

(1) $5x^3 - 25x^2 + 30x = 0$ ← ADD $30x$ to each side.

(2) $5x(x^2 - 5x + 6) = 0$ — Factor out $5x$.

(3) $5x(x-2)(x-3) = 0$ — Factor trinomial.

(4) $5x = 0$ or $x - 2 = 0$ or $x - 3 = 0$ — Zero-product property

(5) $x = 0$, $x = 2$, $x = 3$ — Solve for x.

Remember that you can check your answers by substituting each solution for x in the original equation.

STEPS TO SOLVE

1. EQ. MUST = 0 $(Ax^2 + Bx + C = 0)$
2. FACTOR ANY GCF'S
3. FACTOR COMPLETELY
4. SET EVERY FACTOR TO 0 AND SOLVE.
5. USE CALC + CHECK IN ORIG EQ

There are 3 solutions

$x = 0, 2, 3$

(5) C: $x = 0$ $\quad 0 = 0$ ✓

C: $x = 2$ $\quad -60 = -60$ ✓

C: $x = 3$ $\quad -90 = -90$ ✓

Checkpoint **Complete the following exercises.**

7. Solve $2x^3 + 2x^2 = 40x$.

$-40x \quad -40x$

$2x^3 + 2x^2 - 40x = 0$

$2x(x^2 + x - 20) = 0$

$1 \cdot 20$, $2 \cdot 10$, $4 \cdot 5$

$2x(x + 5)(x - 4) = 0$

$2x = 0$ $\quad x + 5 = 0$ $\quad x - 4 = 0$

$x = 0$ $\quad x = -5$ $\quad x = 4$

C: $0 = 0$ ✓ $\quad$ C: $-200 = -200$ ✓ $\quad$ C: $160 = 160$