

SOLUTIONS, ROOTS, AND ZERO'S MEAN THE SAME THING!
AKA FIND THE VALUE(S) OF X!

9.4 Solve Polynomial Equations in Factored Form

Goal • Solve polynomial equations.

Your Notes

Example

$$f(x) = x^2 + 5x + 6$$

$$x^2 + 5x + 6 = 0$$

FACTORS $(x+3)(x+2) = 0$

SOLVE

$$x+3 = 0 \quad x+2 = 0$$

$$x = -3 \quad x = -2$$

Check: $x = -3$

$$(-3)^2 + 5(-3) + 6 = 0$$

$$0 = 0 \checkmark$$

Check: $x = -2$

$$(-2)^2 + 5(-2) + 6 = 0$$

$$0 = 0 \checkmark$$

$x = -2, -3$ are solutions and are called roots

Roots \iff Solutions

VOCABULARY

*Roots ARE THE SOLUTIONS OF AN EQUATION WHICH ONE SIDE IS ZERO AND THE OTHER SIDE IS A PRODUCT OF FACTORS

ZERO-PRODUCT PROPERTY

$A \cdot B = 0$ (2 FACTORS)

Let a and b be real numbers. If $ab = 0$, then $a \cdot 0 = 0$ or $0 \cdot b = 0$.

Example 1 Use the zero-product property

Solve $(x - 5)(x + 4) = 0$.

Solution

$$(x - 5)(x + 4) = 0$$

$$x - 5 = 0 \quad \text{or} \quad x + 4 = 0$$

$$x = 5 \quad \text{or} \quad x = -4$$

Write original equation.
SET EACH FACTOR TO property
Solve for x .

The solutions of the equation are $x = -4, 5$

CHECK Substitute each solution into the original equation to check.

$$(\underline{5} - 5)(\underline{5} + 4) \stackrel{?}{=} 0 \quad (\underline{-4} - 5)(\underline{-4} + 4) \stackrel{?}{=} 0$$

$$0 \cdot 9 \stackrel{?}{=} 0 \quad -9 \cdot 0 \stackrel{?}{=} 0$$

$$0 = 0 \checkmark \quad 0 = 0 \checkmark$$

check $x = 5$

check $x = -4$

Your Notes

1ST STEP IN
FACTURING
IS TO TAKE OUT
ANY COMMON
FACTORS
(aka GCF)

GCF = GREATEST
COMMON
FACTOR

Example 2 Find the greatest common monomial factor

Factor out the greatest common monomial factor.

a. $\frac{16x}{8} + \frac{40y}{8}$

b. $\frac{6x^2}{6x^2} + \frac{30x^3}{6x^2}$

Solution Numbers - the largest number that goes evenly into all terms!

a. The GCF of 16 and 40 is 8. The variables x and y have No Common Factors. So, the greatest common monomial factor of the terms is 8. **FACTOR OUT 8!**

$16x + 40y = 8(2x + 5y)$ ← **FACTORED INTO 2 TERMS**
arc EQUAL

b. The GCF of 6 and 30 is 6. The GCF of x^2 and x^3 is x^2 . So, the greatest common monomial factor of the terms is $6x^2$. **FACTOR $6x^2$:**

$6x^2 + 30x^3 = 6x^2(1 + 5x)$

Check mentally by multiplying

For variables
 $x^2 = \underbrace{x \cdot x}_x = x$
 $x^3 = \underbrace{x \cdot x \cdot x}_x = x^2$
 GCF for variables is the factor with the smallest exponent.

Example 3 Solve an equation by factoring

Solve the equation.

a. $3x^2 + 15x = 0$

$3x(x+5) = 0$

$3x = 0$ or $x+5 = 0$

$x = 0$ or $x = -5$

The solutions of the equation are $x = 0, -5$. **Tip: Use calc to check roots**

Original equation

① Factor left side.

② Zero-product property **SET EACH FACTOR = 0**

Solve for x.

To use the zero-product property, you must write the equation so that one side is 0. For this reason, must be subtracted from each side of the equation.

b. $9b^2 = 24b$

$9b^2 - 24b = 0$

$3b(3b-8) = 0$

$3b = 0$ or $3b-8 = 0$

$b = 0$ or $b = \frac{8}{3}$

The solutions of the equation are $b = 0, \frac{8}{3}$

Original equation

① Subtract 24b from each side. **To set the EQ to be 0.**

② Factor left side.

③ Zero-product property **SET FACTORS = 0**

Solve for b.

$3b - \frac{8}{3} = 0$
 $+ \frac{8}{3} + \frac{8}{3}$
 $\frac{3b}{3} = \frac{8}{3}$
 $b = \frac{8}{3}$

FOR FRACTIONS

- ① DO NOT CHANGE TO DECIMALS
- ② Reduce (ex $\frac{4}{6} \rightarrow \frac{2}{3}$)
- ③ LEAVE AS IMPROPER FRACTION (ex $\frac{5}{2}$)

c. $9(\frac{8}{3})^2 = 24(\frac{8}{3})$
 $64 = 64 \checkmark$

Your Notes

✓ Checkpoint Solve the equation.

1. $(x + 6)(x - 3) = 0$

$$x + 6 = 0$$

$$x = -6$$

$$C: 0 \cdot -6 = 0 \\ 0 = 0 \checkmark$$

$$x - 3 = 0$$

$$x = 3$$

$$C: 9 \cdot 0 = 0 \\ 0 = 0 \checkmark$$

2. $(x - 8)(x - 5) = 0$

$$x - 8 = 0$$

$$x = 8$$

$$C: 0 \cdot 3 = 0 \\ 0 = 0 \checkmark$$

$$x - 5 = 0$$

$$x = 5$$

$$-3 \cdot 0 = 0 \\ 0 = 0 \checkmark$$

✓ Checkpoint Factor out the greatest common monomial factor.

3. $\frac{10x^2}{2} - \frac{24y^2}{2} \Leftrightarrow 2(5x^2 - 12y^2)$

↑
EQUIVALENT EXPRESSIONS

4. $\frac{3t^6}{t^4} + \frac{8t^4}{t^4}$

$$t^4(3t^2 + 8)$$