9.4 Solve Polynomial Equations in Factored Form

- Solve polynomial equations.

VOCABULARY

**Example**

\[ f(x) = x^2 + 5x + 6 \]
\[ x^2 + 5x + 6 = 0 \]

**Factors**

\[ (x+3)(x+2) = 0 \]

**Solve**

\[ x+3 = 0 \quad x+2 = 0 \]
\[ x = -3 \quad x = -2 \]

**Zero-Product Property**

\[ A \cdot B = 0 \]

Let \( a \) and \( b \) be real numbers. If \( ab = 0 \), then \( a \cdot 0 = 0 \) or \( 0 \cdot b = 0 \).

**Example 1**

**Use the zero-product property**

Solve \((x - 5)(x + 4) = 0\).

**Solution**

\[ (x - 5)(x + 4) = 0 \]

Write original equation.

\[ x - 5 = 0 \quad \text{or} \quad x + 4 = 0 \]

Set each factor to property.

\[ x = 5 \quad \text{or} \quad x = -4 \]

Solve for \( x \).

The solutions of the equation are \( x = -4, 5 \).

CHECK Substitute each solution into the original equation to check.

\[ (5 - 5)(5 + 4) \neq 0 \quad (-4 - 5)(-4 + 4) \neq 0 \]

Check \( x = 5 \)

\[ 0 \cdot 9 \neq 0 \]  \( \checkmark \)

Check \( x = -4 \)

\[ -9 - 0 \neq 0 \]  \( \checkmark \)

\( x = -3, 3 \) are Solutions and are called roots.
Example 2  **Find the greatest common monomial factor**

Factor out the greatest common monomial factor.

a. \( \frac{16x + 40y}{8} \)

\[ 16x + 40y = 8(2x + 5y) \]

**Solution**

Numbers — the largest number that goes evenly into all terms.

1. The GCF of 16 and 40 is \( 8 \). The variables \( x \) and \( y \) have **no common factors**. So, the greatest common monomial factor of the terms is \( 8 \).

\[ 16x + 40y = \frac{8(2x + 5y)}{8} \]  **Factored into 2 terms**

b. The GCF of 6 and 30 is \( 6 \). The GCF of \( x^2 \) and \( x^3 \) is \( x^2 \). So, the greatest common monomial factor of the terms is \( 6x^2 \).

\[ 6x^2 + 30x^3 = \frac{6x^2(1 + 5x)}{6x^2} \]

**Check mentally by multiplying**

**Solution**

2. The GCF of 6 and 30 is \( 6 \). The GCF of \( x^2 \) and \( x^3 \) is \( x^2 \). So, the greatest common monomial factor of the terms is \( 6x^2 \).

\[ 6x^2 + 30x^3 = \frac{6x^2(1 + 5x)}{6x^2} \]

**Check mentally by multiplying**

Example 3  **Solve an equation by factoring**

**Solve the equation.**

a. \( 3x^2 + 15x = 0 \)

**Original equation**

\[ 3x(x + 5) = 0 \]

**Factor left side.**

**Zero-product property**

\[ 3x = 0 \quad \text{or} \quad x + 5 = 0 \]

\[ x = 0 \quad \text{or} \quad x = -5 \]

**Solve for** \( x \).

**The solutions of the equation are**

\[ x = 0, -5 \]

**Tip:** Use a calculator to check roots.

b. \( 9b^2 = 24b \)

**Original equation**

\[ \frac{9b^2 - 24b}{3b} = 0 \]

**Subtract** \( 24b \) **from each side.**

**To set the equation to 0.**

\[ 3b(3b - 8) = 0 \]

**Factor left side.**

**Zero-product property**

\[ 3b = 0 \quad \text{or} \quad 3b - 8 = 0 \]

\[ b = 0 \quad \text{or} \quad b = \frac{8}{3} \]

**Solve for** \( b \).

**The solutions of the equation are**

\[ b = 0, \frac{8}{3} \]

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**For Fractions**

1. **Do not change to decimals**
2. **Reduce** (ex \( \frac{4}{6} \rightarrow \frac{2}{3} \))
3. **Leave as an improper fraction** (ex \( \frac{5}{2} \))

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\[ c: q \left( \frac{8}{3} \right)^2 = 24 \left( \frac{8}{3} \right) \]

\[ \frac{64}{9} = \frac{64}{9} \]
Your Notes

Checkpoint Solve the equation.

1. \((x + 6)(x - 3) = 0\)
   - \(x + 6 = 0\)
     - \(x = -6\)
     - \(C: \boxed{0} \cdot \boxed{-6} = 0\)
   - \(x - 3 = 0\)
     - \(x = 3\)
     - \(C: \boxed{9} \cdot \boxed{0} = 0\)

2. \((x - 8)(x - 5) = 0\)
   - \(x - 8 = 0\)
     - \(x = 8\)
   - \(C: \boxed{0} \cdot \boxed{3} = 0\)
   - \(x - 5 = 0\)
     - \(x = 5\)
     - \(C: \boxed{-3} \cdot \boxed{0} = 0\)

Checkpoint Factor out the greatest common monomial factor.

3. \(10x^2 - 24y^2 \quad \text{<=7} \quad 2 \left(5x^2 - 12y^2\right)\)
   - \(\text{Equivalent Expressions}\)

4. \(3t^6 + 8t^4 \quad \boxed{t^4} \left(3t^2 + 8\right)\)