

# Honors Algebra 1 Notes...

## 8.6 Write and Graph Exponential Decay Functions

Goal • Write and graph exponential decay functions.

### VOCABULARY

a) Exponential decay is a function which begins decreasing at a <sup>very</sup> slow rate, then keeps declining faster and faster.

Example  $f(x) = \left(\frac{1}{2}\right)^x$  EVALUATE!

$f(-2) = 4$        $f(-1) = 2$        $f(0) = 1$        $f(3) = .125$        $f(9) = .00195$

### EXAMPLE 1: Graph an exponential decay function

Graph the function  $y = \left(\frac{1}{3}\right)^x$  and identify its domain and range.

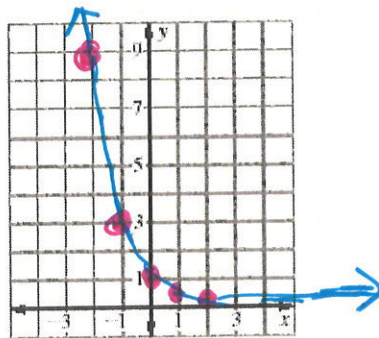
Step 1 **Make** a table by choosing a few values for  $x$  and finding the values of  $y$ .

$x$	-2	-1	0	1	2
$y$	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$

$\left(\frac{1}{3}\right)^{-2}$     $\left(\frac{1}{3}\right)^{-1}$     $\left(\frac{1}{3}\right)^0$     $\left(\frac{1}{3}\right)^1$     $\left(\frac{1}{3}\right)^2$

$\left(\frac{1}{3}\right)^{-2} = (3)^2 = 9$   
 $\left(\frac{1}{3}\right)^{-1} = 3^1 = 3$

Step 2 **Plot** the points; and **Draw** a smooth curve through the points.



Step 3 From either the table or the graph, you can see that **the domain** is

D:  $x = \text{all real numbers}$

Step 4 From either the table or the graph, you can see that **the range** is

R:  $y = \text{all positive real numbers}$

# Honors Algebra 1 Notes...

## EXAMPLE 2: Compare graphs of exponential decay and exponential growth functions

Graph  $y = 3^x$ . Compare the graph with the graph of  $y = \left(\frac{1}{3}\right)^x$

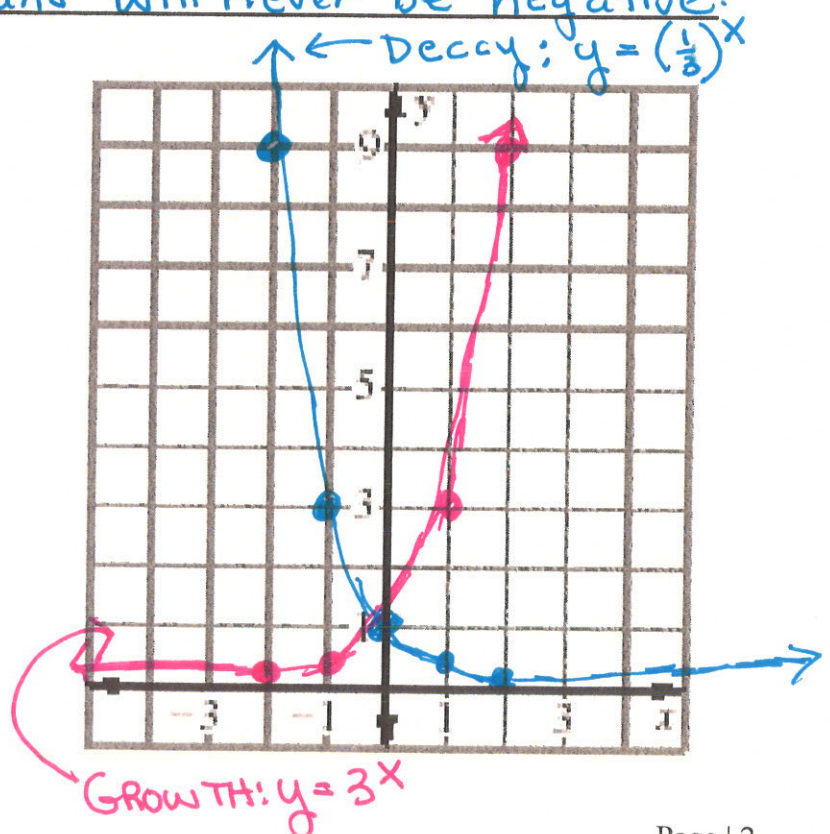
Steps:

- a) Graph the function:  $y = \left(\frac{1}{3}\right)^x$
- Fill in the table of values, plot the points, and draw a smooth curve through the points.
  - What is the **domain** and **range** for the function?
    - **D:** X = all real numbers
    - **R:** y = all positive real numbers
- b) Graph the function:  $y = 3^x$
- Fill in the table of values, plot the points, and draw a smooth curve through the points.
  - What is the **domain** and **range** for the function?
    - **D:** X = all real numbers
    - **R:** y = all positive real numbers
- c) How are the graphs different?

- \* They have the same domain and range
- \* Growth starts very small and then explodes when x's positive (3, 9, 27, 81...)
- \* Decay starts very large then becomes very small and will never be negative.

x	$y = \left(\frac{1}{3}\right)^x$	$y = 3^x$
-2	9	$3^{-2} = \frac{1}{9}$
-1	3	$3^{-1} = \frac{1}{3}$
0	1	$3^0 = 1$
1	$\frac{1}{3}$	$3^1 = 3$
2	$\frac{1}{9}$	$3^2 = 9$

↑  
prior  
Example  
#1





# Honors Algebra 1 Notes...

## EXAMPLE 3: Compare graphs of exponential decay functions

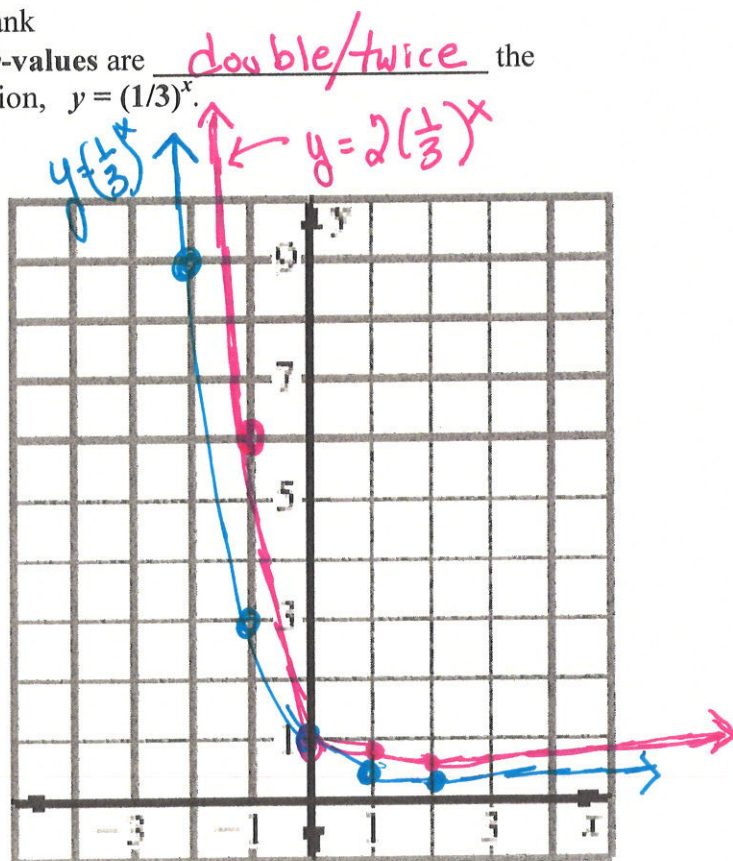
Graph  $y = 2 \cdot \left(\frac{1}{3}\right)^x$ . Compare the graph with the graph of  $y = \left(\frac{1}{3}\right)^x$

Steps:

- Graph the function:  $y = \left(\frac{1}{3}\right)^x$ 
  - Fill in the table of values, plot the points, and draw a smooth curve through the points.
- Graph the function:  $y = 2 \cdot \left(\frac{1}{3}\right)^x$ 
  - Fill in the table of values, plot the points, and draw a smooth curve through the points.
- What is the domain and range for each function? *They are the same*
  - D: *x = all real numbers*
  - R: *y = all positive real numbers*
- How are the graphs different? Fill in the blank
  - For the function,  $y = 2 \cdot \left(\frac{1}{3}\right)^x$ , the y-values are *double/twice* the corresponding y-values for the function,  $y = \left(\frac{1}{3}\right)^x$ .

x	$y = \left(\frac{1}{3}\right)^x$	$y = 2 \cdot \left(\frac{1}{3}\right)^x$
-2	9	$2 \cdot 9 = 18$
-1	3	$2 \cdot 3 = 6$
0	1	$2 \cdot 1 = 2$
1	$\frac{1}{3}$	$2 \cdot \frac{1}{3} = \frac{2}{3}$
2	$\frac{1}{9}$	$2 \cdot \frac{1}{9} = \frac{2}{9}$

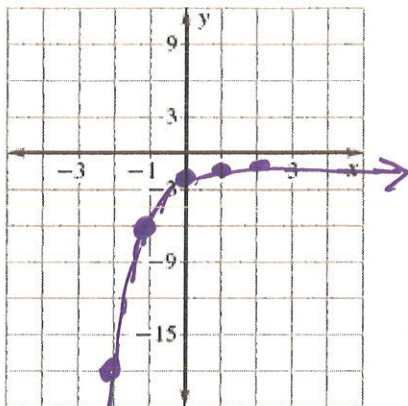
*FROM  
EXAMPLE  
#1*



# Honors Algebra 1 Notes...

✓ **Checkpoint** Complete the following exercise.

1. Graph  $y = -2 \cdot \left(\frac{1}{3}\right)^x$ . Identify its domain and range.



x	y
-2	$-2(9) = -18$
-1	$-2(3) = -6$
0	$-2(1) = -2$
1	$-2(1/3) = -2/3$
2	$-2(1/9) = -2/9$

D:  $x = \text{all real \#s}$   
 R:  $y = \text{all real negative \#s}$

2. How are graphs  $y = \left(\frac{1}{3}\right)^x$  and  $y = -2 \cdot \left(\frac{1}{3}\right)^x$  different?

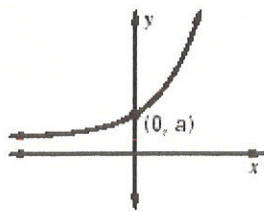
→ ① y values have doubled  
 ② Graph has flipped and reflects around the x-axis

## SUMMARY EXPONENTIAL GROWTH AND DECAY

### EXPONENTIAL GROWTH MODEL

$y = ab^x, a > 0$  and  $b > 1$

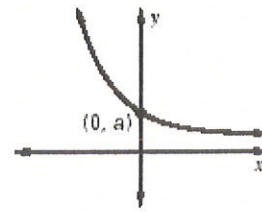
Growth is when the base is greater than 1



### EXPONENTIAL DECAY MODEL

$y = ab^x, a > 0$  and  $0 < b < 1$

Decay is when the base is a positive fraction.



### EXAMPLE 4: Classify functions

For each exponential function, tell whether the graph represents growth or decay; AND WHY!

a)  $y = \left(\frac{4}{3}\right)^x$   $B = 4/3$  which is greater than 1 (GROWTH)

b)  $y = \frac{1}{2}(6)^x$   $B = 6 > 1$  (GROWTH)

c)  $y = 8(.25)^x$   $B = .25 = 1/4$  which is between  $0 < b < 1$  (Decay)