Honors Algebra 1 Notes...

8.5 Write and Graph Exponential Growth Functions

Goal • *Write and graph exponential growth models.*

VOCABULARY

* **Exponential function** is a function which begins increasing at a very slow rate, then keeps growing faster and faster.

EXAMPLE: $2^x = 1^{\leftarrow 2^0}, 2^{\leftarrow 2^1}, 4^{\leftarrow 2^2}, 8, 16, 32, 64, 128, 256, 512 \ldots$

What is 2^{12} ⟶ $2 \wedge 12 =$ **4,096**

What is 2^{18} ⟶ $2 \wedge 18 =$ **262,144**

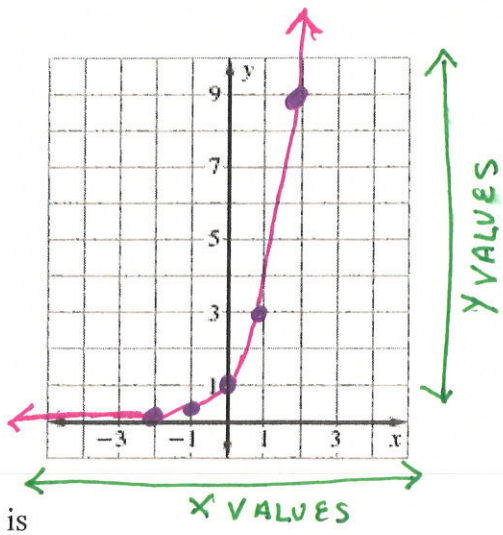
EXAMPLE 1: ***Graph an exponential function***

Graph the function $y = 3^x$. Identify its domain and range.

Step 1 Make a table by choosing a few values for x and finding the values of y.

x	−2	−1	0	1	2
y	1/9	1/3	1	3	9

Mental Work: 3^{-2} ⇓ $1/3^2$; 3^{-1} ⇓ $1/3^1$; 3^0 ; 3^1 ; 3^2



Step 2 Plot the points; and **Draw** a smooth curve through the points.

Step 3 From either the table or the graph, you can see that **the domain** is

D: X = all real numbers .

Step 4 From either the table or the graph, you can see that **the range** is

R: Y = all positive real numbers .

Honors Algebra 1 Notes...

VOCABULARY

y = the value of the investment

* **Exponential Growth Model** $y = a(1 + r)^t$

a is the initial amount. r is the Growth Rate.

$1 + r$ is the growth factor. t is the Time Period.

VOCABULARY

* **Compound interest problems** are an application of the "Exponential Growth Model"

EXAMPLE 3: ***Solve a compound interest problem***

Investment You put $250 in a savings account that earns 4% annual interest compounded yearly. You do not make any deposits or withdrawals. How much will your investment be worth in 10 years?

Key information:

- The initial amount is $250,
- the interest rate is 4%, or .04, and
- the time period is 10 years

$y = a(1 + r)^t$ ←	*Use the "Exponential Growth Formula"*
$y = 250(1 + .04)^{10}$	*Substitute :* $a = 250$ $r = .04$ $t = 10$
$y = 250\ (1.04)^{10}$ →	*Simplify.*
≈ $370.06	*Use a calculator.* 250 * 1.04 ^ 10 = 370.061
You will have 1 **in 10 years.** your investment has increased by $120.06	

Checkpoint **Complete the following exercise.**

4. In Example 3, suppose the annual interest rate is 5%. How much will your investment be worth in 10 years?

$a = \$250$ $r = 5\% = .05$ $t = 10$ yrs

$y = 250\ (1 + .05)^{10} = 250\ (1.05)^{10} \approx 407.223$

Investment would be about $407.22; an increase of $157.22.

EXAMPLE 2: ***Compare graphs of exponential functions***

Graph $y = 2 \cdot 3^x$. Compare the graph with the graph of $y = 3^x$.

Steps:

a) Graph the function: $y = 3^x$
- Fill in the table of values, plot the points, and draw a smooth curve through the points.

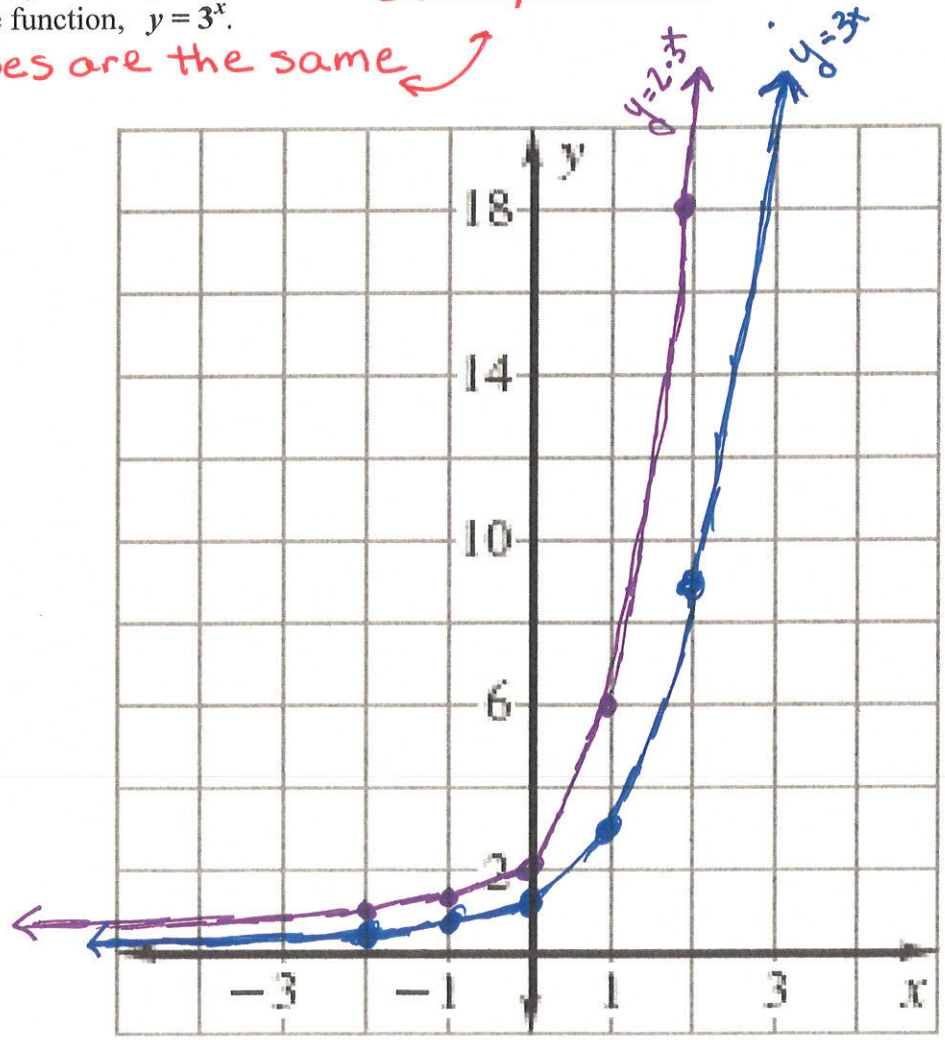
b) Graph the function: $y = 2 \cdot 3^x$
- Fill in the table of values, plot the points, and draw a smooth curve through the points.

c) What is the **domain** is and **range** for each function? THEY ARE THE SAME
- **D:** x = all real numbers.
- **R:** y = all positive real numbers.

d) How are the graphs different? Fill in the blank
- For the function, $y = 2 \cdot 3^x$, the ***y*-values** are double/twice the corresponding ***y*-values** for the function, $y = 3^x$.
- the shapes are the same

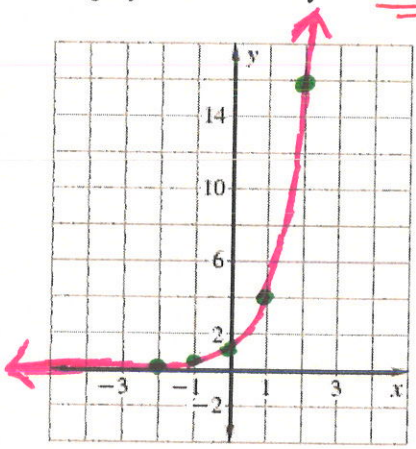
x	$y = 3^x$	$y = 2 \cdot 3^x$
−2	1/9	2/9
−1	1/3	2/3
0	1	2
1	3	6
2	9	18

SEE example #1



✔ *Checkpoint* **Complete the following exercises.**

1. Graph $y = 4^x$. Identify its domain and range.

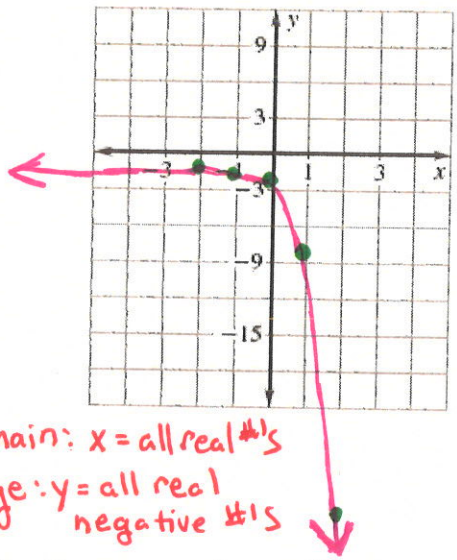


x	y
−2	$4^{-2} = 1/16$
−1	$4^{-1} = 1/4$
0	$4^0 = 1$
1	$4^1 = 4$
2	$4^2 = 16$

Domain: x = all real numbers

Range: y = all positive real numbers

2. Graph $y = -2 \cdot 4^x$. Identify its domain and range.



x	y
−2	$1/16 \cdot -2 = -2/16 = -1/8$
−1	$1/4 \cdot -2 = -2/4 = -1/2$
0	$1 \cdot -2 = -2$
1	$4 \cdot -2 = -8$
2	$16 \cdot -2 = -32$

Domain: x = all real #'s

Range: y = all real negative #'s

3. How are graphs $y = 4^x$ and $y = -2 \cdot 4^x$ different?

① the y-coordinate is double for $y = -2 \cdot 4^x$

② the graph is a mirror image with the leading coef. being negative (−2) and this is a reflection about the X-axis.