8.3 Define and Use Zero and Negative Exponents

Goal • Use zero and negative exponents.

Your Notes

(1) RULE FOR ZERO

(2) NEGATIVE EXPONENTS

NEG EXP → POS. EXP.

(A) $a^{-n} = \boxed{\frac{1}{a^n}}$

why?

$\frac{a^{-n}}{1} \cdot \frac{a^n}{a^n} = \frac{a^0}{a^n} = \boxed{\frac{1}{a^n}}$

(B) $\frac{1}{a^{-n}} = \boxed{a^n}$

$\frac{1}{a^{-n}} \cdot \frac{a^n}{a^n} = \frac{a^n}{a^0} = \boxed{a^n}$

DEFINITION OF ZERO AND NEGATIVE EXPONENTS

Words	Algebra	Example
(1) a to the zero power is 1.	$a^0 = 1$, $a \neq 0$	(a) $5^0 = 1$
(2) a^{-n} is the reciprocal of a^n.	$\frac{a^{-n}}{1} = \boxed{\frac{1}{a^n}}$, $a \neq 0$	$\frac{2^{-1}}{1} = \frac{1}{2^1} = \boxed{\frac{1}{2}}$
	$\frac{1}{a^{-n}} = \frac{a^n}{1} = a^n$	$\frac{1}{2^{-1}} = 2^1 = \boxed{2}$

(b) 0^0 = UNDEFINED

Example 1 *Use definition of zero and negative exponents*

Evaluate the expression.

a. $\frac{2^{-3}}{1} = \frac{1}{2^3}$ ← (1) Change to a positive exponent

$= \boxed{\frac{1}{8}}$ ← (2) **Evaluate exponent.**

*b. $(5x^2)^0 = 5^0 x^0 = 1 \cdot 1 = 1$ — **Definition of** Zero Rule

$x^0 = 1$

$4^0 = 1$

*c. $\left(\frac{5}{4}\right)^{-3} =$ Change to a positive exponent

METHOD 1

$\left(\frac{5}{4}\right)^{-3} = \frac{5^{-3}}{4^{-3}}$ (1) DISTRIB (2) Change − to +

$\frac{4^3}{5^3} = \boxed{\frac{64}{125}}$

METHOD 2

$\left(\frac{5}{4}\right)^{-3} = \left(\frac{4}{5}\right)^{+3}$

$= \frac{4^3}{5^3} = \boxed{\frac{64}{125}}$

(1) FLIP THE FRACTION TO ELIMINATE THE NEG. EXPONENT

(2) Simplify

d. $\frac{0^{-7}}{1} = \frac{1}{0^7} = \frac{1}{0} =$ UNDEFINED

*e. $\frac{5x^{-2}y^3}{2z} =$

$\frac{5y^3}{2x^2z}$

← rewrite with positive exponents

← x^{-2} is the only exponENT that needs to move

*2015 CHANGES

IMPORTANT: SUMMARY OF RULES

FOR NUMBERS (CONSTANTS)

- TREAT NUMBERS LIKE NUMBERS
- LEAVE AS SIMPLIFIED FRACTIONS

Your Notes

$a^0 = 1$

$0^0 =$ UNDEFINED

$a^1 \Leftrightarrow a$

$a^{-N} = \frac{1}{a^N}$

$\frac{1}{a^{-N}} = a^N$

PROPERTIES OF EXPONENTS

Let *a* and *b* be real numbers, and let *m* and *n* be integers.

Describe in your words:

$a^m \cdot a^n = a^{m+n}$ — MULT - same base - add exponents

$(a^m)^n = a^{m \cdot n}$ — NOTICE ( )'s - POWER TO POWER - MULT EXPONENTS

$(ab)^m = a^m b^m$ — NOTICE ( )'s - DISTRIBUTE EXPONENT "m" (mult exp's)

$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ — division - same base - subtract exponents

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ — NOTICE ( )'s - DISTRIBUTE EXPONENT "m" TO ALL FACTORS IN BOTH the NUM. + den.

Example 2 *Evaluate exponential expressions*

Evaluate the expression.

a. $(-5)^4 \cdot (-5)^{-4} = (-5)^{4+-4}$ ← what is the base? (-5) remember ( )'s

$= (-5)^0$ ← Same base + mult → Add exponents

$= \boxed{1}$

b. $(5^{-2})^{-2} = 5^{-2 \cdot -2}$ ← notice ( )'s power to power → mult exp's

written as a power → $= 5^4$

$= 625$ ← EVALUATE CALC: $\boxed{5 \wedge 4}$

*c. $\frac{2^{-3}x}{4^{-2}} = \frac{4^2 x}{2^3} =$ ← STEP I: Change to positive exponents

$= \frac{16x}{8} = \boxed{2x}$ ← simplify

d. $\frac{3^2 x^{-2}}{3^{-1} x^1} = 3^{2-(-1)} x^{-2-1}$

numbers can be bases (ie 3)

$= \frac{3^3 x^{-3}}{1}$

$=$

$$\boxed{\frac{27}{x^{+3}}}$$

Your Notes

Checkpoint Evaluate the expression.

1. $\left(\frac{1}{8}\right)^{-1}$ $\frac{1^{-1}}{8^{-1}} = \frac{8}{1} = 8$	2. $\frac{1}{3^{-2}x^2} = \frac{3^2}{x^2} = \boxed{\frac{9}{x^2}}$
3. $\frac{6^{-1}}{6x}$ $\frac{1}{6 \cdot 6x} = \boxed{\frac{1}{36x}}$	4. $(5^{-1})^2$ $\frac{5^{-2}}{1} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$

Example 3 *Use properties of exponents*

Simplify the expression $\frac{2w^{-3}x}{(2wx)^2}$. Write your answer using only positive exponents.

Solution

$$\frac{2w^{-3}x}{(2wx)^2} = \frac{2w^{-3}x^1}{2^2w^2x^2}$$

$$= \frac{2w^{-3-2}x^{1-2}}{4}$$

$$= \frac{2w^{-5}x^{-1}}{4}$$

$$= \boxed{\frac{1}{2w^5x}}$$

① Do ( )'s

② Same base, subtract exponents

③ Change to positive exponents

Checkpoint Simplify the expression.

5. $\frac{6f^1g^{-4}}{2f^2g}$	6. $(3yz^2)^{-2}$
$3F^{1-2}G^{-4-1}$	$\frac{3^{-2}y^{-2}z^{-4}}{1} =$
$\frac{3F^{-1}G^{-5}}{1}$	$\frac{1}{3^2y^2z^4} =$
$\boxed{\frac{3}{FG^5}}$	$\boxed{\frac{1}{9y^2z^4}}$

Daily Homework Quiz For use after Lesson 8.3

When evaluating, write answer as a fraction.

1. **Evaluate** $\left(\frac{5}{2}\right)^{-3}$

$\left(\frac{5^{-3}}{2^{-3}}\right) = \frac{2^3}{5^3} = \boxed{\frac{8}{125}}$

2. **Evaluate** $4^{-7} \cdot 4^3$

$4 \wedge 4 = 256$

Same base, add exponents

$4^{-7+3} = \frac{4^{-4}}{1} = \frac{1}{4^4} = \boxed{\frac{1}{256}}$

3. **Simplify** $6a^{-4}b^0$.

When simplifying, use only positive exponents.

$\frac{6a^{-4}}{1}$

$\boxed{\frac{6}{a^4}}$

4. **Simplify** $\frac{8x^3y^{-4}}{12x^2y^{-3}}$.

OPTION 1 make + exp's

$\frac{8x^3y^3}{12x^2y^4}$

$\boxed{\frac{2x}{3y}}$

OPTION 2 same base subtract exp's

$\frac{8x^{3-2}\,y^{-4-(-3)}}{12}$

$\frac{8xy^{-1}}{12} =$

$\boxed{\frac{2x}{3y}}$