7.5 Solve Special Types of Linear Systems

Goal
- Identify the number of solutions of a linear system.

Systems have 3 Type of Solutions:
1) One Solution - 2 LINES THAT INTERSECT AT ONE POINT.
2) No Solution - ARE PARALLEL LINES
3) Infinite Solutions ARE THE SAME LINE

Consistent dependent system - A linear system with infinitely many solutions
Example 1  
**A linear system with no solutions**

Show that the linear system has no solution.

\[-2x + y = 1\]  \hspace{1cm} \text{Equation 1} \\
\[-2x + y = -3\]  \hspace{1cm} \text{Equation 2}

**Solution**

**Method 1** Graphing

Graph the linear system.

The lines are ______ because they have the same slope but different y-intercepts. Parallel lines do ________, so the system has ________.

To ease graphing, write each equation in slope intercept form.

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**Example 1**  
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**Solution**

**Method 1** Graphing

Graph the linear system.

The lines are \underline{parallel} because they have the same slope but different y-intercepts. Parallel lines do \underline{not intersect}, so the system has \underline{no solution}.

**Method 2** Élimination

Subtract the equations.

\[-2x + y = 1\] \\
\[-2x + y = -3\] \\
\[0 = 4\]

The variables are \underline{eliminated} and you are left with a \underline{false statement}, regardless of the values of x and y. This tells you that the system has \underline{no solution}.

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**Solution to THIS SYSTEM:**

\[y = 2x + 1\]  \\
\[y = 2x - 3\]  \\
\[\text{NO SOLUTION}\]
Example 2: A linear system with infinitely many solutions

Show that the linear system has infinitely many solutions.

\[ \begin{align*}
  x + 3y &= -3 & \text{Equation 1} \\
  3x + 9y &= -9 & \text{Equation 2}
\end{align*} \]

Solution

Method 1: Graphing

Graph the linear system.
The equations represent the \underline{same line}, so any point on the line is a solution.
So, the linear system has \underline{infinitely many solutions}.
NUMBER OF SOLUTIONS OF A LINEAR SYSTEM

- One solution
- No solution
- Infinitely many solutions

The lines intersect.
The lines have different slopes.
The lines are parallel.
The lines have the same slope and different y-intercepts.
The lines have the same slope and the same y-intercept.

∞ SOLUTIONS