Solve Linear Systems by Adding or Subtracting

**Goal**  
- Solve linear systems using elimination. *(also called linear combination)*

**SOLVING A LINEAR SYSTEM USING THE ELIMINATION METHOD**

**Step 1**  
ADD the equations to **eliminate** one variable.

**Step 2**  
Solve the resulting equation for the other variable.

**Step 3**  
Substitute in either original equation to **find** the value of the other variable.

**Step 4**  
Check $(x, y)$ in both original equations.

---

**Example 1** Use addition to eliminate a variable

Solve the linear system:

- $x + 5y = 9$  
- $4x - 5y = -14$

**Solution**

1. **ADD** the equations to eliminate one variable.
   
   $x + 5y = 9$
   
   $4x - 5y = -14$

   \[
   \begin{align*}
   5x & = -5 \\
   x & = -1
   \end{align*}
   \]

2. Solve for $x = -1$

3. Substitute $x = -1$ in either equation
   
   \[
   \begin{align*}
   x + 5y & = 9 \\
   -1 + 5y & = 9 \\
   5y & = 10 \\
   y & = 2
   \end{align*}
   \]

   The solution is $(x, y) = (-1, 2)$.

4. **CHECK** in the original Eqs.
   
   $c: -1 + 5(2) = 9$
   
   $9 = 9 √$

   $c: 4(-1) - 5(2) = -14$
   
   $-4 - 10 = -14$
   
   $-14 = -14 √$

---

**Notes**

- **Equations** must be in **standard form**
  
  $Ax + By = C$
  
  $A, B, C$ are **integers**

- **Flavors**
  - Addition
  - Subtraction
  - Multiplication

Make sure to check your solution by substituting it into each of the original equations.
Example 2

Use subtraction to eliminate a variable

Solve the linear system:

\[ 3x - 4y = 2 \quad \text{Equation 1} \]
\[ 3x + 2y = 26 \quad \text{Equation 2} \]

Solution

1. Multiply 1 equation by -1:
   \[ 3x - 4y = 2 \quad \rightarrow \quad -3x + 4y = -2 \]
   to eliminate one variable.

2. Solve for \( y \):
   \[ 3x + 2y = 26 \quad \rightarrow \quad y = 4 \]

3. Substitute \( y = 4 \) into either equation and solve for the other variable (x):
   \[ 3x + 2(4) = 26 \quad \rightarrow \quad x = 6 \]

   The solution is \((6, 4)\).

4. Check:
   \[ 3(6) - 4(4) = 2 \]
   \[ 3(6) + 2(4) = 26 \]

Checkpoint

Solve the linear system.

1. \(-8x + 3y = 12\)
   \[ 8x - 9y = 12 \]
   \[ -8y = 24 \quad \rightarrow \quad y = -4 \]

2. \(x + 6y = 13\)
   \[ -2x + 6y = -8 \]
   \[ 2x + 4y = 12 \]
   \[ 3x = 2y \quad \rightarrow \quad x = -3 \]

   \[ -2(3) + 4(-4) = 12 \]
   \[ 12 = 12 \]

3. \(7x + 6y = 13\)
   \[ -2(7) + 6(1) = -8 \]
   \[ 13 = 13 \]
   \[-8 = -8 \]
Example 3: Arrange like terms

Solve the linear system: 6x + 7y = 16  
\[ y = 6x - 32 \]

Solution

1. **Rewrite** Equation 2 so that the like terms are arranged in columns. **Put in Standard Form**

   \[
   6x + 7y = 16 \\
   y = 6x - 32 \\
   \frac{6x}{6} + \frac{7y}{6} = \frac{16}{6} \\
   \]

2. **Add** the equations.

   \[
   6x + 7y = 16 \\
   \underline{y = 6x - 32} \quad \frac{8y}{8} = \frac{-16}{8} \\
   \]

   The solution is \( (5, -2) \).

3. **Checkpoint** Solve the linear system.

   3. \( 4x - 5y = 5 \Rightarrow 4x - 5y = 5 \quad \frac{3x}{3} = \frac{15}{3} \quad x = 5 \)

   \[
   \begin{align*}
   4(5) - 5y &= 5 \\
   20 - 5y &= 5 \\
   -5y &= -15 \\
   y &= 3
   \end{align*}
   \]

   4. \( 7y = 4 - 2x \Rightarrow 2x + 7y = 4 \quad \frac{2x + 7y}{2} = \frac{12}{2} \quad y = 2 \)

   \[
   \begin{align*}
   7(2) &= 4 - 2x \\
   14 &= 4 - 2x \\
   -2x &= -10 \\
   x &= 5
   \end{align*}
   \]
When to use \textbf{Method} \\
(\textit{Equations must be in standard form}) \\
\textit{Ax + By = C}

\begin{enumerate}
\item \textbf{Addition:} USE when 1 of the variables has \underline{opposite coefficients}. \textit{Ex:}
\begin{align*}
2x + 2y &= 10 \\
3x - 2y &= 40
\end{align*}

\item \textbf{Subtraction:} Use when 1 of the variables has the same coefficient \textit{Ex:}
\begin{align*}
5x + 2y &= 10 \\
5x + 5y &= 50 \quad \textarrow{Multiply 1 equation by -1; Then Follow Addition method.}
\end{align*}

\item \textbf{Multiplication:} Use when you cannot use \underline{Addition or Subtraction methods}
\begin{align*}
2x + 4y &= 10 \\
-6x - 6y &= -12
\end{align*}
\textit{Goal is to eliminate 1 variable by multiplying for both equations to get opposite coefs for 1 of the variables}
\end{enumerate}