

7.3

Solve Linear Systems by Adding or Subtracting

Goal • Solve linear systems using elimination. (also called linear combination)

Your Notes

① EQUATIONS MUST BE IN STANDARD FORM

$$Ax + By = C$$

A, B, C are integers.

② 3 FLAVORS
• ADDITION
• SUBTRACTION
• MULTIPLICATION

SOLVING A LINEAR SYSTEM USING THE ELIMINATION METHOD

Step 1 ADD the equations to ELIMINATE one variable.

Step 2 Solve the resulting equation for the other variable.

Step 3 Substitute in either original equation to FIND THE VALUE OF THE OTHER VARIABLE

STEP 4: CHECK (x, y) IN BOTH ORIGINAL EQUATIONS

Example 1 Use addition to eliminate a variable

Solve the linear system: $x + 5y = 9$ Equation 1
 $4x - 5y = -14$ Equation 2

NOTICE!
1 variable (y)
has opposite coefficients

Solution

1. ADD the equations to eliminate one variable.

$$\begin{array}{r} x + 5y = 9 \\ 4x - 5y = -14 \\ \hline 5x = -5 \\ \frac{5x}{5} = \frac{-5}{5} \\ x = -1 \end{array}$$

y is eliminated

2. Solve for X

3. Substitute X = -1 in either equation

$$\begin{array}{r} x + 5y = 9 \\ -1 + 5y = 9 \\ \hline 5y = 10 \\ \frac{5y}{5} = \frac{10}{5} \\ y = 2 \end{array}$$

← PICK AN EQUATION.
Substitute X = -1
Solve for Y

The solution is (-1, 2).

Make sure to check your solution by substituting it into each of the original equations.

4. CHECK IN THE ORIGINAL EQ'S

$$\begin{aligned} a: -1 + 5(2) &= 9 \\ 9 &= 9 \checkmark \end{aligned}$$

$$\begin{aligned} c: 4(-1) - 5(2) &= -14 \\ -4 - 10 &= -14 \\ -14 &= -14 \checkmark \end{aligned}$$

(AKA MULTIPLY BY -1)

Example 2 Use subtraction to eliminate a variable

Solve the linear system:

$$3x - 4y = 2 \quad \text{Equation 1}$$

$$3x + 2y = 26 \quad \text{Equation 2}$$

NOTICE! 1 variable has the same COEF

Solution

1. **MULTIPLE 1 EQUATION BY -1** $3x - 4y = 2 \rightarrow 3x - 4y = 2$
 to eliminate one variable. $-1(3x + 2y = 26) \rightarrow -3x - 2y = -26$

$$\begin{array}{r} 3x - 4y = 2 \\ -3x - 2y = -26 \\ \hline -6y = -24 \\ \frac{-6y}{-6} = \frac{-24}{-6} \\ y = 4 \end{array}$$

2. Solve for y

3. Substitute 4 for y in either equation and SOLVE FOR THE OTHER VARIABLE (x)

Solve for x

$$\begin{array}{r} 3x + 2(4) = 26 \\ 3x + 8 = 26 \\ -8 \quad -8 \\ \hline 3x = 18 \\ \frac{3x}{3} = \frac{18}{3} \rightarrow x = 6 \end{array}$$

The solution is (6, 4).

4. CHECK:

$$c: 3(6) - 4(4) = 2 \quad 2 = 2 \checkmark$$

$$c: 3(6) + 2(4) = 26 \quad 26 = 26 \checkmark$$

Checkpoint Solve the linear system.

1. $-8x + 3y = 12$
 $8x - 9y = 12$

$$\begin{array}{r} -8x + 3y = 12 \\ 8x - 9y = 12 \\ \hline -6y = 24 \\ \frac{-6y}{-6} = \frac{24}{-6} \\ y = -4 \end{array}$$

NOTICE X has opposite COEF
ADDITION METHOD

FIND x:

$$\begin{array}{r} 8x - 9(-4) = 12 \\ 8x + 36 = 12 \\ -36 \quad -36 \\ \hline 8x = -24 \\ \frac{8x}{8} = \frac{-24}{8} \\ x = -3 \end{array}$$

c: $-8(-3) + 3(-4) = 12 \quad 12 = 12 \checkmark$

c: $8(-3) + -9(-4) = 12 \quad -24 + 36 = 12 \quad 12 = 12 \checkmark$

2. $x + 6y = 13$
 $-2x + 6y = -8$

NOTICE Y has same COEF
MULT 1 EQ BY -1

FIND x:

$$\begin{array}{r} x + 6y = 13 \\ -2x - 6y = -8 \\ \hline -x = 5 \\ x = -5 \end{array}$$

FIND Y

$$\begin{array}{r} x + 6y = 13 \\ -x - 6y = -5 \\ \hline 12y = 8 \\ \frac{12y}{12} = \frac{8}{12} \\ y = \frac{2}{3} \end{array}$$

c: $7 + 6(1) = 13 \quad 13 = 13 \checkmark$

c: $-2(7) + 6(1) = -8 \quad -8 = -8 \checkmark$

Example 3 Arrange like terms

Solve the linear system: $6x + 7y = 16$ Equation 1

NOT STD FORM $y = 6x - 32$ Equation 2

Solution

1. Rewrite Equation 2 so that the like terms are arranged in columns. POT IN STANDARD FORM

$6x + 7y = 16$

$6x + 7y = 16$ ↓ +

$y = 6x - 32$ → $-6x + y = -32$ ↓ +

2. ADD the equations.

$\frac{8y}{8} = \frac{-16}{8}$

3. Solve for OTHER VARIABLE (x)

$y = -2$

$6x + 7(-2) = 16$

$$\begin{array}{r} 6x - 14 = 16 \\ +14 \quad +14 \\ \hline 6x = 30 \\ \frac{6x}{6} = \frac{30}{6} \quad x = 5 \end{array}$$

The solution is $(5, -2)$.

4. CHECK: $C: 6(5) + 7(-2) = 16$

$30 - 14 = 16$

$+14 + 14$

$30 = 30 \checkmark$

$C: -2 = 6(5) - 32$

$-2 = -2 \checkmark$

$C: 4(5) - 5(3) = 5$

$5 = 5 \checkmark$

$C: 5(3) = 5 + 10$

$15 = 15 \checkmark$

Checkpoint Solve the linear system.

3. $4x - 5y = 5$

$\rightarrow 4x - 5y = 5$ ↓ +

$5y = x + 10$

$\rightarrow -x + 5y = 10$ ↓ +

$-x - x$

$3x = 15$

$x = 5$

FIND Y

$4(5) - 5y = 5$

$-20 - 5y = -15$

$y = 3$

4. $7y = 4 - 2x$

$\rightarrow 2x + 7y = 4$ ↓ +

$2x + y = -8$

$\rightarrow -1(2x + y = -8)$

$2x + 7y = 4$

$-2x - y = 8$ ↓ +

$6y = 12$

$y = 2$

FIND X:

$7(2) = 4 - 2x$

$10 = -2x$

$x = -5$

$C: 7(2) = 4 - 2(-5)$

$14 = 14 \checkmark$

$C: 2(-5) + 2 = -8$

$-8 = -8 \checkmark$

ELIMINATION

When to use [↑] Method

(EQUATIONS MUST BE IN STANDARD FORM)
 $Ax + By = C$

[1] Addition: USE when 1 of the variables has opposite coefficients. EX:

$$2x + 2y = 10$$

$$3x - 2y = 40$$

[2] Subtraction: Use when 1 of the variables has the same coefficient

EX] $5x + 2y = 10$

$$5x + 5y = 50$$

← multiply 1 equation by -1; then follow addition method.

[3] MULTIPLICATION: Use when you cannot use Addition or Subtraction Methods

$$\begin{array}{r} 2x + 4y = 10 \\ -6x - 6y = -12 \end{array}$$

Goal is to eliminate 1 variable by multiplying 1 or BOTH EQUATIONS TO GET OPPOSITE COEF'S FOR 1 OF THE VARIABLES