

# 6.4 Notes Solve Compound Inequalities

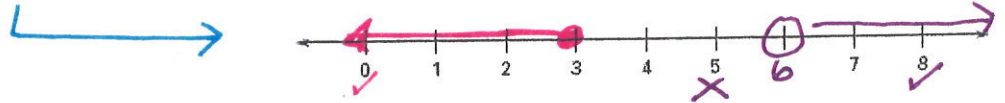
Goal • Solve and graph compound inequalities.

**VOCABULARY:** Compound inequality 2 INEQUALITIES  
CONNECTED WITH "AND" or "OR"

## Example 1 Write and graph compound inequalities

Translate the verbal phrase into a COMPOUND INEQUALITY. Then graph the inequality.

a. All real numbers that are less than or equal to 3 or greater than 6.  
 $x \leq 3$  OR  $x > 6$

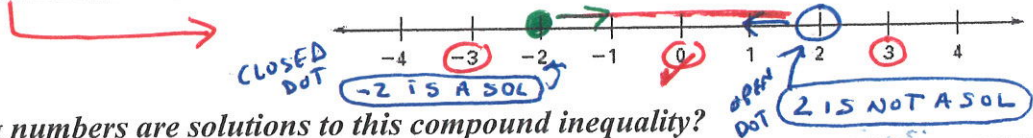


Which of the following numbers are solutions to this compound inequality?

For "OR" ONLY 1 OF THE INEQ MUST BE TRUE

$x=0$ $0 \leq 3$ OR $0 > 6$ T F <u><math>x=0</math> IS A SOLUTION</u>	$x=5$ $5 \leq 3$ OR $5 > 6$ F F	$x=8$ $8 \leq 3$ OR $8 > 6$ F T <u><math>x=8</math> IS A SOLUTION</u>
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b. All real numbers that are greater than or equal to -2 and less than 2.  
 $x \geq -2$  AND  $x < 2$



Which of the following numbers are solutions to this compound inequality?

For "AND" BOTH INEQ'S MUST BE TRUE

$x=-3$ $-3 \geq -2$ AND $-3 < 2$ F T	$x=0$ $0 \geq -2$ AND $0 < 2$ T T <u><math>x=0</math> IS A SOLUTION</u>	$x=3$ $3 \geq -2$ AND $3 < 2$ T F
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Translate the verbal phrase into a COMPOUND INEQUALITY. 2) Graph the inequality. 3) Write the COMPOUND INEQUALITY as a SINGLE INEQUALITY.

c. All real numbers that are greater than -8 and less than or equal to -3.

"AND" may be written as a compound or single INEQ.

Compound Inequality:  $x > -8$  AND  $x \leq -3$

Graph it:

Single Inequality:  $-8 < x \leq -3$

Labels: "small #", "big #", "EQUIVALENT STATE"

## VOCABULARY

- The solution of an "AND" COMPOUND INEQUALITY is the INTERSECTION of the 2 individual inequalities. To find the INTERSECTION, determine where the two graphs of the individual inequalities OVERLAP.
- The solution of an "OR" COMPOUND INEQUALITY is the UNION of the 2 individual inequalities. Solve the 2 inequalities separately and COMBINE the solutions TOGETHER.

**Example 2** Solve a compound inequality with "and"

Solve  $3x - 3 \geq 15$  and  $3x - 3 < 24$ . Graph your solution.

$$\begin{array}{r} 3x - 3 \geq 15 \quad \text{and} \quad 3x - 3 < 24 \\ \hline +3 \quad +3 \\ \hline 3x \geq 18 \quad \text{and} \quad 3x < 27 \\ \hline \frac{3x}{3} \geq \frac{18}{3} \quad \text{and} \quad \frac{3x}{3} < \frac{27}{3} \\ \hline x \geq 6 \quad \text{and} \quad x < 9 \end{array}$$

Solve the compound inequality.

Add 3 to each expression.

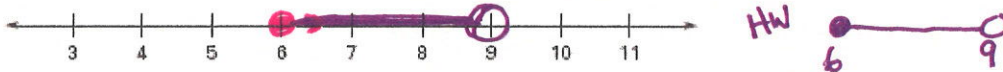
Divide each expression by 3.

$x \geq 6$  AND  $x < 9$

**THE SOLUTION MUST INCLUDE "AND"!!!**

The solutions are all real numbers Greater than or equal to 6 and less than 9.

Graph your solution.



The solution can also be written as a single "and" inequality:  $6 \leq x < 9$ .

**Example 3** Solve a single inequality with "and"

Solve  $15 < -7x + 1 < 50$ . Graph your solution.

$$\begin{array}{r} 15 < -7x + 1 < 50 \\ \hline -1 \quad -1 \quad -1 \\ \hline 14 < -7x < 49 \\ \hline \frac{14}{-7} < \frac{-7x}{-7} < \frac{49}{-7} \\ \hline -2 > x > -7 \end{array}$$

Solve the inequality.

Subtract 1 from each of the 3 parts of the inequality.

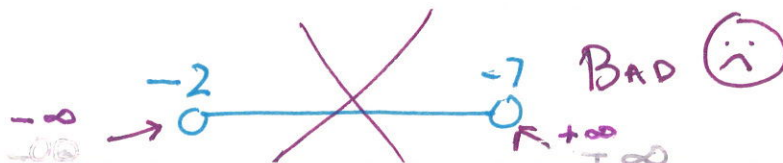
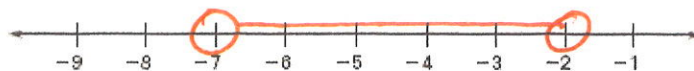
Divide each expression by -7 and reverse the symbols

Rewrite the inequality with "<" and "≤" symbols.

↳ Rewrite  $-7 < x < -2$  Read inside out

The solutions are all real numbers Greater than -7 and Less than -2.

Graph your solution.



**Example 4** Solve a compound inequality with "or"

Solve  $5x + 6 \leq -9$  or  $2x - 8 > 12$ . Graph your solution.

$$\begin{array}{r} 5x + 6 \leq -9 \\ -6 \quad -6 \\ \hline 5x \leq -15 \\ \frac{5x}{5} \leq \frac{-15}{5} \\ x \leq -3 \end{array} \quad \text{or} \quad \begin{array}{r} 2x - 8 > 12 \\ +8 \quad +8 \\ \hline 2x > 20 \\ \frac{2x}{2} > \frac{20}{2} \\ x > 10 \end{array}$$

Solve the inequality.

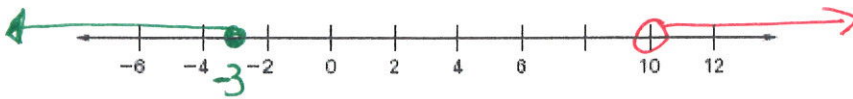
Use addition or subtraction property of inequality.

Use division property of inequality.

$x \leq -3$  OR  $x > 10$

← You must include "or"!!!

The solutions are all real numbers less than or equal to -3 or greater than 10.



✓ **Checkpoint** Solve the inequality. Graph your solution.

**DO HW LIKE THIS**

1.  $-3 \leq -2x + 1 < 11$

$$\begin{array}{r} -3 \leq -2x + 1 < 11 \\ -1 \quad -1 \quad -1 \\ \hline -4 \leq -2x < 10 \\ \frac{-4}{-2} \leq \frac{-2x}{-2} < \frac{10}{-2} \\ 2 \geq x > -5 \end{array}$$



remember to switch symbols

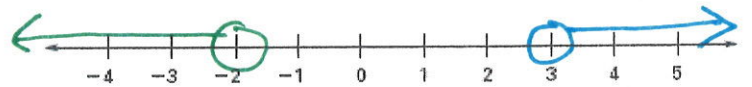
$2 \geq x > -5$

rewrite with  $<, \leq$  !!!

$-5 < x \leq 2$

2.  $9x + 1 < -17$  or  $7x - 12 > 9$

$$\begin{array}{r} 9x + 1 < -17 \\ -1 \quad -1 \\ \hline 9x < -18 \\ \frac{9x}{9} < \frac{-18}{9} \\ x < -2 \end{array} \quad \text{or} \quad \begin{array}{r} 7x - 12 > 9 \\ +12 \quad +12 \\ \hline 7x > 21 \\ \frac{7x}{7} > \frac{21}{7} \\ x > 3 \end{array}$$



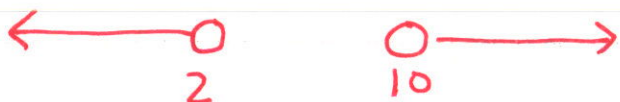
$x < -2$  OR  $x > 3$

## Special Cases

GRAPH AND  
DETERMINE  
SOLUTION

### 1) "And" Special Case

$$X < 2 \text{ AND } X > 10$$



∴ "AND" means INTERSECT!

Since the 2 inequalities do NOT INTERSECT THERE IS NO SOLUTION.

$X = \text{NO SOLUTION}$  ← is the answer

### 2) "Or" Special Case

$$X < 10 \text{ OR } X > 2$$



∴ "OR" means the UNION!

Since the 2 inequalities OVER LAP, THE ENTIRE NUMBER LINE IS COVERED THEREFORE:

$X = \text{ALL REAL NUMBERS}$