Write Equations of Parallel and Perpendicular Lines

**Goal**  •  Write equations of parallel and perpendicular lines.

**VOCABULARY**

<table>
<thead>
<tr>
<th>reciprocal</th>
<th>reciprocal with opposite signs</th>
</tr>
</thead>
<tbody>
<tr>
<td>negative reciprocal</td>
<td>negative reciprocal with opposite signs</td>
</tr>
</tbody>
</table>

Perpendicular lines  •  2 lines that intersect and form a right (90°) angle (⊥)

**Examples**

<table>
<thead>
<tr>
<th>Given m</th>
<th>negative reciprocal</th>
<th>reciprocal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = -\frac{5}{3} )</td>
<td>( \frac{1}{5} )</td>
<td>( \frac{3}{5} )</td>
</tr>
<tr>
<td>( m = \frac{1}{2} )</td>
<td>-2</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( m = \frac{3}{4} )</td>
<td>-( \frac{4}{3} )</td>
<td>( \frac{4}{3} )</td>
</tr>
<tr>
<td>( m = -\frac{4}{6} )</td>
<td>( \frac{6}{4} )</td>
<td>( \frac{3}{2} )</td>
</tr>
</tbody>
</table>

**PARALLEL LINES** // lines

If two nonvertical lines have the same slope, then they are parallel.

If two nonvertical lines are //, then they have the same slope.

**Example 1**  Write an equation of a parallel line

Write an equation of the line that passes through \((2, 4)\) and is parallel to the line \(y = 4x + 1\).

**Solution**

1. **Step 1**  Identify the slope: The graph of the given equation has a slope of \(4\). So, the parallel line through \((2, 4)\) has a slope of \(4\).

2. **Step 2**  Write equation in point-slope form: \(y - y_1 = m(x - x_1)\).

   \[ m = 4 \]
   \[ y = 4 = 4(x - 2) \]

3. **Step 3**  Write an equation in slope-intercept form: \(y = mx + b\).

   \[ y - 4 = 4x - 8 \]
   \[ y = 4x - 4 \]

4. **Step 4**  Write equation in standard form: \(Ax + By = C\).

   \[ 4x - 4y = -4 \]

**Notice**  // lines:
- Given: \(y = 4x + 1\)
- Found // line: \(y = 4x - 4\)
**Checkpoint** Complete the following exercises.

1. Write an equation of the line that passes through \((-4, 6)\) and is parallel to the line \(y = -3x + 2\). Given Line

   \[
   \text{We want a line through pt } (-4, 6)
   \]

   \[
   \text{1. } m = -3
   \]

   \[
   \text{2. put into p1s: } y - 6 = -3(x + 4)
   \]

   \[
   \text{3. if asked put into slope-intercept}
   \]

   \[
   y - 6 = -3x - 12
   \]

   \[
   \frac{+6}{+6}
   \]

   \[
   y = -3x - 6
   \]

**Perpendicular Lines**

If two nonvertical lines have the slopes that are **negative reciprocals**, then the lines are **perpendicular**. A horizontal line and a vertical line are **always perpendicular**.

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**Example 2** Determine parallel or perpendicular lines

Determine which of the following lines, if any, are parallel or perpendicular:

Line a: \(12x - 3y = 3\) \(\leftarrow m = 4\)

Line b: \(y = 4x + 2\) \(\leftarrow y = mx + b\) \(m = 4\)

Line c: \(4y + x = 8\) \(\leftarrow m = -\frac{1}{4}\)

**Solution**

**Find the slopes of the lines.**

Line b: The equation is in slope-intercept form. The slope is \(\frac{4}{1}\).

**Write the equations for lines a and c in slope-intercept form.**

Line a: \(12x - 3y = 3\)

\[
\frac{-12x}{-3} = \frac{-3y}{-3} = \frac{3}{-3}
\]

\[
y = 4x - 1
\]

Line c: \(4y + x = 8\)

\[
\frac{4y}{4} = \frac{-x}{4} + \frac{8}{4}
\]

\[
y = -\frac{1}{4}x + 2
\]

Lines a and b have a slope of \(\frac{4}{1}\), so they are **parallel**.

Line c has a slope of \(-\frac{1}{4}\), the negative reciprocal of \(\frac{1}{4}\), so it is **perpendicular** to lines a and b.
1. Find slope:

IF NECESSARY PUT INTO S/I \( y = mx + b \)

2. Determine which of the following lines, if any, are parallel or perpendicular.

Line a: \( 4x + y = 2 \) \( m = -4 \)
Line b: \( 5y + 20x = 10 \) \( m = -4 \)
Line c: \( 8y = 2x + 8 \) \( m = 1/4 \)

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Example 3 Determine whether lines are perpendicular

Determine if the following lines are perpendicular.

Line a: \( 6y = 5x + 8 \)
Line b: \( -10y = 12x + 10 \)

Solution

Find the slopes of the lines. Write the equations in slope-intercept form.

Line a: \( 6y = 5x + 8 \)
\[ y = \frac{5}{6}x + \frac{8}{6} \]
\[ m = \frac{5}{6} \]

Line b: \( -10y = 12x + 10 \)
\[ y = -\frac{6}{5}x - 1 \]
\[ m = -\frac{6}{5} \]

The slope of line a is \( \frac{5}{6} \). The slope of line b is \( -\frac{6}{5} \).

The two slopes \( \text{are} \) negative reciprocals, so lines a and b \( \text{are} \) perpendicular.

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Check your answer using slopes.

\( M = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \)

Checkpoint Complete the following exercises.

3. Determine whether line a through (1, 3) and (3, 4) is perpendicular to line b through (1, -3) and (2, -5).

Justify your answer using slopes.

Line a: \( (1,3) \) (3, 4)
\[ m = \frac{3 - 4}{1 - 3} = \frac{-1}{-2} \]
\[ m = \frac{1}{2} \]

Line b: \( (1,-3) \) (2, -5)
\[ m = \frac{-3 + 5}{1 - 2} = \frac{2}{-1} \]
\[ m = -2 \]

Lines A and B are \( \bot \)
Example 4: Write an equation of a perpendicular line in all 3 linear eq. forms.

Write an equation of the line that passes through \((-3, 4)\) and is perpendicular to the line \(y = \frac{1}{3}x + 2\).

Solution:

1. **Step 1**: Identify the slope. The graph of the given equation has a slope of \(\frac{1}{3}\). Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line through \((-3, 4)\) is \(-3\) (\(\perp m = -3\)).

2. **Step 2**: Write equation in point-slope form: \(P/S\) \(y - y_1 = m(x - x_1)\)

\[P/S: \quad y - 4 = -3(x + 3)\]

3. **Step 3**: Write equation in slope-intercept form: \(S/I\) \(y = mx + b\)

\[S/I: \quad y = -3x - 5 \quad \text{b} = -5\]

4. **Step 4**: Write equation in standard form: \(S/T\) \(Ax + By = C\)

\[S/T: \quad 3x + y = -5\]

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Homework:

5.5 HW:

- Pg 321
- #5 1, 2, 3, 24 (3x), and 35

4. Write an equation of the line that passes through \((4, -2)\) and is perpendicular to the line \(y = 5x + 2\).

\[\perp m = -\frac{1}{5}\]

\[P:\quad (4, -2)\]

**IN ALL 3 FORMS:**

- **P/S**: \(y + 2 = -\frac{1}{5}(x - 4)\)
  \[y + 2 = -\frac{1}{5}x + \frac{4}{5}\]

- **S/I** \(\frac{1}{5}x + y = -\frac{6}{5}\)

- **STANDARD**: \(x + 5y = -6\)