

# Write Equations of Parallel and Perpendicular Lines

Goal • Write equations of parallel and perpendicular lines.

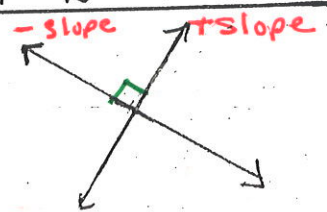
EXAMPLES

GIVEN $m$	NEGATIVE RECIPROCAL $m$
$m = -5$	$\perp m = +1/5$
$m = 1/2$	$\perp m = -2$
$m = 3/4$	$\perp m = -4/3$
$m = -4/5$	$\perp m = 5/4$

VOCABULARY

NEGATIVE RECIPROCAL: **RECIPROCAL WITH OPPOSITE SIGNS.**

Perpendicular lines **2 LINES THAT INTERSECT AND FORM A RIGHT (90°) ANGLE (⊥)**

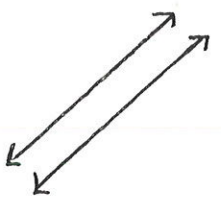


⊥ LINES

PARALLEL LINES // lines

If two nonvertical lines have the same slope, then they are parallel

If two nonvertical lines are //, then they have the same m.



Example 1 Write an equation of a parallel line

Write an equation of the line that passes through (2, 4) and is parallel to the line  $y = 4x + 1$ . (LI) ← Given

**Solution**  
 Step 1 Identify the slope. The graph of the given equation has a slope of 4. So, the parallel line through (2, 4) has a slope of 4. (LI)

Step 2 WRITE EQUATION IN POINT-SLOPE FORM:  $y - y_1 = m(x - x_1)$

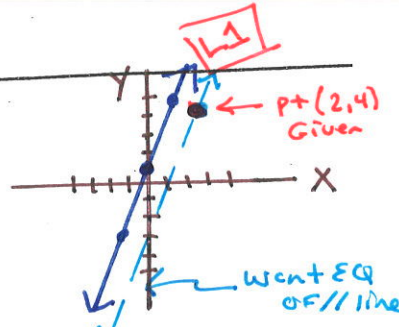
$m = 4$   
 pt (2, 4)  
 $y - 4 = 4(x - 2)$

Step 3 Write an equation in SLOPE-INTERCEPT FORM:  $y = mx + b$

$y - 4 = 4x - 8$   
 $+4 \quad +4$   
 $y = 4x - 4$

STEP 4 WRITE EQUATION IN STANDARD FORM:  $AX + BY = C$

$y = 4x - 4$   
 $-4x \quad -4x$   
 $-4x + y = -4$



Your Notes  
 Reciprocal -  
 FLIP THE FRACTION  
 AND KEEP THE  
 SIGN

MULTIPLY

$m = -5 \quad \perp m = 1/5$   
 $-5(1/5) = -1$   


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 $1/2(-2) = -1$   


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 $3/4(-4/3) = -1$   
 $-4/5(5/4) = -1$

Rule ⊥ lines  
 $m \cdot \perp m = -1$

NOTICE // lines:  
 Given:  $y = 4x + 1$   
 Found // line  
 $y = 4x - 4$

Checkpoint Complete the following exercises.

1. Write an equation of the line that passes through  $(-4, 6)$  and is parallel to the line  $y = -3x + 2$ . **Given Line**

We want a // line through pt  $(-4, 6)$

① //  $m = -3$

② put into PLS:  $y - 6 = -3(x + 4)$

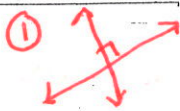
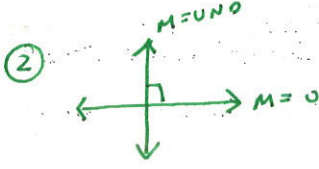
③ IF ASKED PUT INTO SLOPE-INTERCEPT

$$\begin{array}{r} y - 6 = -3x - 12 \\ +6 \quad \quad +6 \\ \hline y = -3x - 6 \end{array}$$

**PERPENDICULAR LINES ( $\perp$ )**

① If two nonvertical lines have the slopes that are **NEGATIVE RECIPROCAL**s then the lines are **PERPENDICULAR**.

② A **HORIZONTAL** LINE AND A **VERTICAL** LINE ARE ALWAYS **PERPENDICULAR**.

**Example 2** Determine parallel or perpendicular lines

Determine which of the following lines, if any, are parallel or perpendicular:

Line a:  $12x - 3y = 3$   $\leftarrow m = 4$

Line b:  $y = 4x + 2$   $\leftarrow y = mx + b$   $m = 4$

Line c:  $4y + x = 8$   $\leftarrow m = -\frac{1}{4}$

**Solution**

Find the slopes of the lines.

Line b: The equation is in slope-intercept form. The slope is  $4$ .

Write the equations for lines a and c in slope-intercept form.

Line a:  $12x - 3y = 3$

$$\begin{array}{r} -12x \quad -14 \\ \hline -3y = \frac{-12x}{-3} + \frac{3}{-3} \\ y = \quad \quad \quad \end{array} \rightarrow y = 4x - 1$$

Line c:  $4y + x = 8$

$$\begin{array}{r} -x \quad -x \\ \hline 4y = \frac{-x}{4} + \frac{8}{4} \\ y = \quad \quad \quad \end{array} \rightarrow y = -\frac{1}{4}x + 2$$

\* Lines a and b have a slope of  $4$ , so they are **parallel**.

\* Line c has a slope of  $-\frac{1}{4}$ , the negative reciprocal of  $4$ , so it is **perpendicular** to lines a and b.

FIND THE SLOPES

// lines have the same slope

$\perp$  lines have slopes that are negative reciprocals.

STEP I

STEP II

PUT IN  $y = mx + b$

Checkpoint Complete the following exercises.

- ① FIND SLOPE  
 ✓ ② IF NEEDED PUT INTO S/I ( $y = mx + b$ )

2. Determine which of the following lines, if any, are parallel or perpendicular.

Line a:  $4x + y = 2$   $m = -4$   
 Line b:  $5y + 20x = 10$   $m = -4$   
 Line c:  $8y = 2x + 8$   $m = 1/4$

**A**  $4x + y = 2$   
 $-4x \quad -4x$   
 $y = -4x + 2$

**B**  $5y + 20x = 10$   
 $-20x \quad -20x$   
 $5y = -20x + 10$   
 $\frac{5y}{5} = \frac{-20x}{5} + \frac{10}{5}$   
 $y = -4x + 2$

**C**  $8y = 2x + 8$   
 $\frac{8y}{8} = \frac{2x}{8} + \frac{8}{8}$   
 $y = \frac{1}{4}x + 1$

**Example 3** Determine whether lines are perpendicular

Determine if the following lines are perpendicular.

Line a:  $6y = 5x + 8$   
 Line b:  $-10y = 12x + 10$

**Solution**

Find the slopes of the lines. Write the equations in slope-intercept form.

Line a:  $6y = 5x + 8$   
 $\frac{6y}{6} = \frac{5x}{6} + \frac{8}{6}$   
 $y = \frac{5}{6}x + \frac{8}{6}$   $m = \frac{5}{6}$

Line b:  $-10y = 12x + 10$   
 $\frac{-10y}{-10} = \frac{12x}{-10} + \frac{10}{-10}$   
 $y = -\frac{6}{5}x - 1$   $m = -\frac{6}{5}$

The slope of line a is  $\frac{5}{6}$ . The slope of line b is  $-\frac{6}{5}$ .

The two slopes **ARE** negative reciprocals, so lines a and b **ARE** perpendicular.

↑ opposite signs

$\frac{5}{6}$  and  $\frac{6}{5}$  are reciprocals (flip fraction)

**Answer**

- Lines A and B are  $\parallel$
- Line A and C are  $\perp$
- Line B and C are  $\perp$

**Notice**

\* MULTIPLY THE SLOPES

$\frac{5}{6} \cdot -\frac{6}{5} = -1$

ALL  $\perp$  slopes with NEGATIVE RECIPROCALLS MULTIPLIED = -1

Checkpoint Complete the following exercises.

3. Determine whether line a through (1, 3) and (3, 4) is perpendicular to line b through (1, -3) and (2, -5). Justify your answer using slopes.

LINE A: (1, 3) (3, 4)

$m = \frac{3-4}{1-3} = \frac{-1}{-2}$

$m = 1/2$

LINE B: (1, -3) (2, -5)

$m = \frac{-3-5}{1-2} = \frac{-8}{-1}$

$m = -8$

Lines A and B are  $\perp$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

**Example 4** Write an equation of a perpendicular line in ALL 3 LINEAR EQ. FORMS

Write an equation of the line that passes through  $(-3, 4)$  and is perpendicular to the line  $y = \frac{1}{3}x + 2$ .

**Solution**

**Step 1** Identify the slope. The graph of the given equation has a slope of  $\frac{1}{3}$ . Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line through  $(-3, 4)$  is  $-\frac{3}{1}$ .  $\perp m = -3$

**Step 2** WRITE EQUATION IN POINT-SLOPE FORM:  $\boxed{\text{P/S}} y - y_1 = m(x - x_1)$

P/S:  $y - 4 = -3(x + 3)$

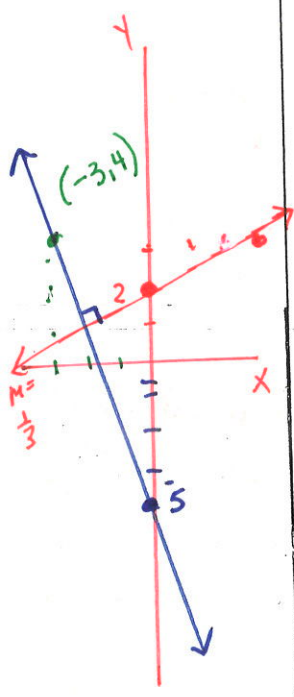
**STEP 3** WRITE EQUATION IN SLOPE-INTERCEPT FORM:  $\boxed{\text{S/I}} y = mx + b$

$$\begin{array}{r} y - 4 = -3x - 9 \\ +4 \qquad \qquad +4 \\ \hline \end{array}$$

S/I  $\boxed{y = -3x - 5}$   $b = -5$

**STEP 4** WRITE EQUATION IN STANDARD FORM:  $\boxed{\text{STD}} Ax + By = C$

STD Form  $\boxed{3x + y = -5}$



**Homework**

5.5 HW:

Pg 321

#1's 1, 2, 1

3-24 (3x)

AND 35

4. Write an equation of the line that passes through  $(4, -2)$  and is perpendicular to the line  $y = 5x + 2$ .

$\perp m = -\frac{1}{5}$

pt  $(4, -2)$

IN ALL 3 FORMS:

P/S:  $y + 2 = -\frac{1}{5}(x - 4)$   
 $y + 2 = -\frac{1}{5}x + \frac{4}{5}$   
 $-2 \qquad \qquad -2$

S/I  $y = -\frac{1}{5}x - \frac{1}{5}$   
 $+\frac{1}{5}x \quad +\frac{1}{5}x$

5  $(\frac{1}{5}x + y = -\frac{6}{5})$

STD Form  $x + 5y = -6$

$\perp$  P/S:  $y + 2 = -\frac{1}{5}(x - 4)$

$\perp$  S/I:  $y = -\frac{1}{5}x - \frac{1}{5}$

$\perp$  STANDARD:  $x + 5y = -6$