5.3 Write Linear Equations in Point-Slope Form

Goal 
- Write linear equations in point-slope form.

VOCABULARY

2 Forms of Linear Equations That Describe a Given Line.

Point-slope form: \[ y - y_1 = m(x - x_1) \]
- Coordinates of a given point \((x_1, y_1)\)
- \(m = \text{slope}\)

Slope-intercept form: \[ y = mx + b \]
- \(m = \text{slope}\)
- \(b = \text{y-intercept}\)
- \(x\) and \(y\) variables

POINT-SLOPE FORM

The point-slope form of the equation of the nonvertical line through a given point \((x_1, y_1)\) with a slope of \(m\) is

\[ y - y_1 = m(x - x_1) \]

Example 1 Write an equation in point-slope form

Write an equation in point-slope form on the line that passes through the point \((3, 2)\) and has a slope of 2.

Solution

Write point-slope form.

\[ y - y_1 = m(x - x_1) \]

Substitute \(2\) for \(m\), \(3\) for \(x_1\), and \(2\) for \(y_1\).

Point \((3, 2)\)

\[ m = 2 \]
Example 2

Graph the equation \( y - 2 = \frac{1}{2}(x - 2) \).

Solution

Because the equation is in point-slope form, you know that the line has a slope of \( \frac{1}{2} \) and passes through the point \( (2, 2) \).

\[ x_1 = 2 \quad y_1 = 2 \]

Plot the point \( (2, 2) \). Find a second point on the line using the slope. Draw a line through the points.

**Checkpoint** Complete the following exercises.

1. Write an equation in point-slope form of the line that passes through the point \((-3, 5)\) and has a slope of 4.

\[ y - 5 = 4(x + 3) \]

2. Graph the equation \( y - 4 = 2(x - 1) \).

\[ \text{m} = \frac{3}{1} \]

\[ \text{point} \ (1, -1) \]
Example 3 Use point-slope form to write an equation

Write an equation in point-slope form of the line shown.

Solution

Step 1 Find the slope of the line.

\[ m = \frac{\text{Rise}}{\text{Run}} \]

\[ m = \frac{5}{6} \]

Step 2 Write the equation in point-slope form.

You can use either given point.

Method 1 Use \((-2, -3)\)

\[ y - y_1 = m(x - x_1) \]

\[ y - (-3) = \frac{5}{6}(x - (-2)) \]

Method 2 Use \((4, 2)\)

\[ y - y_1 = m(x - x_1) \]

\[ y - 2 = \frac{5}{6}(x - 4) \]

CHECK Uncheck that the equations are equivalent by writing them in slope-intercept form.

\[ y + \frac{3}{2} = \frac{5}{6}x + \frac{5}{3} \]

\[ y = \frac{5}{6}x - \frac{4}{3} \]

\[ y - \frac{2}{2} = \frac{5}{6}x - \frac{10}{3} + \frac{2}{2} \]

\[ y = \frac{5}{6}x - \frac{4}{3} \]

Same

\[ Y_{\text{int}} = b = -\frac{4}{3} = -1 \frac{1}{3} \]

Scrappy Work: \(-3 \frac{1}{2} + 2 = -1 \frac{1}{2} = -\frac{4}{3} \)
### 5.2 Use Linear Equations in Slope-Intercept Form (Cont.)

**Goal:** Write an equation of a line using points on the line.

**Example 4** Write an equation given two points

Write an equation of the line that passes through \((2, -3)\) and \((\text{-}2, 1)\).

**Solution**

**Step 1** Calculate the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{-3 - 1}{2 + 2} = \frac{-4}{4} = -1
\]

**Step 2** Put in Point Slope Form.

Pick either point. Use the slope and the point \((2, -3)\).

\[y - y_1 = m(x - x_1)\]

\[y + 3 = -1(x - 2)\]

**Step 3** Write an equation of the line. In Slope Intercepts (When Asked)

Write slope-intercept form.

\[y = -x - 1\]

What is a 3rd equation to describe this line?

\[y - 1 = -1(x + 2)\]

\[y = -x - 3\]
**Checkpoint** Complete the following exercise.

2. Write an equation for the line that passes through \((-8, -13)\) and \((4, 2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13 - 2}{-8 - 4} = \frac{-15}{-12} = \frac{5}{4}
\]

\[
\text{Pls } y - 2 = \frac{5}{4}(x - 4)
\]

\[
y = \frac{5}{4}x - 3
\]

3. Write an equation for the line that passes through \((-3, 4)\) and \((1, -2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{1 - (-3)} = \frac{-6}{4} = \frac{-3}{2}
\]

\[
\text{Pls } y - 4 = \frac{-3}{2}(x + 3)
\]

\[
y = \frac{-3}{2}x - \frac{1}{2}
\]

**HOW TO WRITE EQUATIONS IN SLOPE-INTERCEPT FORM**

1. **Given slope** \(m\) and **y-intercept** \(b\).

   \[
   m + b \text{ into S/I Eq } y = mx + b
   \]

2. **Given slope** \(m\) and one point.
   
   \[a\] plug \(m\) and \((x_1, y_1)\) into Pls
   
   \[b\] use algebra to put into \(y = mx + b\)

3. **Given two points.**
   
   \[a\] use the points to find the slope \(m = \frac{\Delta y}{\Delta x}\)
   
   \[b\] pick a point and plug into Pls \(y - y_1 = m(x - x_1)\)
   
   \[c\] use algebra to put into \(y = mx + b\)

\[\text{STEP 1}
\text{To find slope}
\]

[Given a graph]

\[M = \frac{\text{Rise}}{\text{Run}}\]

[Given 2 points]

\[M = \frac{\Delta y}{\Delta x}\]
Writing Equations for Horizontal and Vertical Lines

Your Notes:

- The equation of a **Horizontal Line** is $y = b$, where $b = y$-intercept and the slope is **Zero**.
- The equation of a **Vertical Line** is $x = a$, where $a = x$-intercept and the slope is **Undefined**.

**Example 1**

- Write the equation of the line that goes through the points (5, -2) and (5, 2)

  **Step 1:** Find $m = \frac{\Delta y}{\Delta x} = \frac{-2 - 2}{5 - 5} = \frac{-4}{0}$  
  $m = \text{Undefined}$

  **Step 2:** Sketch a graph

  **Step 3:** All vertical lines have the eq $x = a$ where $a = x$-int

**Example 2**

- Write the equation of the line that goes through the points (-3, 2) and (3, 2)

  **Step 1:** Find $m = \frac{\Delta y}{\Delta x} = \frac{2 - 2}{-3 - 3} = \frac{0}{-6} = 0$

  **Step 2:** Sketch a graph

  **Step 3:** Fill in $y = mx + b$
  
  $m = 0$, $b = 2$
  
  $y = 0x + 2$
  
  $Y = 2$

**Example 3**

- Write the equation of for the Horizontal and Vertical Lines that go through the point (5, 10)

  **Horizontal Line** $y = 10$

  **Vertical Line** $x = 5$