

# 12.7

## Solve Rational Equations

Goal • Solve rational equations.

Your Notes

### VOCABULARY

Rational equation HAVE 2 OR MORE RATIONAL EXPRESSIONS (AKA FRACTIONS WITH VARIABLES)

EX:  $\frac{5}{x-1} = \frac{x}{4}$

### Review Proportions

Solve

$$\frac{2}{5} = \frac{x}{60}$$

$$2(60) = 5 \cdot x$$

$$\frac{120}{5} = \frac{5x}{5}$$

$$x = 24$$

C:  $\frac{2}{5} = \frac{24}{60}$   
 $.4 = .4$  ✓

### STEPS

- ① Cross multiply
- ② Divide
- ③ Check

### Example 1 Use the cross products property

Solve  $\frac{5}{x-1} = \frac{x}{4}$ . Check your solution.

Solution

$$\frac{5}{x-1} = \frac{x}{4}$$

$$20 = x(x-1)$$

$$20 = x^2 - x$$

$$0 = x^2 - x - 20$$

$$0 = (x+4)(x-5)$$

$$x+4 = 0 \text{ or } x-5 = 0$$

$$x = -4 \text{ or } x = 5$$

The solutions are  $-4$  and  $5$ .

CHECK If  $x = -4$ :

$$\frac{5}{-4-1} \stackrel{?}{=} \frac{-4}{4}$$

$$\frac{5}{-5} = \frac{-4}{4}$$

$$-1 = -1 \checkmark$$

If  $x = 5$ :

$$\frac{5}{5-1} \stackrel{?}{=} \frac{5}{4}$$

$$\frac{5}{4} = \frac{5}{4} \checkmark$$

Write original equation.

Cross products property

Subtract 20 from each side.

Factor polynomial.

Zero-product property

Solve for x.

SOLVE THE Q.E.  $Ax^2 + Bx + C = 0$   
 SOLVE BY FACTORING OR USE Q.F.  
 $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Checkpoint Solve the equation. Check your solution.

1.  $\frac{-2}{x+9} = \frac{x}{7}$

$-14 = x^2 + 9x$

$0 = x^2 + 9x + 14$

$0 = (x+7)(x+2)$

$x = -7, -2$

C:  $\frac{-2}{2} = \frac{-7}{7}$   
 $-1 = -1$

C:  $\frac{-2}{7} = \frac{-2}{7}$  ✓

2.  $\frac{6}{x-4} = \frac{3}{x}$

$6x = 3(x-4)$   
 $6x = 3x - 12$   
 $-3x - 3x$

$3x = -12$   
 $\frac{3x}{3} = \frac{-12}{3}$

$x = -4$

C:  $\frac{6}{-8} = \frac{3}{-4}$

$-.75 = -.75$  ✓

**Example 2** FACTOR TO FIND LCD

Solve

$\frac{x+6}{2x+12} = \frac{4}{x+6}$   
 $\frac{x+6}{2(x+6)}$

STEP I

Write each denominator in factored form. The LCD is

LCD = 2(x+6)

THE LCD MUST INCLUDE EVERY FACTOR!

STEP II

MULT. THE EQUATION BY THE LCD TO ELIMINATE THE DEN.

$2(x+6) \left[ \frac{x+6}{2(x+6)} = \frac{4}{x+6} \right]$

Mental Step

$2(x+6) \cdot \frac{x+6}{2(x+6)} = 2(x+6) \cdot \frac{4}{(x+6)}$

STEP III

Solve the transformed EQUATION

$x+6 = 2 \cdot 4$

$x+6 = 8$   
 $-6 -6$

$x = 2$

STEP IV

Check:

$\frac{2+6}{2(2)+12} = \frac{4}{2+6}$   
 $\frac{8}{16} = \frac{4}{8} = \frac{1}{2}$  ✓

ANSWER

$x = 2$

D:  $x \neq -6$

**Example 3** SOLVE RATIONAL EQUATIONS

Solve:

$$4x^2 \left[ \frac{1}{4} - \frac{1}{2x^2} = \frac{1}{4x} \right]$$

STEP 1: MULTIPLY THE EQUATION BY THE LCD.

LCD =  $\frac{4x^2}{}$

$$\frac{4x^2}{1} \left( \frac{1}{4} \right) + \frac{4x^2}{1} \left( \frac{-1}{2x^2} \right) = \frac{4x^2}{1} \left( \frac{1}{4x} \right)$$

$$x^2 - 2 = x$$

STEP 2: SOLVE THE TRANSFORMED EQUATION

$$x^2 - x - 2 = 0$$

SOLVE BY FACTORING OR QF

$$(x-2)(x+1) = 0$$

STEP 3: CHECK FOR EXTRANEOUS SOLUTIONS IN THE ORIGINAL EQUATION

$x=2$        $x=-1$

C:  $\frac{1}{4} - \frac{1}{2(2)^2} = \frac{1}{4(2)}$   
 $\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$   
 $\frac{1}{8} = \frac{1}{8} \checkmark$   
 $.125 = .125 \checkmark$

C:  $\frac{1}{4} - \frac{1}{2(-1)^2} = \frac{1}{4(-1)}$   
 $\frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$   
 $-\frac{1}{4} = -\frac{1}{4} \checkmark$

Checkpoint Complete the following exercise.

3. Solve:

Check your solution.

$$\frac{3b}{1} \left[ \frac{b+2}{3} + \frac{b+5}{3b} = \frac{1}{b} \right]$$

LCD =  $3b$

$$3b \left( \frac{b+2}{3} \right) + 3b \left( \frac{b+5}{3b} \right) = 3b \left( \frac{1}{b} \right)$$

C:  $B = -1$   
 $\frac{1}{3} + \frac{4}{-3} = \frac{1}{-1}$   
 $-1 = -1 \checkmark$

$b(b+2) + (b+5) = 3$  ← NOW SOLVE

$$b^2 + 2b + b + 5 = 3$$

$$b^2 + 3b + 5 = 3$$

$$b^2 + 3b + 2 = 0$$

Solve the QE

$$(b+2)(b+1) = 0$$

$b = -1, -2$

C:  $B = -2$   
 $\frac{0}{2} + \frac{3}{-6} = \frac{1}{-2}$   
 $-\frac{1}{2} = -\frac{1}{2}$

12.7 Practice Problems

Solve each equation. State the LCD. Remember to check for extraneous solutions.

$$1) \left[ \frac{x+4}{3x^2} - \frac{1}{2x} = \frac{1}{6x} \right]$$

**LCD =  $6x^2$**

$$2(x+4) - 3x = x$$

$$2x + 8 - 3x = x$$

$$-x + 8 = x$$

$$\frac{8}{2} = \frac{2x}{2}$$

**$x = 4$**

C:  $\frac{8}{48} - \frac{1}{8} = \frac{1}{24}$

$.041\bar{6} = .041\bar{6}$

$$2) \left[ \frac{x+5}{x^2} - \frac{x+6}{x^2} = \frac{1}{x} \right]$$

**LCD =  $x^2$**

$$(x+5) - (x+6) = x$$

$$x+5 - x - 6 = x$$

**$x = -1$**

C:  $\frac{4}{1} - \frac{5}{1} = -1$

$-1 = -1 \checkmark$

$$3) \left[ \frac{1}{2r^2} + \frac{1}{6} = \frac{2}{3} \right]$$

**LCD =  $6r^2$**

$$3 + \frac{R^2}{-R^2} = \frac{4R^2}{-R^2} \leftarrow \text{ISOLATE } R^2$$

$$\frac{3}{3} = \frac{3R^2}{3}$$

$$R^2 = 1$$

**$R = \pm 1$**

C:  $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$.6\bar{6} = .6\bar{6}$

$$4) \left[ \frac{1}{n} - \frac{1}{4} = \frac{1}{n^2} \right]$$

**LCD =  $4n^2$**

$$4n - n^2 = 4 \leftarrow \text{Tip: Want } -n^2 \text{ to have a +coef}$$

$$0 = n^2 - 4n + 4$$

$$0 = (n-2)(n-2)$$

**$n = 2$**

$$5) \left[ \frac{1}{4} - \frac{2}{n^2} = \frac{1}{4n^2} \right]$$

**LCD =  $4n^2$**

$$n^2 - 8 = 1$$

$$n^2 = 9$$

**$n = \pm 3$**

FACTOR  $(n+3)(n-3) = 0$

OR  
ISOLATE  $n^2$

C:  $\frac{1}{4} - \frac{2}{9} = \frac{1}{36}$

$.028 = .028 \checkmark$

C:  $\frac{1}{2} - \frac{1}{4} = \frac{1}{2}$

$.25 = .25 \checkmark$

$$6) \left[ \frac{x^2 - 3x + 2}{x} = \frac{6}{5} - \frac{8}{5x} \right]$$

**LCD =  $5x$**

$$5(x^2 - 3x + 2) = 6x - 8$$

$$5x^2 - 15x + 10 = 6x - 8$$

$$5x^2 - 21x + 18 = 0$$

USE QF  $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$x = \frac{21 \pm \sqrt{441 - 4(5)(18)}}{2(5)}$$

$$x = \frac{21 \pm \sqrt{81}}{10}$$

$$x = \frac{21+9}{10}$$

**$x = 3$**

$$x = \frac{21-9}{10}$$

**$x = 1.2$**

C:  $|x = 3|$

$$\frac{2}{3} = \frac{6}{9} - \frac{8}{15}$$

$.6\bar{6} = .6\bar{6} \checkmark$

C:  $|x = 1.2|$

$$\frac{-16}{1.2} = \frac{6}{5} - \frac{8}{6}$$

$-13 = -13 \checkmark$